More with functions

Functions as values

Functions as arguments

A flexible sort

Curried functions

Functions as values

A fundamental characteristic of a functional language is that functions are values that can be used as flexibly as values of other types.

In essence, the fun declaration creates a function value and binds it to a name. Additional names can be bound to a function value with a val declaration.

```
- fun double(n) = 2*n;
val double = fn : int -> int

- val twice = double;
val twice = fn : int -> int

- twice;
val it = fn : int -> int

- twice 3;
val it = 6 : int
```

Note that unlike values of other types, no representation of a function is shown. Instead, "fn" is displayed. (Think flexibly: What could be shown instead of only fn?)

Functions as values, continued

Just as values of other types can appear in lists, so can functions:

```
- val convs = [floor, ceil, trunc];
val convs = [fn,fn,fn] : (real -> int) list

- hd convs;
val it = fn : real -> int

- it 4.3;
val it = 4 : int

- (hd (tl convs)) 4.3;
val it = 5 : int
```

What is the type of the list [hd]?

What is the type of [size, length]?

Functions as values, continued

It should be no surprise that functions can be elements of a tuple:

```
- (hd, 1, size, "x", length);
val it = (fn,1,fn,"x",fn)
  : ('a list -> 'a) * int * (string -> int) * string * ('b list -> int)

- [it];
val it = [(fn,1,fn,"x",fn)]
  : (('a list -> 'a) * int * (string -> int) * string * ('b list -> int)) list
```

Using the "op" syntax we can work with operators as functions:

```
- swap(pair(op~, op^));
val it = (fn,fn) : (string * string -> string) * (int -> int)

- #2(it) 10;          (* #n(tuple) produces the nth value of the tuple *)
val it = ~10 : int
```

What are some other languages that allow functions to be treated as values, at least to some extent?

Functions as arguments

A function may be passed as an argument to a function.

This function simply *applies* a given function to a value:

```
- fun apply(F,v) = F(v);
val apply = fn : ('a -> 'b) * 'a -> 'b
```

Usage:

```
- apply(size, "abcd");
val it = 4 : int

- apply(swap, (3,4));
val it = (4,3) : int * int

- apply(length, apply(m_to_n, (5,7)));
val it = 3 : int
```

A function that uses other functions as values is said to be a *higher-order function*.

Could apply be written in Java? In C?

Functions as arguments, continued

Consider the following function:

```
fun f(f', x, 1) = f'(x)
 |  f(f', x, n) = f(f', f'(x), n - 1)
```

What does it do?

What is its type?

Give an example of a valid use of the function.

Functions as arguments, continued

Here is a function that applies a function to every element of a list and produces a list of the results:

```
fun applyToAll(_, [ ]) = [ ]
  |  applyToAll(f, x::xs) = f(x)::applyToAll(f, xs);
```

Usage:

```
- applyToAll(double, [10, 20, 30]);
val it = [20,40,60] : int list

- applyToAll(real, iota(5));
val it = [1.0,2.0,3.0,4.0,5.0] : real list

- applyToAll(length, [it, it@it]);
val it = [5,10] : int list

- applyToAll(implode,
      applyToAll(rev,
          applyToAll(explode, ["one", "two", "three])));
val it = ["eno","owt","eerht"] : string list
```

Functions as arguments, continued

Here's a roundabout way to calculate the length of a list:

```
- val L = explode "testing";
val L = [#"t",#"e",#"s",#"t",#"i",#"n",#"g"] : char list

- fun one _ = 1;
val one = fn : 'a -> int

- sumInts(applyToAll(one, L));
val it = 7 : int
```

Problem: Create a list like ["x", "xx", "xxx", ... "xxxxxxxxxx"]. (One to ten "x"s.)

We'll see later that applyToAll is really the map function from the library, albeit in a slightly different form.

Functions that produce functions

Consider a function that applies two specified functions to the same value and returns the function producing the larger integer result:

```
- fun larger(f1, f2, x) = if f1(x) > f2(x) then f1 else f2;
val larger = fn : ('a -> int) * ('a -> int) * 'a -> 'a -> int

- val g = larger(double, square, 5);
val g = fn : int -> int

- g(5);
val it = 25 : int

- val h = larger(sum, len, [0, 0, 0]);
val h = fn : int list -> int

- h([10,20,30]);
val it = 3 : int

- (larger(double, square, ~4)) (10);
val it = 100 : int
```

A flexible sort

Recall order(ed)_pair, insert, and sort from tally (slide 112). They work together to sort a (char * int) list.

```
fun ordered_pair((_, v1), (_, v2)) =  v1 > v2

fun insert(v, [ ]) = [v]
  |  insert(v, x::xs) = if ordered_pair(v,x) then v::x::xs else x::insert(v, xs)

fun sort([ ]) = [ ]
  |  sort(x::xs) = insert(x, sort(xs))
```

Consider eliminating ordered_pair and instead supplying a function to test whether the values in a 2-tuple are the desired order.

