# Functional Programming with Haskell

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# Programming Paradigms

# Paradigms

Thomas Kuhn's *The Structure of Scientific Revolutions* (1962) describes a *paradigm* as a scientific achievement that is...

- "...sufficiently unprecedented to attract an enduring group of adherents away from competing modes of scientific activity."
- "...sufficiently open-ended to leave all sorts of problems for the redefined group of practitioners to resolve."

Kuhn cites works such as Newton's *Principia*, Lavoisier's *Chemistry*, and Lyell's *Geology* as serving to document paradigms.

# Paradigms, continued

A paradigm provides a conceptual framework for understanding and solving problems.

A paradigm has a world view, a vocabulary, and a set of techniques that can be applied to solve a problem.

(Another theme for us.)

A question to keep in mind:

What are the problems that programming paradigms attempt to solve?

# The procedural programming paradigm

From the early days of programming into the 1980s the dominant paradigm was *procedural programming*:

> Programs are composed of bodies of code (procedures) that manipulate individual data elements or structures.

Much study was focused on how best to decompose a large computation into a set of procedures and a sequence of calls.

Languages like FORTRAN, COBOL, Pascal, and C facilitate procedural programming.

Java programs with a single class are typically examples of procedural programming.

# The object-oriented programming paradigm

In the 1990s, object-oriented programming became the dominant paradigm. Problems are solved by creating systems of objects that interact.

> "*Instead of a bit-grinding processor plundering data structures, we have a universe of well-behaved objects that courteously ask each other to carry out their various desires."*—Dan Ingalls

Study shifted from how to decompose computations into procedures to how to model systems as interacting objects.

Languages like C++ and Java facilitate use of an object-oriented paradigm.

# The influence of paradigms

The programming paradigm(s) we know affect how we approach problems.

If we use the procedural paradigm, we'll first think about breaking down a computation into a series of steps.

If we use the object-oriented paradigm, we'll first think about modeling the problem with a set of objects and then consider their interactions.

# Language support for programming paradigms

If a language makes it easy and efficient to use a particular paradigm, we say that the language supports the paradigm.

What language features are required to support procedural programming?

- The ability to break programs into procedures.

What language features does OO programming require, for OO programming as you know it?

- Ability to define classes that comprise data and methods
- Ability to specify inheritance between classes

# Multiple paradigms

Paradigms in a field of science are often incompatible.

Example: geocentric vs. heliocentric model of the universe

Can a programming language support multiple paradigms?

Yes!  We can do procedural programming with Java.

The programming language Leda fully supports the procedural, imperative, object-oriented, functional, and logic programming paradigms.

Wikipedia's **Programming_paradigm** cites 60+ paradigms!

But, are "programming paradigms" really paradigms by Kuhn's definition or are they just characteristics?

# The imperative programming paradigm

The imperative paradigm has its roots in programming at the machine level, usually via assembly language.

Machine-level programming:

- Instructions change memory locations or registers
- Instructions alter the flow of control

Programming with an imperative language:

- Expressions compute values based on memory contents
- Assignments alter memory contents
- Control structures guide the flow of control

# The imperative programming paradigm

Both the procedural and object-oriented paradigms typically make use of the imperative programming paradigm.

Two fundamental characteristics of languages that support the imperative paradigm:

- "Variables"—data objects whose values typically change as execution proceeds.

- Support for iteration—a “while” control structure, for example.

# Imperative programming, continued

Here's an imperative solution in Java to sum the integers in an array:

```
int sum(int a[])
{
    int sum = 0;
    for (int i = 0; i < a.length; i++)
        sum += a[i];

    return sum;
}
```

The **for** loop causes **i** to vary over the indices of the array, as the variable **sum** accumulates the result.

How can the above solution be improved?

# Imperative programming, continued

With Java's "enhanced **for**", also known as a for-each loop, we can avoid array indexing.

```
int sum(int a[])
{
    int sum = 0;
    for (int val: a)
        sum += val;

    return sum;
}
```

Is this an improvement? If so, why?

Can we write **sum** in a non-imperative way?

# Imperative programming, continued

We can use recursion instead of a loop, but...ouch!

```
int sum(int a[]) { return sum(a, 0); }

int sum(int a[], int i)
{
    if (i == a.length)
        return 0;
    else
        return a[i] + sum(a, i+1);
}
```

Wrt. correctness, which of the three versions would you bet your job on?

# Sidebar: The level of a paradigm

Programming paradigms can apply at different levels:

- Making a choice between procedural and object-oriented programming fundamentally determines the high-level structure of a program.

- The imperative paradigm is focused more on the small aspects of programming—how code looks at the line-by-line level.

Java combines the object-oriented and imperative paradigms.

The procedural and object-oriented paradigms apply to *programming in the large*.

The imperative paradigm applies to *programming in the small*.

# Imperative vs. applicative methods in Java

Java methods can be classified as imperative or *applicative*.

- An imperative method changes an object.

  "Change this."

- An applicative method produces a new object.

  "Make me something new from this."

In some cases we have an opportunity to choose between the two.

# Imperative vs. applicative methods, continued

Consider a Java class representing a 2D point:

```
class Point {
   private int x, y;
}
```

An imperative method to translate by an x and y displacement:

```
public void translate(int dx, int dy) {
      x += dx; y += dy;
}
```

An applicative translate:

```
public Point translate(int dx, int dy) {
      return new Point(x + dx, y + dy);
}
```

What are the pros and cons?

# Imperative vs. applicative methods, continued

Imagine a **Line** class, whose instances are constructed with two **Point**s. Example: **Line A = new Line(p1, p2);**

Two blocks of code follow. Left half of class: Look at only the first block. Right half: Look at only the second block. Raise your hand when you understand what **Line L** represents.

```
Point end = p.clone();
end.translate(10,20);
Line L = new Line(p, end);
```

```
Line L = new Line(p, p.translate(10,20));
```

Note: Slide redone after copies!

# Side effects

An expression is a sequence of symbols that can be evaluated to produce a value. Here's a Java expression:

```
i + j * k
```

If evaluating an expression also causes an observable change somewhere ~~else~~, we say that expression has a *side effect*.

Here's a Java expression with a side effect:

```
i + j++ * k
```

Do these two expressions have the same value?

What's the side effect?

# Side effects, continued

Which of these Java expressions have a side effect?

`x = 10`

`p1.translate(10, 20) // Consider imp. & app. cases...`

`"testing".toUpperCase()`

`L.add("x")`, where `L` is an `ArrayList`

`System.out.println("Hello!")`

# Side effects, continued

Side effects are a hallmark of imperative programing.

Programs written in an imperative style are essentially an orchestration of side effects.

Can we program without side effects?

