HW 2

due at the beginning of lecture: Tue 27 Feb 2018

1 Induction
Use induction to prove that $2^n > n^2$ for $n > 4$ ($n$ is an integer).

2 Structural Induction
Use structural induction to prove that the number of nodes, in a complete (non-empty) binary tree of height $h$, is $2^{h+1} - 1$.

(Remember, a “complete” tree is a tree where every layer is full.)

NOTE: Simply summing up the size of each layer is a direct proof, not inductive. Instead, you must break a large tree down into one or more smaller trees!

3 Some Properties of Asymptotic Notation
Prove or disprove the following conjectures about asymptotic notation (use the formal definitions of asymptotic notation when appropriate):

(a)
$f(n) + g(n) = \Theta(min(f(n), g(n)))$

(b)
If $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$, then $f_1(n) + f_2(n) = O(g_1(n) + g_2(n))$.

(c)
If $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$, then $f_1(n) \cdot f_2(n) = O(g_1(n) \cdot g_2(n))$. 
4 Deleting from a BST

In the below problems, you will delete nodes from this BST. If you hit Case 3, you may choose either the successor or the predecessor - but clearly describe which one you chose.

After each deletion, you must:

- Re-draw the tree
- Print out an in-order traversal of the tree
- Print out a pre-order traversal of the tree

(a) Delete B from the above tree, and draw how it looks afterwards.

(b) Now, reset the tree to the original state (that is, ignore part a) and delete Q.

(c) Now, reset the tree again, and delete K.