A flexible sort, continued

Here are versions of insert and sort that use a function to test the order of elements in a 2-tuple:

```
fun insert(v, [ ], isInOrder) = [v]
 |  insert(v, x::xs, isInOrder) =
        if isInOrder(v,x) then v::x::xs
                          else x::insert(v, xs, isInOrder)

fun sort([ ], isInOrder) = [ ]
 |  sort(x::xs, isInOrder) = insert(x, sort(xs, isInOrder), isInOrder)
```

Types:

```
- insert;
val it = fn : 'a * 'a list * ('a * 'a -> bool) -> 'a list

- sort;
val it = fn : 'a list * ('a * 'a -> bool) -> 'a list
```

What C library function does this version of sort resemble?

A flexible sort, continued

Sorting integers:

```
- fun intLessThan(a,b) = a < b;
val intLessThan = fn : int * int -> bool

- sort([4,10,7,3], intLessThan);
val it = [3,4,7,10] : int list
```

We might sort (int * int) tuples based on the sum of the two values:

```
fun sumLessThan( (a1, a2), (b1, b2) ) = a1 + a2 < b1 + b2;

- sort([(1,1), (10,20), (2,~2), (3,5)], sumLessThan);
val it = [(2,~2),(1,1),(3,5),(10,20)] : (int * int) list
```

Problem: Sort an int list list based on the largest value in each of the int lists. Sorting

```
[[3,1,2],[50],[10,20],[4,3,2,1]]
```

would yield

```
[[3,1,2],[4,3,2,1],[10,20],[50]]
```

Curried functions

It is possible to define a function in *curried* form:

```
- fun add x y = x + y;        (Two arguments, x and y, not (x,y), a 2-tuple )
val add = fn : int -> int -> int
```

The function add can be called like this:

```
- add 3 5;
val it = 8 : int
```

Note the type of add: int -> (int -> int) (Remember that -> is right-associative.)

What add 3 5 means is this:

```
- (add 3) 5;
val it = 8 : int
```

add is a function that takes an int and produces a function that takes an int and produces an int. add 3 produces a function that is then called with the argument 5.

Is add(3,5) valid?

Curried functions, continued

For reference: fun add x y = x + y. The type is int -> (int -> int).

More interesting than add 3 5 is this:

```
- add 3;
val it = fn : int -> int

- val plusThree = add 3;
val plusThree = fn : int -> int
```

The name plusThree is bound to a function that is a *partial instantiation* of add. (a.k.a. *partial application*)

```
- plusThree 5;
val it = 8 : int

- plusThree 20;
val it = 23 : int

- plusThree (plusThree 20);
val it = 26 : int
```

Curried functions, continued

For reference:

```
fun add x y = x + y
```

As a conceptual model, think of this expression:

```
val plusThree = add 3
```

as producing a result similar to this:

```
fun plusThree(y) = 3 + y
```

The idea of a partially applicable function was first described by Moses Schönfinkel. It was further developed by Haskell B. Curry. Both worked wtih David Hilbert in the 1920s.

What prior use have you made of partially applied functions?

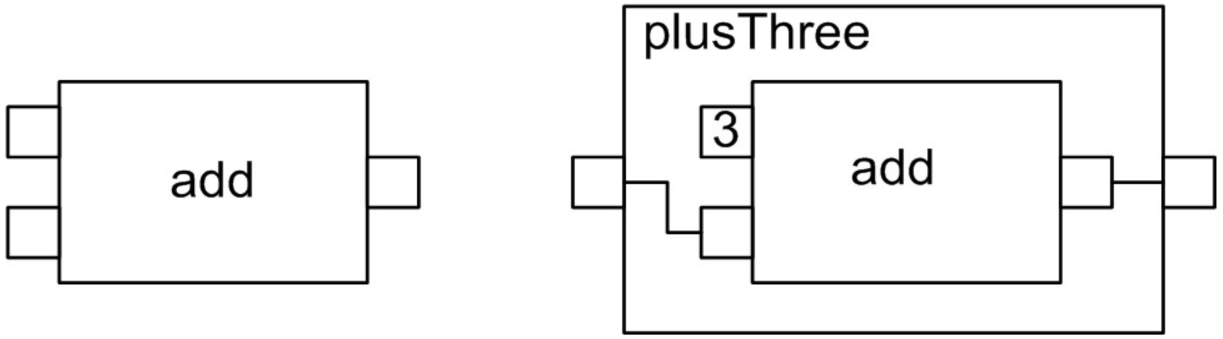
Curried functions, continued

For reference:

```
- fun add x y = x + y;
val add = fn : int -> int -> int

- val plusThree = add 3;
val plusThree = fn : int -> int
```

Analogy: A partially instantiated function is like a machine with a hardwired input value.