# The Functional Paradigm

# The functional programming paradigm

A key characteristic of the functional paradigm is writing functions that are like pure mathematical functions.

Pure mathematical functions:

- Always produce the same value for given input(s)
- Have no side effects
- Can be combined brainlessly to produce more powerful functions

Ideally, functions are specified with notation that's similar to what you see in math books—cases and expressions.

# Functional programming, continued

Other characteristics of the functional paradigm:

- Values are <u>never</u> changed but lots of new values are created.
- Recursion is used in place of iteration.
- <u>Functions are values</u>. Functions are put into ~~in~~ data structures, passed to functions, and returned from functions. LOTS of temporary functions are created.

Based on the above, how well would Java support functional programming? How about C?

# Haskell basics

# What is Haskell?

Haskell is a pure functional programming language; it has no imperative features.

Was designed by a committee with the goal of creating a standard language for research into functional programming.

First version appeared in 1990. Latest version is known as Haskell 2010.

Is said to be *non-strict*—it supports *lazy evaluation*.

It is not object-oriented in any way.

My current opinion: it has a relatively large mental footprint.

# Haskell resources

Website: **haskell.org** *(sluggish as of 1/19/15)*

All sorts of resources!

Books: (on Safari, too)

*Learn You a Haskell for Great Good!*, by Miran Lipovača

**http://learnyouahaskell.com** (Known as LYAH.)

*Real World Haskell*, by O'Sullivan, Stewart, and Goerzen

**http://realworldhaskell.org** (I'll call it RWH.)

*Programming in Haskell*, by Hutton

Note: See appendix B for mapping of non-ASCII chars!

Haskell 2010 Report (I'll call it H10.)

**http://haskell.org/definition/haskell2010.pdf**

# Interacting with Haskell

On lectura we can interact with Haskell by running **ghci**:

```
$ ghci
GHCi, version 7.4.1: ...more...  :? for help
Loading package ghc-prim ... linking ... done.
Loading package integer-gmp ... linking ... done.
Loading package base ... linking ... done.
>
```

With no arguments, **ghci** starts a read-eval-print loop (REPL)—expressions that we type at the prompt (**>**) are evaluated and the result is printed.

Note: the standard prompt is **Prelude>** but I've got

```
:set prompt "> "
```

in my **~/.ghci** file.

# Interacting with Haskell, continued

Let's try some expressions with **ghci**:

```
> 3+4
7

> 3 * 4.5
13.5

> (3 > 4) || (5 < 7)
True

> 2 ^ 200
160693804425899027554196209234116260252220299378
2792835301376

> "abc" ++ "xyz"
"abcxyz"
```

# Interacting with Haskell, continued

We can use **:help** to see available commands:

```
> :help
 Commands available from the prompt:
   <statement>                 evaluate/run <statement>
   :                           repeat last command
   :{\n ..lines.. \n:}\n       multiline command
   ...lots more...
```

The command **:set +t** causes types to be shown:

```
> :set +t
> 3+4
7
it :: Integer
```

"**::**" is read as "has type". The value of the expression is "bound" to the name **it**.

# Interacting with Haskell, continued

We can use **it** in subsequent computations:

```
> 3+4
7
it :: Integer

> it + it * it
56
it :: Integer

> it /= it
False
it :: Bool
```

# Extra Credit Assignment 1

For two assignment points of extra credit:

1. Run **ghci** (or WinGHCi) somewhere and try ten Haskell expressions with some degree of variety. (Not just ten additions, for example!) Do a **:set +t** at the start.

2. Capture the output and put it in a plain text file, eca1.txt, and turn it in via the eca1 D2L dropbox. (No need for your name, NetID, etc. in the file.)

Due: At the start of the next lecture after we hit this slide.

Needless to say, feel free to read ahead in the slides and show experimentation with the following material, too.

# Getting Haskell

# Getting Haskell

You can either get Haskell for your machine or use Haskell on lectura.

To work on your own machine, get a copy of the Haskell Platform for your operating system from haskell.org.

On OS X, I'm using *Haskell Platform 2014.2.0.0 for Mac OS X, 64bit* from www.haskell.org/platform/mac.html

On Windows, use *Haskell Platform 2014.2.0.0 for Windows* from http://www.haskell.org/platform/windows.html The 32-bit version should be fine but if you have trouble, (1) let me know and (2) go ahead and try the 64-bit version.

You'll need an editor that can create plain text files. Sublime Text is very popular.

# Using Haskell on lectura

To work on lectura from a Windows machine, you might login with PuTTY. (See following slide.)

OS X, do **ssh YOUR-NETID@lectura.cs.arizona.edu**

You might edit Haskell files on lectura with **vim**, **emacs**, or **nano** (ick!), or use something like **gedit** on a Linux machine in a CS lab.

Alternatively, you might edit on your machine with something like Sublime Text and use a synchronization tool (like WinSCP on Windows) to keep your copy on lectura constantly up to date.

If you go the route of editing on your machine and running on lectura, let me know if you have trouble figuring out how to do automatic synchronization. It's a terrible waste of time to do a manual copy of any sort in the middle of your edit/run cycle.

# Getting and running PuTTY

If you Google for "putty", the first hit should be this:

PuTTY Download Page

- www.chiark.greenend.org.uk/~sgtatham/putty/download.html

Download **putty.exe**. It's just an executable—no installer!

