What Is Logic?

**Definition: Philosophical Logic**

**Definition: Mathematical Logic**
Propositional Logic

Propositional Logic is part of Mathematical Logic. Versions include:

- **First Order Logic** (FOL, a.k.a. *First Order Predicate Calculus* (FOPC)) includes simple term variables and quantifications.
- **Second Order Logic** allows its variables to represent more complex structures (in particular, predicates).
- **Modal Logic** adds support for modalities; that is, concepts such as possibility and necessity.

Well-Formed Formulae

**Definition: Well-Formed Formula (wff)**

Example(s):
Why Are We Studying Logic?

A few of the many reasons:

- Logic is the foundation for computer operation.
- Logical conditions are common in programs:
  - Selection:
    ```java
    if (score <= max) { ... }
    ```
  - Iteration:
    ```java
    while (i<limit && list[i]!=sentinel) ...
    ```
- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).
  - Examples: Trees, Graphs, Recursive Algorithms, ...
- Programs can be proven correct.
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

Simple Propositions (1 / 2)

**Definition:** Proposition

```
... ...
```

**Definition:** Simple Proposition

```
... ...
```
Proposition Labels

To save writing, it is traditional to label propositions with lower-case letters called *proposition labels* or *statement letters*.

Example(s):
Compound Propositions

Definition: Compound Proposition

And with what do we combine them?

Conjunctions (1 / 2)

Remember ABC’s “Schoolhouse Rock” education series?

“Conjunction Junction” (1973)
(Music/Lyrics by Bob Dorough; Performed by Jack Sheldon)
Conjunctions (2 / 2)

Conjunctions are:

- compound propositions formed in English with the word “and”,
- formed in logic with the caret symbol (\( \wedge \)), and
- true only when both participating propositions are true.

Example(s):

Disjunctions (1 / 3)

Consider this compound proposition:

Under which circumstances is that claim true? Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If all three are acceptable, the disjunction is

_________________ ( ).
Consider the same example and possibilities:

3 is the number of sides of a triangle or the number of times this class meets per week.

Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If the third possibility is not acceptable, the disjunction is ______________ ( ).
Negation

Negating a proposition simply flips its value.

Symbols representing negation include:

\[ \neg x \quad \bar{x} \quad \sim x \quad x' \]

Example(s):

Truth Tables (1 / 2)

Truth tables aid in the evaluation of compound propositions.

Structure of a Truth Table:

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>q</th>
<th>p \land q</th>
<th>(p \land q) \lor p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables (2 / 2)

Truth Tables of $\land$, $\lor$, $\oplus$, and $\neg$:

NOT ($\neg$)

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
</table>

AND ($\land$)

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
</tr>
</thead>
</table>

OR ($\lor$)

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
</tr>
</thead>
</table>

XOR ($\oplus$)

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \oplus q$</th>
</tr>
</thead>
</table>

Precedence of Logical Operators

Total agreement is hard to come by:

Rosen 7/e | Gersting 5/e | Hein 2/e | Epp 1/e
---|---|---|---
Precedence | p. 11 | p. 6 | p. 351 | p. 24

Highest

$\neg$, $\sim$

$\land$, $\land$, $\lor$

$\lor$

$\rightarrow$, $\leftrightarrow$

Lowest

$\leftrightarrow$

(Note: We’ll cover $\rightarrow$ and $\leftrightarrow$ soon.)

In this class:
Operator Associativity

Consider evaluating: \( a = b = -2 \times 3 \times 7; \) in Java

Example(s):

---

Equivalence of Propositions

**Definition:** Logically Equivalent

Example(s):
Review: Is There isn’t a cloud in the sky a proposition?

Question: Is the following sentence a proposition?

Step 1: Identify the simple propositions.

Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 2: Assign easy-to-remember statement labels.
Step 3: Identify the logical operators.

Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 4: Construct the matching logical expression.

So . . . what’s the point? Three examples:

- Expressing Program Conditions
- Natural Language Understanding
- Proof Setup
Three Categories of Propositions (1 / 2)

**Definition: Tautology**

**Definition: Contradiction**

**Definition: Contingency**

Example(s):

\[ d \oplus (\neg k \land m) \]

Three Categories of Propositions (2 / 2)

Example(s):
## Digression: Logical Bit Operations in Java

<table>
<thead>
<tr>
<th>Operator</th>
<th>Name</th>
<th>Example (Dec.)</th>
<th>Example (Bin.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∼</td>
<td>Complement</td>
<td>∼ 12 = −13</td>
<td>∼ 00001100 = 11110011</td>
</tr>
<tr>
<td>&amp;</td>
<td>AND</td>
<td>12 &amp; 10 = 8</td>
<td>&amp; 1010 1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>10 = 14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OR</td>
</tr>
<tr>
<td>∧</td>
<td>XOR</td>
<td>12 ∧ 10 = 6</td>
<td>∧ 1010 0110</td>
</tr>
<tr>
<td>&gt;&gt;</td>
<td>Shift Right</td>
<td>33 &gt;&gt; 1 = 16</td>
<td>&gt;&gt; 1 = 00010000</td>
</tr>
<tr>
<td>&lt;&lt;</td>
<td>Shift Left</td>
<td>33 &lt;&lt; 2 = 132</td>
<td>&lt;&lt; 2 = 10000100</td>
</tr>
</tbody>
</table>