This model assumes that data flows from left to right.

Curried functions, continued

Consider another function:

```
- fun f a b c = a*2 + b*3 + c*4;
val f = fn : int -> int -> int -> int

- val f_a = f 1;
val f_a = fn : int -> int -> int

        fun f_a(b,c) = 1*2 + b*3 + c*4

- val f_a_b = f_a 2;
val f_a_b = fn : int -> int

        fun f_a_b(c) = 1*2 + 2*3 + c*4
                             8       + c*4;

- f_a_b 3;
val it = 20 : int

- f_a 5 10;
val it = 57 : int
```

Curried functions, continued

At hand:

```
- fun f a b c = a*2 + b*3 + c*4;
val f = fn : int -> int -> int -> int
```

Note that the expression

```
f 10 20 30;
```

is evaluated like this:

```
((f 10) 20) 30
```

In C, it's said that "declaration mimics use"—a declaration like `int f()` means that if you see the expression `f()`, it is an `int`. We see something similar with ML function declarations:

```
fun add(x,y) = x + y        Call: add (3, 4)

fun add x y = x + y         Call: add 3 4
```

Curried functions, continued

Problem: Define a curried function named mul to multiply two integers. Using a partial application, use a val binding to create a function equivalent to fun double(n) = 2 * n.

Here is a curried implementation of m_to_n (slide 76):

```
- fun m_to_n m n = if m > n then [ ] else m :: (m_to_n  (m+1)  n);
val m_to_n = fn : int -> int -> int list
```

Usage:

```
- m_to_n 1 7;
val it = [1,2,3,4,5,6,7] : int list

- val L = m_to_n ~5 5;
val L = [~5,~4,~3,~2,~1,0,1,2,3,4,5] : int list
```

Problem: Create the function iota, described on slide 78. (iota(3) produces [1,2,3].)

Curried functions, continued

What's happening here?

```
- fun add x y = x + y;
val add = fn : int -> int -> int

- add double(3) double(4);
Error: operator and operand don't agree [tycon mismatch]
  operator domain: int * int
  operand:         int -> int
  in expression:
    add double
```

Curried functions, continued

Problem—fill in the blanks:

```
fun add x y z = x + y + z;


val x = add 1;


val xy = x 2;


xy 3;


xy 10;


x 0 0;
```

Curried functions, continued

Here is sort from slide 125:

```
fun sort([ ], isInOrder) = [ ]
 |  sort(x::xs, isInOrder) = insert(x, sort(xs, isInOrder), isInOrder)
```

A curried version of sort:

```
fun sort _ [ ] = [ ]
   |  sort isInOrder (x::xs) = insert(x, (sort isInOrder xs), isInOrder)
```

Usage:

```
- val intSort = sort  intLessThan;
val int_sort = fn : int list -> int list

- int_sort [4,2,1,8];
val it = [1,2,4,8] : int list
```

Why does the curried form have the function as the first argument?

Curried functions, continued

Functions in the ML standard library (the "Basis") are often curried.

String.isSubstring returns true iff its first argument is a substring of the second argument:

```
- String.isSubstring;
val it = fn : string -> string -> bool

- String.isSubstring "tan" "standard";
val it = true : bool
```

We can create a partial application that returns true iff a string contains "tan":

```
- val hasTan = String.isSubstring "tan";
val hasTan = fn : string -> bool

- hasTan "standard";
val it = true : bool

- hasTan "library";
val it = false : bool
```

See the Resources page on the website for a link to documentation for the Basis.