**Binaries**

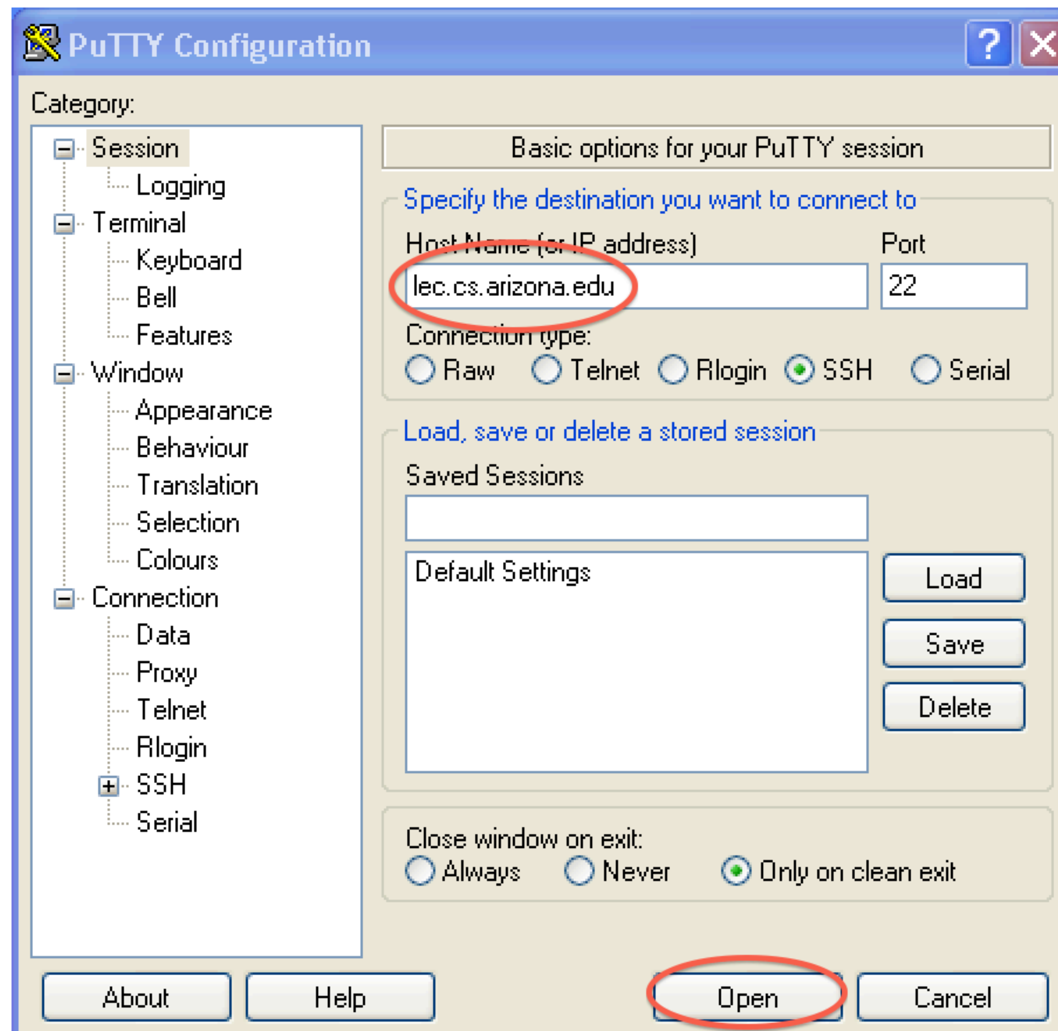
*The latest release version (beta 0.63).*
fixed the bug, before reporting it to me

**For Windows on Intel x86**

| | |
|---|---|
| PuTTY: | putty.exe |
| PuTTYtel: | puttytel.exe |
| PSCP: | pscp.exe |

# PuTTY, continued

Click on **putty.exe** to run it. In the dialog that opens, fill in **lec.cs.arizona.edu** for Host Name and click Open.



# **ghci** on lectura

Login to lectura using your UA NetID. Run **ghci**, and try some expressions:

```
lectura.cs.arizona.edu - PuTTY
lectura ~ 5003 $ ghci
GHCi, version 7.4.1: http://www.haskell.org/ghc/  :? for help
Loading package ghc-prim ... linking ... done.
Loading package integer-gmp ... linking ... done.
Loading package base ... linking ... done.
> 3 + 4
7
> "abc" ++ "xyz"
"abcxyz"
> 2 ^ 200
1606938044258990275541962092341162602522202993782792835301376
> (control-D to exit)
Leaving GHCi.
lectura ~ 5004 $
```

Go to [http://cs.arizona.edu/computing/services](http://cs.arizona.edu/computing/services) and use "Reset my forgotten Unix password" if needed.

# The ~/.ghci file

When **ghci** starts up on Linux or OS X it looks for the file **~/.ghci** – a **.ghci** file in the user's home directory.

Below are a couple of lines that I find handy in my **~/.ghci** file. The first sets the prompt and the second loads a module that allows functions to be printed as values, although just showing **<function>** for function values.

```
:set prompt "> "
:m +Text.Show.Functions
```

# ~/.ghci, continued

The counterpart path on Windows is this:

%APPDATA%\ghc\ghci.conf

(Note: file is named **ghci.conf**, not **.ghci**!)

%APPDATA% represents the location of your Application Data directory. You can find that path by typing **set appdata** in a command window, like this:

```
C:\>set appdata
APPDATA=C:\Users\whm\Application Data
```

Combing the two, the full path to the file would be

C:\Users\whm\Application Data\ghc\ghci.conf

Details on **.ghci** and lots more can be found in

https://downloads.haskell.org/~ghc/latest/docs/users_guide.pdf

# Functions and function types

# Calling functions

In Haskell, *juxtaposition* indicates a function call:

```
> negate 3
-3
it :: Integer

> even 5
False
it :: Bool

> pred 'C'
'B'
it :: Char

> signum 2
1
it :: Integer
```

Note: These functions and many more are defined in the Haskell "Prelude", which is loaded by default when **ghci** starts up.

**ghci** uses The GNU Readline library.

Use TAB to complete names, ^R to incrementally search backwards, ^A/^E for start/end of line, etc.

Lots more:
tiswww.case.edu/php/chet/readline/rluserman.html

# Calling functions, continued

Function call with juxtaposition is left-associative.

**signum negate 2** means **(signum negate) 2**

```
> signum negate 2
<interactive>:40:1:    -- It's an error!
   No instance for (Num (a0 -> a0)) arising from a
use of `signum'
...
```

We add parentheses to call **negate 2** first:

```
> signum (negate 2)
-1
it :: Integer
```

# Calling functions, continued

Function call with juxtaposition has higher precedence than any operator.

```
> negate 3+4
1
it :: Integer
```

negate 3 + 4 means (negate 3) + 4. Use parens to force + first:

```
> negate (3 + 4)
-7
it :: Integer

> signum (negate (3 + 4))
-1
it :: Integer
```

# Function types

Haskell's **Data.Char** module has a number of functions for working with characters. We'll use it to start learning about function types.

```
> :m Data.Char
```
*(:m(odule) loads a module)*

```
> isLower 'b'
True
it :: Bool

> toUpper 'a'
'A'
it :: Char

> ord 'A'
65
it :: Int

> chr 66
'B'
it :: Char
```

We can also reference a function in a module with a *qualified name*:

```
% ghci
GHCi, version 7.6.3: ...
> Data.Char.ord 'G'
71
```

# Function types, continued

We can use **gchi**'s **:type** command to see what the type of a function is:

```
> :type isLower
isLower :: Char -> Bool
```
*(read -> as "to")*

The type **Char -> Bool** means that the function takes an argument of type **Char** and produces a result of type **Bool.**

Using **ghci**, what are the types of **toUpper**, **ord**, and **chr**?

We can use **:browse Data.Char** to see everything in the module.

# Type consistency

Like most languages, Haskell requires that expressions be *type-consistent* (or *well-typed*).

