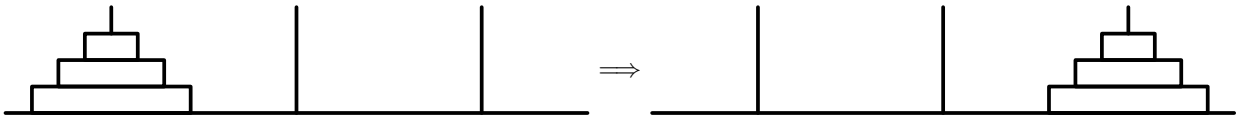
CSc 227 — Program Design and Development
(McCann)

Towers of Hanoi

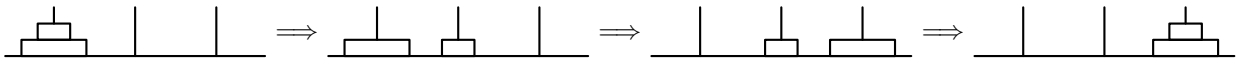
Background: According to legend, monks in a remote monastery are working to move a pile of 64 stone disks from one giant post in the ground to another. When they complete their task, the world will end.

The disks start in a pile with each disk a bit larger than the one above it. The monks can move only one disk at a time (stone disks are heavy!), disks must be stored on posts (to keep the monastery tidy), a larger disk may never be placed on a smaller disk (lest the smaller disk be crushed), and there are only three posts (disk posts and real estate are expensive, and monks aren't made of money, you know).

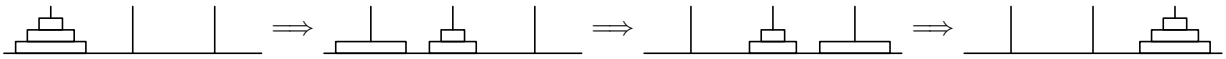
Examples: If we have just three disks, here are how the starting (on the left) and ending (right) configurations would appear:



When the quantities of disks are small, the problem is easily solved. For example, if there are just two disks, only three moves are needed:



When we reconsider the three-disk problem in light of the two-disk solution, a pattern begins to emerge (note that we're skipping a few moves here, both to save space and to highlight the pattern):



Notice – the problem of moving a stack of three disks breaks down into the problem of moving a stack of two disks twice.

Foundation of a Recursive Solution: We start as we always do:

Question: What's easier than . . . moving a stack of n disks?
Answer: Moving a stack of $n - 1$ disks . . . twice!

From there, we can define our base and general cases:

Base Case: Moving a pile of zero disks
General Case: To move n disks from a source post to a destination post:

(a) Move $n - 1$ disks from the source post to the helper post
(b) Move one disk from the source post to the destination post
(c) Move $n - 1$ disks from the helper post to the destimation post

Don't keep us in suspense! When will the world end? It turns out that the monks have to perform $2^{64} - 1 = 18,446,744,073,709,551,615$ disk moves. If the monks are *really* organized and can move disks at a rate of one per second, the job will be complete in a mere $584,942,417,355$ (about 585 billion) years. By comparison, our solar system is estimated to be just 4.5 billion years old. Relax!