Curried functions, continued

In fact, the curried form is syntactic sugar. An alternative to `fun add x y = x + y` is this:

```
- fun add x =
    let
       fun add' y = x + y
    in
       add'
    end
val add = fn : int -> int -> int   (Remember associativity: int -> (int -> int) )
```

A call such as `add 3` produces an instance of `add'` where `x` is bound to `3`. That instance is returned as the value of the `let` expression.

```
- add 3;
val it = fn : int -> int

- it 4;
val it = 7 : int

- add' 3 4;
val it = 7 : int
```

List processing idioms with functions

Mapping

Anonymous functions

Predicate based functions

Reduction/folding

travel, revisited

Mapping

The applyToAll function seen earlier applies a function to each element of a list and produces a list of the results. There is a built-in function called map that does the same thing.

```
- map;
val it = fn : ('a -> 'b) -> 'a list -> 'b list

- map size ["just", "testing"];
val it = [4,7] : int list

- map sumInts [[1,2,3],[5,10,20],[]];
val it = [6,35,0] : int list
```

Mapping is one of the idioms of functional programming.

There is no reason to write a function that performs an operation on each value in a list. Instead create a function to perform the operation on a single value and then map that function onto lists of interest.

Mapping, continued

Contrast the types of applyToAll and map. Which is more useful?

```
- applyToAll;
val it = fn : ('a -> 'b) * 'a list -> 'b list

- map;
val it = fn : ('a -> 'b) -> 'a list -> 'b list
```

Consider a partial application of map:

```
- val sizes = map size;
val sizes = fn : string list -> int list

- sizes ["ML", "Ruby", "Prolog"];
val it = [2,4,6] : int list

- sizes ["ML", "Icon", "C++", "Prolog"];
val it = [2,4,3,6] : int list
```

Mapping, continued

Here's one way to generate a string with all the ASCII characters:

```
- implode (map chr (m_to_n 0 127));
val it =
  "\^@\^A\^B\^C\^D\^E\^F\a\b\t\n\v\f\r\^N\^O\^P\^Q\^R\^S\^T\^U\^V\^W\^X\^Y\^Z\^[
\^\\^]\^^\^_ !\"#$%&'()*+,-./0123456789:;<=>?@ABCDE#"
  : string
```

(Note that the full value is not shown—the trailing # indicates the value was truncated for display.)

Problem: Write a function equalsIgnoreCase of type string * string -> bool. The function Char.toLower (char -> char) converts upper-case letters to lower case and leaves other characters unchanged.

Mapping with curried functions

It is common to map with a partial application:

```
- val addTen = add 10;
val addTen = fn : int -> int

- map addTen (m_to_n 1 10);
val it = [11,12,13,14,15,16,17,18,19,20] : int list

- map (add 100) (m_to_n 1 10);
val it = [101,102,103,104,105,106,107,108,109,110] : int list
```

The partial application "plugs in" one of the addends. The resulting function is then called with each value in the list in turn serving as the other addend.

Remember that map is curried, too:

```
- val addTenToAll = map (add 10);
val addTenToAll = fn : int list -> int list

- addTenToAll [3,1,4,5];
val it = [13,11,14,15] : int list
```

Mapping with anonymous functions

Here's another way to define a function:

```
- val double = fn(n) => n * 2;
val double = fn : int -> int
```

The expression being evaluated, fn(n) => n * 2, is a simple example of a *match expression*. It provides a way to create a function "on the spot".

If we want to triple the numbers in a list, instead of writing a triple function we might do this:

```
- map (fn(n) => n * 3) [3, 1, 5, 9];
val it = [9,3,15,27] : int list
```

The function created by fn(n) => n * 3 never has a name. It is an anonymous function. It is created, used, and discarded.

The term *match expression* is ML-specific. A more general term for an expression that defines a nameless function is a *lambda expression*.

Mapping with anonymous functions, continued

Explain the following:

```
- map (fn(s) => (size(s), s)) ["just", "try", "it"];
val it = [(4,"just"),(3,"try"),(2,"it")] : (int * string) list
```

Problem: Recall this mapping of a partial application:

```
- map (add 100) (m_to_n 1 10);
val it = [101,102,103,104,105,106,107,108,109,110] : int list
```

Do the same thing but use an anonymous function instead.

Predicate-based functions

The built-in function List.filter applies function F to each element of a list and produces a list of those elements for which F produces true. Here's one way to write filter:

```
- fun filter F [ ] = [ ]
   |   filter F (x::xs) = if (F x) then x::(filter F xs)
                                          else (filter F xs);
val filter = fn : ('a -> bool) -> 'a list -> 'a list
```

It is said that F is a *predicate*—inclusion of a list element in the result is predicated on whether F returns true for that value.