Here is an example of an inconsistency:

```
> chr 'x'
<interactive>:32:5:
  Couldn't match expected type Int with actual type Char
  In the first argument of `chr', namely 'x'

> :type chr
chr :: Int -> Char

> :type 'x'
'x' :: Char
```

**chr** requires its argument to be an **Int** but we gave it a **Char**. We can say that **chr 'x'** is *ill-typed*.

# Type consistency, continued

State whether each expression is well-typed and if so, its type.

```
'a'
isUpper
isUpper 'a'
not (isUpper 'a')
not not (isUpper 'a')
toUpper (ord 97)
isUpper (toUpper (chr 'a'))
isUpper (intToDigit 100)
```

```
'a' :: Char
chr :: Int -> Char
digitToInt :: Char -> Int
intToDigit :: Int -> Char
isUpper :: Char -> Bool
not :: Bool -> Bool
ord :: Char -> Int
toUpper :: Char -> Char
```

# Sidebar: Using a REPL to help learn a language

As we've seen, **ghci** provides a REPL (read-eval-print loop).

What are some other languages that have a REPL available?

How does a REPL help us learn a language?

Is there a REPL for Java?

What characteristics does a language need to support a REPL?

If there's no REPL for a language, how hard is it to write one?

# Type classes

# What's the type of **negate**?

Recall the **negate** function:

```
> negate 5
-5
it :: Integer

> negate 5.0
-5.0
it :: Double
```

What is the type of **negate**? (Is it both **Integer -> Integer** and **Double -> Double**??)

# Type classes

**Bool**, **Char**, and **Integer** are examples of Haskell types.

Haskell also has *type classes*. A type class specifies the operations must be supported on a type in order for that type to be a member of that type class.

**Num** is one of the many type classes defined in the Prelude.

**:info Num** shows that for a type to be a **Num,** it must support addition, subtraction, multiplication and four functions: **negate**, **abs**, **signNum**, and **fromInteger**. (The **Num** club!)

The Prelude defines four *instances* of the **Num** type class: **Int** (word-size), **Integer** (unlimited size), **Float** and **Double**.

# Type classes, continued

Here's the type of **negate:**

```
> :type negate
negate :: Num a => a -> a
```

The type of **negate** is specified using a *type variable*, **a**.

The portion **a -> a** specifies that **negate** returns a value having the same type as its argument.

*"If you give me an **Int**, I'll give you back an **Int**."*

The portion **Num a =>** is a *class constraint*. It specifies that the type **a** must be an instance of the type class **Num**.

How can we state the type of **negate** in English?

***negate** accepts any value whose type is an instance of **Num**. It returns a value of the same type.*

# Type classes, continued

What type do integer literals have?

```
> :type 3
3 :: Num a => a

> :type (-27)                -- Note: Parens needed!
(-27) :: Num a => a
```

Literals are typed with a class constraint of **Num**, so they can be used by any function that accepts **Num a => a**.

# Type classes, continued

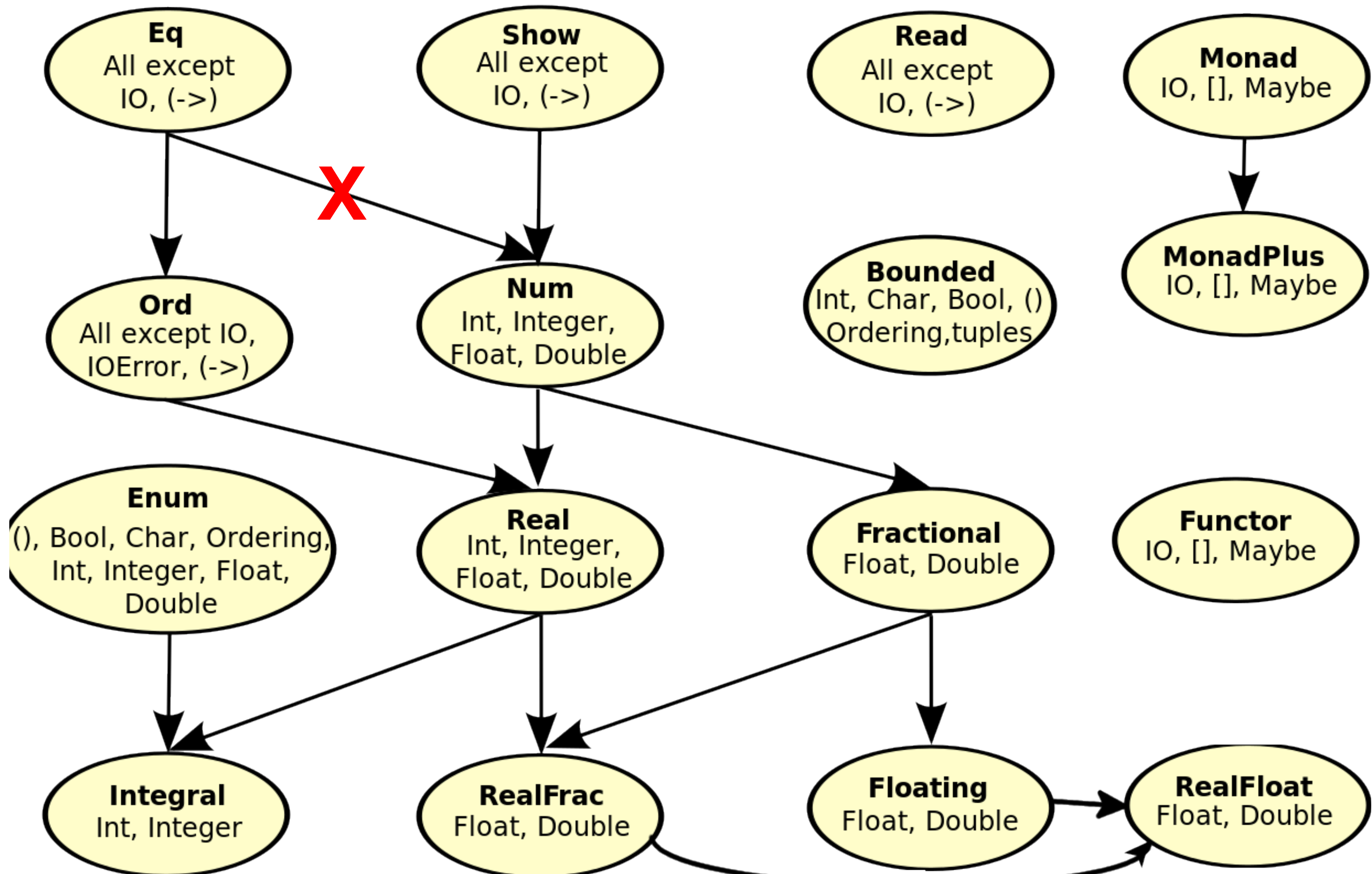
Will **negate 3.4** work?

```
> :type negate
negate :: Num a => a -> a

> :type 3.4
3.4 :: Fractional a => a

> negate 3.4
-3.4
```

Speculate: Why does it work?