### Example: Linux File Permissions

```bash
-rw-rw-r-- 1 mccann mccann 3561 Oct 28 1929 stocktosell
```
Conditional Propositions (1 / 3)

Example:

Definition: Conditional Proposition

Conditional Propositions (2 / 3)

In “if \( p \), then \( q \)”, \( p \) and \( q \) are known by various names:

Common forms of “if \( p \), then \( q \)” (Rosen 7/e, p. 6):

- if \( p \), then \( q \)
- if \( p \), \( q \)
- \( p \) implies \( q \)
- \( p \) only if \( q \)
- \( p \) is sufficient for \( q \)
- a necessary condition for \( p \) is \( q \)
- \( q \) unless \( \neg p \)
- \( q \) if \( p \)
- \( q \) when \( p \)
- \( q \) whenever \( p \)
- \( q \) follows from \( p \)
- \( q \) is necessary for \( p \)
- a sufficient condition for \( q \) is \( p \)
- and I’m sure you could find more!
Truth of Conditional Propositions (1 / 2)

When should this be considered ‘true’?

If you make it through voir dire, you will serve on the jury.

The possibilities:

1. Antecedent true, Consequent true; statement is: ____.

2. Antecedent true, Consequent false; statement is: ____.

3. Antecedent false, Consequent true; statement is: ____.

4. Antecedent false, Consequent false; statement is: ____.
Truth of Conditional Propositions (2 / 2)

Not satisfied? Maybe this will help:

```java
if (y < x) {
    temp = x;  x = y;  y = temp;
}
```

Inverse, Converse, and Contrapositive

**Definition: Inverse**

**Definition: Converse**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
## Contraposition

**Definition:** Contrapositive

<table>
<thead>
<tr>
<th>$r$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

**Examples:** English Translation (1/2)
Example: English → Logic
Another Example: English → Logic

Political Example: “Push” Polling

“Would you be more likely or less likely to vote for John McCain for president if you knew he had fathered an illegitimate black child?”

— Used in 2000 U.S. Presidential Campaign
What is the meaning of:

A triangle is equilateral if and only if all three angles are equal.

Definition: Biconditional Proposition

<table>
<thead>
<tr>
<th>T</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Biconditionals and Logical Equivalence

**Definition:** Logically Equivalent (2)

Example(s):

De Morgan’s Laws

Example(s):
Example: De Morgan’s Laws and Programming

Checking to see if a score is not a ‘B’:

Common Logical Equivalences (1 / 3)

Table I: Some Equivalences using AND (\(\land\)) and OR (\(\lor\)):

| (a) \(p \land p \equiv p\), \(p \lor p \equiv p\) | Idempotent Laws |
| (b) \(p \lor T \equiv T\), \(p \land F \equiv F\) | Domination Laws |
| (c) \(p \land T \equiv p\), \(p \lor F \equiv p\) | Identity Laws |
| (d) \(p \land q \equiv q \land p\), \(p \lor q \equiv q \lor p\) | Commutative Laws |
| (e) \((p \land q) \lor r \equiv (p \land q) \land (p \land r)\) | Associative Laws |
| \((p \lor q) \land r \equiv (p \lor q) \lor (p \lor r)\) | Distributive Laws |
| (f) \(p \land (q \lor r) \equiv (p \land q) \lor (p \land r)\) | Absorption Laws |
| \(p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)\) | |
| \(p \land (p \lor q) \equiv p\), \(p \lor (p \land q) \equiv p\) | |

Table II: Some More Equivalences (adding \(\neg\)):

| (a) \(\neg(\neg p) \equiv p\) | Double Negation |
| (b) \(p \lor \neg p \equiv T\), \(p \land \neg p \equiv F\) | Negation Laws |
| (c) \(\neg(p \land q) \equiv \neg p \lor \neg q\) | De Morgan's Laws |
| \(\neg(p \lor q) \equiv \neg p \land \neg q\) | |
Table III: Still More Equivalences (adding →):

(a) \( p \rightarrow q \equiv \neg p \lor q \) Law of Implication
(b) \( p \rightarrow q \equiv \neg q \rightarrow \neg p \) Law of the Contrapositive
(c) \( T \rightarrow p \equiv p \) "Law of the True Antecedent"
(d) \( p \rightarrow F \equiv \neg p \) "Law of the False Consequent"
(e) \( p \rightarrow p \equiv T \) Self-implication (a.k.a. Reflexivity)
(f) \( p \rightarrow q \equiv (p \land \neg q) \rightarrow F \) Reductio Ad Absurdum

Remember:

You do not need to memorize these tables . . .

... But you do need to know how to use them!
Applications of Logical Equivalences (1 / 5)

**Question:** Is \((p \land q) \rightarrow p\) a tautology? (1)

By use of a Truth Table; we’ve seen this before:

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(p \land q)</th>
<th>((p \land q) \rightarrow p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Because the expression evaluates to true for all possible arrangements of truth values, the expression is a tautology.

Applications of Logical Equivalences (2 / 5)

**Question:** Is \((p \land q) \rightarrow p\) a tautology? (2)
Question: Is \((p \land q) \rightarrow p\) a tautology? (3)

Example(s):

```plaintext
if ((games <= 10 || ties > 2) && games >= 11) ...
```
Question: Are \((p \land q) \lor (p \land r)\) and \(p \land (\overline{q} \land \overline{r})\) logically equivalent?