Problem: Explain the following.

```
- val f = List.filter (fn(n) => n mod 2 = 0);
val f = fn : int list -> int list

- f [5,10,12,21,32];
val it = [10,12,32] : int list

- length (f (m_to_n 1 100));
val it = 50 : int
```

Predicate-based functions, continued

Another predicate-based function is List.partition:

```
- List.partition;
val it = fn : ('a -> bool) -> 'a list -> 'a list * 'a list

- List.partition (fn(s) => size(s) <= 3) ["a", "test", "now"];
val it = (["a","now"],["test"]) : string list * string list
```

String.tokens uses a predicate to break a string into tokens:

```
- Char.isPunct;
val it = fn : char -> bool

- String.tokens Char.isPunct "a,bc:def.xyz";
val it = ["a","bc","def","xyz"] : string list
```

Problem: What characters does Char.isPunct consider to be punctuation?

Real-world application: A very simple grep

The UNIX grep program searches files for lines that contain specified text. Imagine a very simple grep in ML:

```
- grep;
val it = fn : string -> string list -> unit list

- grep "sort" ["all.sml","flexsort.sml"];
all.sml:fun sort1([ ]) = [ ]
all.sml: |  sort1(x::xs) =
all.sml:    insert(x, sort1(xs))
flexsort.sml:fun sort([ ], isInOrder) = [ ]
flexsort.sml: |  sort(x::xs, isInOrder) = insert(x, sort(xs, isInOrder), isInOrder)
val it = [(),()] : unit list
```

We could use SMLofNJ.exportFn to create a file that is executable from the UNIX command line, just like the real grep.

A simple grep, continued

Implementation

```
fun grepAFile text file =
    let
        val inputFile = TextIO.openIn(file);
        val fileText = TextIO.input(inputFile);
        val lines = String.tokens (fn(c) => c = #"\n") fileText
        val linesWithText = List.filter (String.isSubstring text) lines
        val _ = TextIO.closeIn(inputFile);
    in
        print(concat(map (fn(s) =>  file ^ ":" ^ s ^ "\n") linesWithText))
    end;

fun grep text files = map (grepAFile text) files;
```

Notes:

- TextIO.openIn opens a file for reading.
- TextIO.input reads an entire file and returns it as a string.
- Study the use of anonymous functions, mapping, and partial application.
- No loops, no variables, no recursion at this level.

How much code would this be in Java? Do you feel confident the code above is correct?

Reduction of lists

Another idiom is *reduction* of a list by repeatedly applying a binary operator to produce a single value. Here is a simple reduction function:

```
- fun reduce F [ ] = raise Empty
    |   reduce F [x] = x
    |   reduce F (x::xs) = F(x, reduce F xs)
val reduce = fn : ('a * 'a -> 'a) -> 'a list -> 'a
```

Usage:

```
- reduce op+ [3,4,5,6];
val it = 18 : int
```

What happens:

```
op+(3, reduce op+ [4,5,6])
    op+(4, reduce op+ [5,6])
        op+(5, reduce op+ [6])
```

Or,

```
op+(3, op+(4, op+(5,6)))
```

Reduction, continued

More examples:

```
- reduce  op^  ["just", "a", "test"];
val it = "justatest" : string

- reduce op* (iota 5);
val it = 120 : int
```

Problem: How could a list like [[1,2],[3,4,5],[6]] be turned into [1,2,3,4,5,6]?

Reduction, continued

Because reduce is curried, we can create a partial application:

```
- val concat = reduce op^;   (* mimics built-in concat *)
val concat = fn : string list -> string

- concat ["xyz", "abc"];
val it = "xyzabc" : string

- val sum = reduce  op+ ;
val sum = fn : int list -> int

- sum(iota 10);
val it = 55 : int

- val max = reduce (fn(x,y) => if x > y then x else y);
val max = fn : int list -> int

- max [5,3,9,1,2];
val it = 9 : int
```

Reduction, continued

Another name for reduction is "folding"; There are two built-in reduction/folding functions: foldl and foldr. Contrast their types with the implementation of reduce shown above:

```
- foldl;
val it = fn : ('a * 'b -> 'b) -> 'b -> 'a list -> 'b

- foldr;
val it = fn : ('a * 'b -> 'b) -> 'b -> 'a list -> 'b

- reduce;
val it = fn : ('a * 'a -> 'a) -> 'a list -> 'a
```

Here's an example of foldr:

```
- foldr op+ 0 [5,3,9,2];
val it = 19 : int
```

What are the differences between reduce and foldr?

Speculate: What's the difference between foldl and foldr?

Reduction, continued

At hand:

```
- foldr;                                         (* foldl has same type *)
val it = fn : ('a * 'b -> 'b) -> 'b -> 'a list -> 'b
- reduce;
val it = fn : ('a * 'a -> 'a) -> 'a list -> 'a
```

Our reduce has two weaknesses: (1) It can't operate on an empty list. (2) The operation must produce the same type as the list elements.