# Type classes, continued

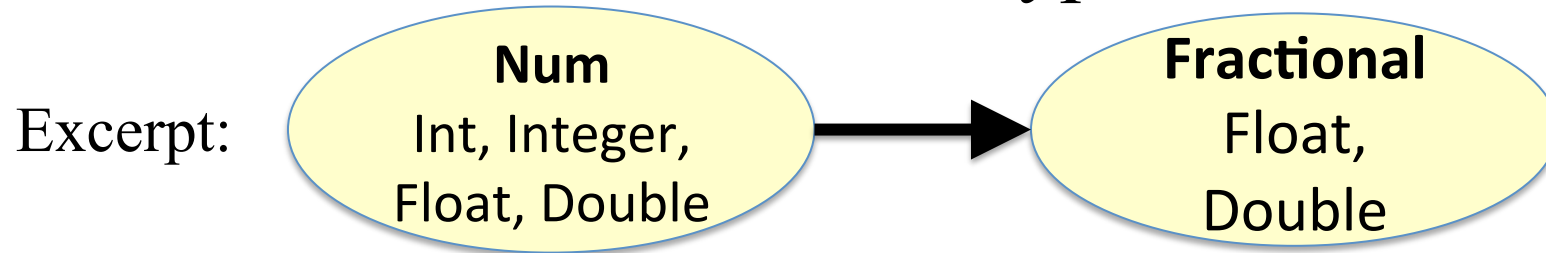
Haskell type classes form a hierarchy. The Prelude has these:




Adapted from http://en.wikibooks.org/wiki/Haskell/Classes_and_types

# Type classes, continued

Excerpt:




The arrow from **Num** to **Fractional** means that a **Fractional** can be used as a **Num**. (What does that remind you of?)

Given

```
negate :: Num a => a -> a
```

and

```
5.0 :: Fractional a => a
```

then

**negate 5.0** is valid.

# Type classes, continued

What's meant by the type of **pz**?

```
pz :: (Bounded a, Fractional b) => a -> b
```

Would **pz 'a'** be valid? How about **pz 5.5**? **pz 7**?

LYAH pp. 27-33 has a good description of the Prelude's type classes. ("Type Classes 101")

RWH uses the term "typeclasses"—one word!

## **negate** is *polymorphic*

In essence, **negate :: Num a => a -> a** describes many functions:

```
negate :: Integer -> Integer
negate :: Int -> Int
negate :: Float -> Float
negate :: Double -> Double
...and more...
```

**negate** is a *polymorphic function*. It handles values of many forms.

If a function's type has any type variables, it's a polymorphic function.

How does Java handle this problem? How about C? C++?

Take a break?

# More on functions

# Writing simple functions

A function can be defined in the REPL by using **let**. Example:

```
> let double x = x * 2
double :: Num a => a -> a

> double 5
10
it :: Integer

> double 2.7
5.4
it :: Double

> double (double (double 1111111111111))
8888888888888
it :: Integer
```

# Simple functions, continued

More examples:

```
> let neg x = -x
neg :: Num a => a -> a

> let isPositive x = x > 0
isPositive :: (Num a, Ord a) => a -> Bool

> let toCelsius temp = (temp - 32) * 5/9
toCelsius :: Fractional a => a -> a
```

The determination of types based on the operations performed is known as *type inferencing*. (More on it later!)

Note: function and parameter names must begin with a lowercase letter or _. (If capitalized they're assumed to be *data constructors*.)

# Simple functions, continued

We can use :: *type* to constrain the type inferred for a function:

```
> let neg x = -x :: Integer
neg :: Integer -> Integer

> let isPositive x = x > (0::Integer)
isPositive :: Integer -> Bool

> let toCelsius temp = (temp - 32) * 5/(9::Double)
toCelsius :: Double -> Double
```

We'll use :: *type* to simplify some following examples.

# Sidebar: loading functions from a file

We can put function definitions in a file. When we do, **we leave off the let**!

I've got four function definitions in the file **simple.hs**, as shown with the UNIX **cat** command:

```
% cat simple.hs
double x = x * 2 :: Integer   -- Note: no "let"!
neg x = -x :: Integer
isPositive x = x > (0::Integer)
toCelsius temp = (temp - 32) * 5/(9::Double)
```

The **.hs** suffix is required.

# Sidebar, continued

Assuming **simple.hs** is in the current directory, we can load it with **:load** and see what we got with **:browse**.

```
% ghci
> :load simple
[1 of 1] Compiling Main          ( simple.hs, interpreted )
Ok, modules loaded: Main.

> :browse
double :: Integer -> Integer
neg :: Integer -> Integer
isPositive :: Integer -> Bool
toCelsius :: Double -> Double
```

Note the colon in **:load**, and that the suffix **.hs** is assumed.

We can use a path, like **:load ~/372/hs/simple**, too.

# Sidebar: Learning a Language

Look for ways to type less, to spend more time learning and less time typing!

Anticipate: How might we type less when loading a file?

```
> :l simple
[1 of 1] Compiling Main            ( simple.hs, interpreted )
Ok, modules loaded: Main.
```

After an initial load, **:reload** is sufficient:

```
> :reload
[1 of 1] Compiling Main            ( simple.hs, interpreted )
Ok, modules loaded: Main.
```

Can we still type less?

# Sidebar: My usual edit-run cycle

**ghci** is clumsy to type! I've got an **hs** alias in my **~/.bashrc**:

```
alias hs=ghci
```

I specify the file I'm working with as an argument to **hs**.

```
% hs simple
GHCi, version 7.6.3 ...
[1 of 1] Compiling Main             ( simple.hs, interpreted )
Ok, modules loaded: Main.
> ... experiment ...
```

After editing in a different window I use **:r** to reload the file.

```
> :r
[1 of 1] Compiling Main             ( simple.hs, interpreted )
Ok, modules loaded: Main.
> ...experiment some more...
```

Lather, rinse, repeat.

# Functions with multiple arguments

Here's a function that produces the sum of its two arguments:

```
> let add x y = x + y :: Integer
```

Here's how we call it: (no commas or parentheses!)

```
> add 3 5
8
```

Here is its type:

```
> :type add
add :: Integer -> Integer -> Integer
```

The operator -> is right-associative, so the above means this:

```
add :: Integer -> (Integer -> Integer)
```

But what does that mean?

# Multiple arguments, continued

Recall our negate function:

```
> let neg x = -x :: Integer
neg :: Integer -> Integer
```

Here's **add** again, with parentheses added to show precedence:

```
> let add x y = x + y :: Integer
add :: Integer -> (Integer -> Integer)
```

**add** is a function that takes an integer as an argument and produces a function as its result!

**add 3 5** means **(add 3) 5**

Call **add** with the value 3, producing a nameless function.

Call that nameless function with the value 5.

# Partial application

When we give a function fewer arguments than it requires, the result is called a *partial application*. It is a function.

We can bind a name to a partial application like this:

```
> let plusThree = add 3
plusThree :: Integer -> Integer
```

The name **plusThree** now references a function that takes an **Integer** and returns an **Integer**.

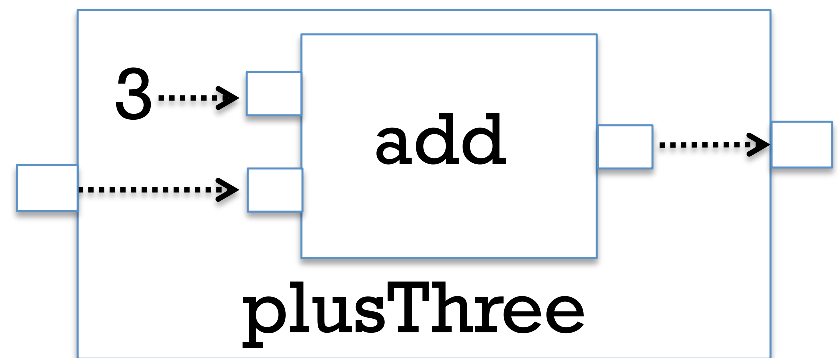
What will **plusThree 5** produce?

```
> plusThree 5
8
it :: Integer
```

# Partial application, continued

At hand:

```
> let add x y = x + y :: Integer
add :: Integer -> (Integer -> Integer)  -- parens added

> let plusThree = add 3
plusThree :: Integer -> Integer
```




Analogy: **plusThree** is like a calculator where you've clicked 3, then +, and handed it to somebody.

# Partial application, continued

At hand:

```
> let add x y = x + y :: Integer
add :: Integer -> (Integer -> Integer)  -- parens added
```

Another: *(with parentheses added to type to aid understanding)*

```
> let add3 x y z = x + y + z :: Integer
add3 :: Integer -> (Integer -> (Integer -> Integer))
```

These functions are said to be defined in *curried* form, which allows partial application of arguments.

The idea of a partially applicable function was first described by Moses Schönfinkel. It was further developed by Haskell B. Curry. Both worked wtih David Hilbert in the 1920s.

$log_2\ n$

What prior use have you made of partially applied functions?

**Note: next set of slides!**

# Some key points

Key points:

- A function with a type like **Integer -> Char -> Char** takes two arguments, an **Integer** and a **Char**. It produces a **Char**.

- A function call like

  **f x y z**

  means

  **((f x) y) z**

  and (conceptually) causes two temporary, unnamed functions to be created.

- Calling a function with fewer arguments that it requires creates a *partial application*.

# Specifying a function's type

It is common practice to specify the type of a function along with its definition in a file.

What's the ramification of the difference in these two type specifications?

```
add1::Num a => a -> a -> a
add1 x y = x + y

add2::Integer -> Integer -> Integer
add2 x y = x + y
```

# Sidebar: Continuation with indentation

A Haskell source file is a series of *declarations*. Here's a file with two declarations:

```
% cat indent1.hs
add::Integer -> Integer -> Integer
add x y = x + y
```

A declaration can be continued across multiple lines by indenting lines more than the first line of the declaration. These weaving declarations are poor style but are valid:

```
add
        ::
   Integer-> Integer-> Integer
add x y
    =
 x
   + y
```

# Indentation, continued

A line that starts in the same column as the previous declaration ends that declaration and starts a new one.

```
% cat indent2.hs
add::Integer -> Integer -> Integer
add x y =
x + y

% ghci indent2
...
indent2.hs:3:1:
   parse error (possibly incorrect indentation or
mismatched brackets)
Failed, modules loaded: none.
```

Note that 3:1 indicates line 3, column 1.

# Function/operator equivalence

Haskell operators are simply functions that can be invoked with an infix form.

We can use **:info** to find out about an operator.

```
> :info (^)
(^) :: (Num a, Integral b) => a -> b -> a
infixr 8 ^
```

**(Num a, Integral b) => a -> b -> a** shows that the first operand must be a number and the second must be an integer.

**infixr 8** shows that it is right-associative, with priority 8.

Explore ==, >, +, *, ||, ^^ and **.

# Function/operator equivalence, continued

To use an operator as a function, enclose it in parentheses:

```
> (+) 3 4
7
```

Conversely, we can use a <u>function</u> as an <u>operator</u> by enclosing it in backquotes:

```
> 3 `add` 4
7

> 11 `rem` 3
2
```

# Function/operator equivalence, continued

Haskell lets us define custom operators.

Example: (loading from a file)

```
(+%) x percentage = x + x * percentage / 100
infixl 6 +%
```

Usage:

```
> 100 +% 1
101.0
> 12 +% 25
15.0
```

The characters ! # $ % & * + . / < = > ? @ \ ^ | - ~ : and non-ASCII Unicode symbols can be used in custom operators.

Modules often define custom operators.

# Reference: Operators from the Prelude

<table>
<tr><th>Precedence</th><th>Left associative operators</th><th>Non associative operators</th><th>Right associative operators</th></tr>
<tr><td>9</td><td>!!</td><td></td><td>.</td></tr>
<tr><td>8</td><td></td><td></td><td>^, ^^, **</td></tr>
<tr><td>7</td><td>*, /, `div`, `mod`, `rem`, `quot`</td><td></td><td></td></tr>
<tr><td>6</td><td>+, -</td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td><td>:, ++</td></tr>
<tr><td>4</td><td></td><td>==, /=, <, <=, >, >=, `elem`, `notElem`</td><td></td></tr>
<tr><td>3</td><td></td><td></td><td>&&</td></tr>
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<tr><td>1</td><td>>>, >>=</td><td></td><td></td></tr>
<tr><td>0</td><td></td><td></td><td>$, $!, `seq`</td></tr>
</table>

Note: From page 51 in Haskell 2010 report

# More functions

The *general form* of a function definition (for now):

```
let name param1 param2 ... paramN = expression
```

Problem: Define a function **min3** that computes the minimum of three values. The Prelude has a **min** function.

```
> min3 5 2 10
2

> let min3 a b c = min a (min b c)
min3 :: Ord a => a -> a -> a -> a
```

Problem: Define a function **eq3** that returns **True** if its three arguments are equal, **False** otherwise.