Consider this identity operation:

```
- foldr op:: [ ] [1,2,3,4];
val it = [1,2,3,4] : int list
```

Here are the op:: (cons) operations that are performed:

```
- op::(1, op::(2, op::(3, op::(4, [ ]))));
val it = [1,2,3,4] : int list
```

Note that the empty list in op::(4, [ ]) comes from the call. (Try it with [10] instead of [ ].)

Reduction, continued

At hand:

```
- foldr op:: [ ] [1,2,3,4];
val it = [1,2,3,4] : int list
```

foldl (note the "L") folds from the left, not the right:

```
- foldl op:: [ ] [1,2,3,4];
val it = [4,3,2,1] : int list
```

Here are the op:: calls that are made:

```
- op::(4, op::(3, op::(2, op::(1, [ ]))));
val it = [4,3,2,1] : int list
```

Reduction, continued

In some cases foldl and foldr produce different results. In some they don't:

```
- foldr op^ "!" ["a","list","of","strings"];
val it = "alistofstrings!" : string

- foldl op^ "!" ["a","list","of","strings"];
val it = "stringsoflista!" : string

- foldr op+ 0 [5,3,9,2];
val it = 19 : int

- foldl op+ 0 [5,3,9,2];
val it = 19 : int

- foldl op@ [ ] [[1,2],[3],[4,5]];
val it = [4,5,3,1,2] : int list

- foldr op@ [ ] [[1,2],[3],[4,5]];
val it = [1,2,3,4,5] : int list
```

What characteristic of an operation leads to different results with foldl and foldr?

travel, revisited

Here's a version of travel (slide 107) that uses mapping and reduction (folding) instead of explicit recursion:

```
fun dirToTuple(#"n") = (0,1)
  | dirToTuple(#"s") = (0,~1)
  | dirToTuple(#"e") = (1,0)
  | dirToTuple(#"w") = (~1,0)

fun addTuples((x1 , y1), (x2, y2)) = (x1 + x2, y1 + y2);

fun travel(s) =
   let
      val tuples = map dirToTuple (explode s)
      val displacement = foldr addTuples (0,0) tuples
   in
      if displacement = (0,0) then "Got home"
                              else "Got lost"
   end
```

How confident are we that it is correct? Would it be longer or shorter in Java?

Even more with functions

Composition

Manipulation of operands

Composition of functions

Given two functions F and G, the *composition* of F and G is a function C that for all values of x, C(x) = F(G(x)).

Here is a primitive compose function that applies two functions in turn:

```
- fun compose(F,G,x) = F(G(x));
val compose = fn :  ('a -> 'b) * ('c -> 'a) * 'c -> 'b
```

Usage:

```
- length;
val it = fn : 'a list -> int

- explode;
val it = fn : string -> char list

- compose(length, explode, "testing");
val it = 7 : int
```

Could we create a function composeAll([f1, f2, ... fn], x) that would call f1(f2(...fn(x)))?

The composition operator (○)

There is a composition operator in ML:

```
- op o;     (* lower-case "Oh" *)
val it = fn : ('a -> 'b) * ('c -> 'a) -> 'c -> 'b
```

Two functions can be composed into a new function:

```
- val strlen = length o explode;
val strlen = fn : string -> int

- strlen "abc";
val it = 3 : int
```

Consider the types with respect to the type of `op o`:

```
'a is 'a list
'b is int
'c is string

(('a list -> int) * (string -> 'a list)) -> (string -> int)
```

Composition, continued

When considering the type of a composed function only the types of the leftmost and rightmost functions come into play.

Note that the following three compositions all have the same type. (Yes, the latter two are doing some "busywork"!)

```
- length o explode;
val it = fn : string -> int

- length o explode o implode o explode;
val it = fn : string -> int

- length o rev o explode o implode o rev o explode;
val it = fn : string -> int
```

A COMMON ERROR is to say the type of `length o explode` is something like this:

`string -> 'a list -> int` **<u>(WRONG!!!)</u>**

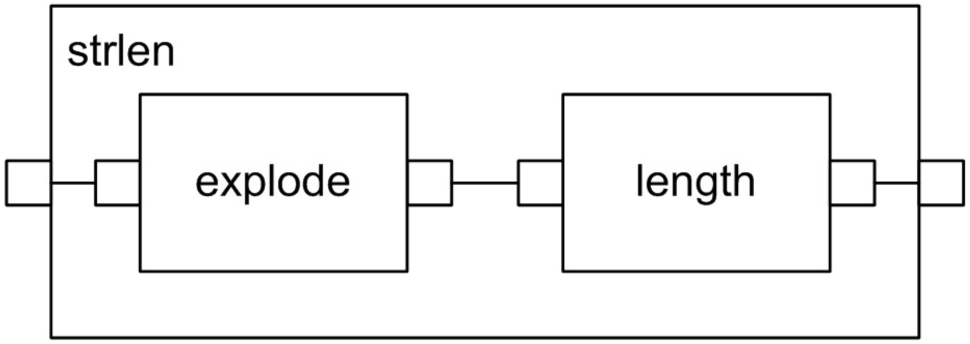
<u>Assuming a composition is valid, the type is based only on the input of the rightmost function and the output of the leftmost function.</u>

Composition, continued

For reference:

```
- val strlen = length o explode;
val strlen = fn : string -> int
```

Analogy: Composition is like bolting machines together.



Because these machine models assume left to right data flow, explode comes first.

Composition, continued

Recall order and swap:

```
fun order(x, y) = if x < y then (x, y) else (y, x)

fun swap(x, y) = (y, x)
```

A descOrder function can be created with composition:

```
- val descOrder = swap o order;
val descOrder = fn : int * int -> int * int

- descOrder(1,4);
val it = (4,1) : int * int
```

Problem: Using composition, create a function to reverse a string.

Problem: Create a function to reverse each string in a list of strings and reverse the order of strings in the list. (Example: f ["one","two","three"] would produce ["eerht","owt","eno"].)

Composition, continued

Problem: Create two functions second and third, which produce the second and third elements of a list, respectively:

```
- second([4,2,7,5]);
val it = 2 : int

- third([4,2,7,5]);
val it = 7 : int
```

Problem: The function xrepl(x, n) produces a list with n copies of x:

```
- xrepl(1, 5);
val it = [1,1,1,1,1] : int list
```

Create a function repl(s, n), of type string * int -> string, that produces a string consisting of n copies of s. For example, repl("abc", 2) = "abcabc".

Problem: Compute the sum of the odd numbers between 1 and 100, inclusive. Use only composition and applications of op+, iota, isEven, foldr, filter, and not (bool -> bool).

Another way to understand composition

Composition can be explored by using functions that simply echo their call.

Example:

```
- fun f(s) = "f(" ^ s ^ ")";
val f = fn : string -> string

- f("x");
val it = "f(x)" : string
```

Two more:

```
fun g(s) = "g(" ^ s ^ ")";

fun h(s) = "h(" ^ s ^ ")";
```

Compositions:

```
- val fg = f o g;
val fg = fn : string -> string

- fg("x");
val it = "f(g(x))" : string

- val ghf = g o h o f;
val ghf = fn : string -> string

- ghf("x");
val it = "g(h(f(x)))" : string

- val q = fg o ghf;
val q = fn : string -> string

- q("x");
val it = "f(g(g(h(f(x)))))" : string
```

"Computed" composition

Because composition is just an operator and functions are just values, we can write a function that computes a composition. compN f n composes f with itself n times:

```
- fun compN f 1 = f
   |   compN f n = f o compN f (n-1);
val compN = fn : ('a -> 'a) -> int -> 'a -> 'a
```

Usage:

```
- val f = compN double 3;
val f = fn : int -> int

- f 10;
val it = 80 : int

- compN double 10 1;
val it = 1024 : int

- map (compN double) (iota 5);
val it = [fn,fn,fn,fn,fn] : (int -> int) list
```

Could we create compN using folding?

Manipulation of operands

Consider this function:

```
- fun c f x y = f (x,y);
val c = fn : ('a * 'b -> 'c) -> 'a -> 'b -> 'c
```

Usage:

```
- c op+ 3 4;
val it = 7 : int

- c op^ "a" "bcd";
val it = "abcd" : string
```

What is it doing?

What would be produced by the following partial applications?

```
c op+

c op^
```

Manipulation of operands, continued

Here's the function again, with a revealing name:

```
- fun curry f x y = f (x,y);
val curry = fn : ('a * 'b -> 'c) -> 'a -> 'b -> 'c
```

Consider:

```
- op+;
val it = fn : int * int -> int

- val add = curry op+;
val add = fn : int -> int -> int

- val addFive = add 5;
val addFive = fn : int -> int

- map addFive (iota 10);
val it = [6,7,8,9,10,11,12,13,14,15] : int list

- map (curry op+ 5) (iota 10);
val it = [6,7,8,9,10,11,12,13,14,15] : int list
```

Manipulation of operands, continued

For reference:

```
- fun curry f x y = f (x,y);
val curry = fn : ('a * 'b -> 'c) -> 'a -> 'b -> 'c
```

For a moment, think of a partial application as textual substitution:

`val add = curry op+`	is like	`fun add x y = op+(x, y)`
`val addFive = curry op+ 5`	is like	`fun addFive y = op+(5, y)`

Bottom line:

If we have a function that takes a 2-tuple, we can easily produce a curried version of the function.