# Guards

Recall this characteristic of functional programming:

"Ideally, functions are specified with notation that's similar to what you see in math books—cases and expressions."

This function definition uses *guards* to specify three cases:

```
sign x | x < 0 = -1
       | x == 0 = 0
       | otherwise = 1
```

Notes:

- No **let**—this definition is loaded from a file with **:load**
- sign x appears just once. First guard might be on next line.
- The *guard* appears between | and =, and produces a **Bool**
- What is **otherwise**?

# Guards, continued

Problem: Using guards, define a function **smaller**, like **min**:

```
> smaller 7 10
7

> smaller 'z' 'a'
'a'
```

Solution:

```
smaller x y
    | x <= y = x
    | otherwise = y
```

# Guards, continued

Problem: Write a function **weather** that classifies a given temperature as hot if 80+, else nice if 70+, and cold otherwise.

```
> weather 95
"Hot!"
> weather 32
"Cold!"
> weather 75
"Nice"
```

A solution that takes advantage of the fact that guards are tried in turn:

```
weather temp | temp >= 80 = "Hot!"
             | temp >= 70 = "Nice"
             | otherwise = "Cold!"
```

# Haskell's **if-else**

Here's an example of Haskell's **if-else**:

```
> if 1 < 2 then 3 else 4
3
```

How does this compare to the **if-else** in Java?

## Sidebar: Java's **if-else**

Java's **if-else** is a statement. It cannot be used where a value is required.

Java's conditional operator is the analog to Haskell's **if-else**.

`1 < 2 ? 3 : 4` (Java conditional, a.k.a ternary operator)

It's an expression that can be used when a value is required.

Java's if-else statement has an else-less form but Haskell's **if-else** does not. Why doesn't Haskell allow it?

Java's **if-else** vs. Java's conditional operator provides a good example of a *statement* vs. an *expression*.

Pythoners: What's the if-else situation in Python?

`3 if 1 < 2 else 4`

# Haskell's **if-else**, continued

What's the type of these expressions?

```
> :type if 1 < 2 then 3 else 4
if 1 < 2 then 3 else 4 :: Num a => a

> :type if 1 < 2 then 3 else 4.0
if 1 < 2 then 3 else 4.0 :: Fractional a => a

> if 1 < 2 then 3 else '4'
  <interactive>:12:15:
   No instance for (Num Char) arising from the literal `3'

> if 1 < 2 then 3
     <interactive>:13:16:
     parse error (possibly incorrect indentation or
mismatched brackets)
```

# Guards vs. **if-else**

Which of the versions of **sign** below is better?

```
sign x
    | x < 0 = -1
    | x == 0 = 0
    | otherwise = 1
```

```
sign x = if x < 0 then -1
                  else if x == 0 then 0
                                 else 1
```

We'll later see that *patterns* add a third possibility for expressing cases.

# Recursion

A recursive function is a function that calls itself either directly or indirectly.

Computing the factorial of a integer (N!) is a classic example of recursion. Write it in Haskell (and don't peek below!) What is its type?

```
factorial n
    | n == 0 = 1          -- Base case, 0! is 1
    | otherwise = n * factorial (n - 1)

> :type factorial
factorial :: (Eq a, Num a) => a -> a

> factorial 40
815915283247897734345611269596115894272000000000
```

# Recursion, continued

One way to manually trace through a recursive computation is to underline a call, then rewrite the call with a textual expansion:

```
factorial n
    | n == 0 = 1
    | otherwise = n * factorial (n – 1)
```

<u>factorial 4</u>

4 * <u>factorial 3</u>

4 * 3 * <u>factorial 2</u>

4 * 3 * 2 * <u>factorial 1</u>

4 * 3 * 2 * 1 * <u>factorial 0</u>

4 * 3 * 2 * 1 * 1

# Recursion, continued

Consider repeatedly dividing a number until the quotient is 1:

```
> 28 `quot` 3
9
> it `quot` 3
3
> it `quot` 3
1
```

*(Note backquotes to use* **quot** *as infix op.)*

*(Remember that* **it** *is previous result.)*

Problem: Write a recursive function **numDivs divisor x** that computes the number of times **x** must be divided by **divisor** to reach a quotient of 1.

```
> numDivs 3 28
3
> numDivs 2 7
2
```

# Recursion, continued

A solution:

```
numDivs divisor x
    | (x `quot` divisor) < 1 = 0
    | otherwise =
            1 + numDivs divisor (x `quot` divisor)
```

Example:

```
> numDivs 3 28
3
```

What is its type?

```
numDivs :: (Integral a, Num a1) => a -> a -> a1
```

Will **numDivs 2 3.4** work?

```
> numDivs 2 3.4
<interactive>:93:1:
  No instance for (Integral a0) arising from a use of
`numDivs'
```

# Sidebar: Fun with partial applications

Let's compute two partial applications of **numDivs**, using **let** to bind them to identifiers:

```
> let f = numDivs 2
> let g = numDivs 10
> f 9
3
> g 1001
3
```

What are more descriptive names than **f** and **g**?

```
> let floor_log2 = numDivs 2
> floor_log2 1000
9

> let floor_log10 = numDivs 10
> floor_log10 1000
3
```

# Lists

# List basics

In Haskell, a list is a sequence of values of the same type.

Here's one way to make a list. Note the type of **it** for each.

```
> [7, 3, 8]
[7,3,8]
it :: [Integer]

> [1.3, 10, 4, 9.7]
[1.3,10.0,4.0,9.7]
it :: [Double]

> ['x', 10]
<interactive>:20:7:
    No instance for (Num Char) arising from the literal
`10'
```

# List basics, continued

The function **length** returns the number of elements in a list:

```
> length [3,4,5]
3

> length []
0
```

What's the type of **length**?

```
> :type length
length :: [a] -> Int
```

With no class constraint specified, **[a]** indicates that **length** operates on lists containing elements of any type.

# List basics, continued

The **head** function returns the first element of a list.

```
> head [3,4,5]
3
```

What's the type of **head**?

```
head :: [a] -> a
```

Here's what **tail** does. How would you describe it?

```
> tail [3,4,5]
[4,5]
```

What's the type of **tail**?

```
tail :: [a] -> [a]
```

# List basics, continued

The ++ operator concatenates two lists, producing a new list.

```
> [3,4] ++ [10,20,30]
[3,4,10,20,30]

> it ++ it
[3,4,10,20,30,3,4,10,20,30]

> let f = (++) [1,2,3]
> f [4,5]
[1,2,3,4,5]

> f [4,5] ++ reverse (f [4,5])
[1,2,3,4,5,5,4,3,2,1]
```

What are the types of ++ and reverse?

```
> :type (++)
(++) :: [a] -> [a] -> [a]

> :type reverse
reverse :: [a] -> [a]
```

# List basics, continued

A range of values can be specified with a dot-dot notation:

```
> [1..20]
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20]
it :: [Integer]

> [-5,-3..20]
[-5,-3,-1,1,3,5,7,9,11,13,15,17,19]

> length [-1000..1000]
2001

> [10..5]
[]
it :: [Integer]
```

# List basics, continued

The !! operator produces a list's Nth element, zero-based:

```
> :type (!!)
(!!) :: [a] -> Int -> a

> [10,20..100] !! 3
40
```

Sadly, we can't use a negative value to index from the right.

```
> [10,20..100] !! (-2)
*** Exception: Prelude.(!!): negative index
```

Should that be allowed?