Manipulation of operands, continued

Recall repl from slide 165:

```
- repl("abc", 4);
val it = "abcabcabcabc" : string
```

Let's create some partial applications of a curried version of it:

```
- val stars = curry repl "*";
val stars = fn : int -> string

- val arrows = curry repl " ---> ";
val arrows = fn : int -> string

- stars 10;
val it = "**********" : string

- arrows 5;
val it = " --->  --->  --->  --->  ---> " : string

- map arrows (iota 3);
val it = [" ---> "," --->  ---> "," --->  --->  ---> "] : string list
```

Manipulation of operands, continued

Sometimes we have a function that is curried but we wish it were not curried. For example, a function of type 'a -> 'b -> 'c that would be more useful if it were 'a * 'b -> 'c.

Consider a curried function:

```
- fun f x y = g(x,y*2);
val f = fn : int -> int -> int
```

Imagine that we'd like to map f onto an (int * int) list. We can't! (Why?)

Problem: Write an uncurry function so that this works:

```
- map (uncurry f) [(1,2), (3,4), (5,6)];
```

Important: The key to understanding functions like curry and uncurry is that without partial application they wouldn't be of any use.

Manipulation of operands, continued

The partial instantiation curry repl "x" creates a function that produces some number of "x"s, but suppose we wanted to first supply the replication count and then supply the string to replicate.

Example:

```
- five;      (Imagine that 'five s' will call 'repl(s, 5)'.)
val it = fn : string -> string

- five "*";
val it = "*****" : string

- five "<x>";
val it = "<x><x><x><x><x>" : string
```

Manipulation of operands, continued

Consider this function:

```
- fun swapArgs f x y = f y x;
val swapArgs = fn : ('a -> 'b -> 'c) -> 'b -> 'a -> 'c
```

Usage:

```
- fun cat s1 s2 = s1 ^ s2;
val cat = fn : string -> string -> string

- val f = swapArgs cat;
val f = fn : string -> string -> string

- f  "a"  "b";
val it = "ba" : string

- map (swapArgs (curry op^) "x") ["just", "a", "test"];
val it = ["justx","ax","testx"] : string list
```

Manipulation of operands, continued

```
- val curried_repl = curry repl;
val curried_repl = fn : string -> int -> string

- val swapped_curried_repl = swapArgs curried_repl;
val swapped_curried_repl = fn : int -> string -> string

- val five = swapped_curried_repl 5;
val five = fn : string -> string

- five "*";
val it = "*****" : string

- five "<->";
val it = "<-><-><-><-><->" : string
```

Or,

```
- val five = swapArgs (curry repl) 5;
val five = fn : string -> string

- five "xyz";
val it = "xyzxyzxyzxyzxyz" : string
```

Example: optab

Function optab(F, N, M) prints a table showing the result of F(n,m) for each value of n and m from 1 to N and M, respectively. F is always an int * int -> int function.

Example:

```
- optab;
val it = fn : (int * int -> int) * int * int -> unit

- optab(op*, 5, 7);
          1    2    3    4    5    6    7
     1    1    2    3    4    5    6    7
     2    2    4    6    8   10   12   14
     3    3    6    9   12   15   18   21
     4    4    8   12   16   20   24   28
     5    5   10   15   20   25   30   35
val it = () : unit
```

optab, continued

```
val repl = concat o xrepl;

fun rightJustify width value =
    repl(" ", width-size(value)) ^ value

fun optab(F, nrows, ncols) =
    let
        val rj = rightJustify 4  (* assumes three-digit results at most *)

        fun intsToRow (L) = concat(map (rj o Int.toString) L) ^ "\n"

        val cols = iota ncols

        fun mkrow nth = intsToRow(nth::(map (curry F nth) cols))

        val rows = map mkrow (iota nrows)
    in
        print((rj "") ^ intsToRow(cols) ^ concat(rows))
    end
```

```
- optab(add, 3, 4);
       1   2   3   4
   1   2   3   4   5
   2   3   4   5   6
   3   4   5   6   7
val it = () : unit
```