# Comparing lists

Haskell lists are <u>values</u> and can be compared as values:

```
> [3,4] == [1+2, 2*2]
True

> [3] ++ [] ++ [4] == [3,4]
True

> tail (tail [3,4,5,6]) == [last [4,5]] ++ [6]
True
```

Conceptually, how many lists are created by each of the above?

A programmer using a functional language writes complex expressions using lists (and more!) as freely as a Java programmer might write **`f(x) * a == g(a,b) + c`**.

# Comparing lists, continued

Lists are compared *lexicographically*: Corresponding elements are compared until an inequality is found. The inequality determines the result of the comparison.

Example:

```
> [1,2,3] < [1,2,4]
True
```

Why: The first two elements are equal, and `3 < 4`.

More examples:

```
> [1,2,3] < [1,1,1,1]
False
> [1,2,3] > [1,2]
True
> [1..] < [1,3..]     -- Comparing infinite lists!
True
```

LATER...

# Lists of Lists

We can make lists of lists.

```
> let x = [[1], [2,3,4], [5,6]]
x :: [[Integer]]
```

Note the type: **x** is a list of **Integer** lists.

**length** counts elements at the top level.

```
> length x
3
```

Recall that **length :: [a] -> Int** Given that, what's the type of **a** for **length x**?

What's the value of **length (x ++ x ++ [3])**?

# Lists of lists, continued

```
> let x = [[1], [2,3,4], [5,6]]

> head x
[1]

 > tail x
[[2,3,4],[5,6]]

> x !! 1 !! 2
4

> let y = [[1..],[10,20..]] ++ [[2,3]]
> take 5 (head (tail y))
[10,20,30,40,50]
```

LATER...

# Strings are [Char]

Strings in Haskell are simply lists of characters.

```
> "testing"
"testing"
it :: [Char]

> ['a'..'z']
"abcdefghijklmnopqrstuvwxyz"
it :: [Char]

> ["just", "a", "test"]
["just","a","test"]
it :: [[Char]]
```

What's the beauty of this?

# Strings, continued

All list functions work on strings, too!

```
> let asciiLets = ['A'..'Z'] ++ ['a'..'z']
asciiLets :: [Char]

> length asciiLets
52

> reverse (drop 26 asciiLets)
"zyxwvutsrqponmlkjihgfedcba"

> :type elem
elem :: Eq a => a -> [a] -> Bool

> let isAsciiLet c = c `elem` asciiLets
isAsciiLet :: Char -> Bool
```

# Strings, continued

The Prelude defines **String** as **[Char]** (a *type synonym*).

```
> :info String
type String = [Char]
```

A number of functions operate on **String**s. Here are two:

```
> :type words
words :: String -> [String]

> :type putStr
putStr :: String -> IO () -- an "action" (more later!)
```

What's the following doing?

```
> putStr (unwords (tail (words "Just some words!")))
some words!it :: ()
```

# Strings, continued

What's the following expression computing?

```
> length [(Data.Char.chr 0)..]
1114112
```

Another way:

```
> length ([minBound..maxBound]::[Char])
1114112
```

LATER...

# "cons" lists

Like most functional languages, Haskell's lists are "cons" lists.

A "cons" list has two parts:
- head: a value
- tail: a list of values (possibly empty)

The : ("cons") operator creates a list from a value and a list of values that same type (or an empty list).

```
> 5 : [10, 20,30]
[5,10,20,30]
```

What's the type of the cons operator?

```
> :type (:)
(:) :: a -> [a] -> [a]
```

# "cons" lists, continued

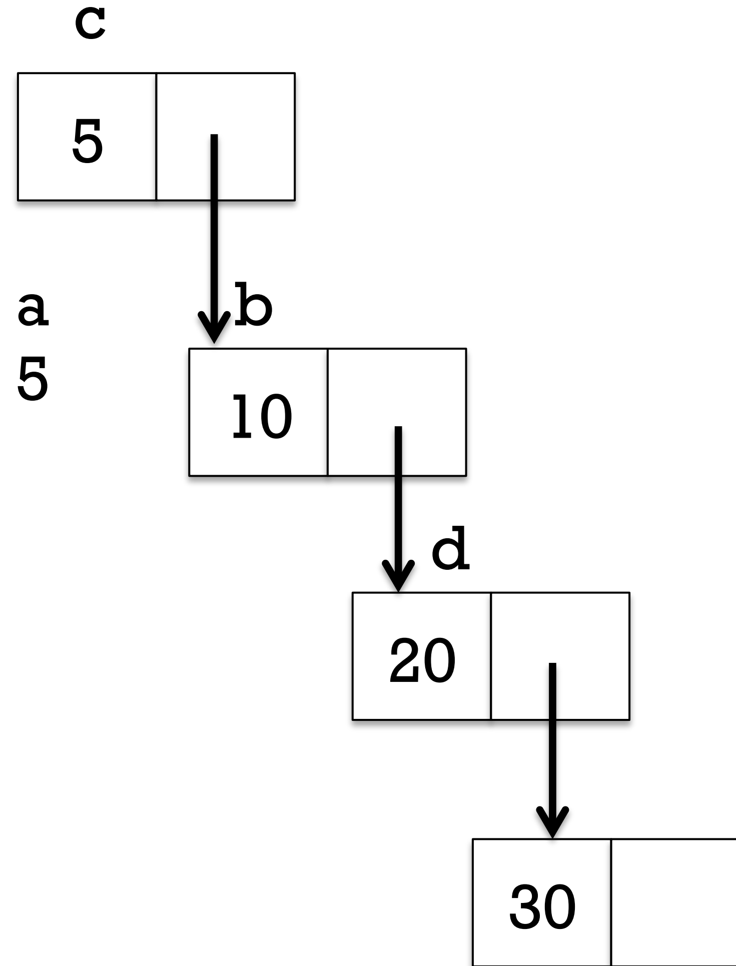
The cons (:) operation forms a new list from a value and a list.

```
> let a = 5
> let b = [10,20,30]
> let c = a:b
[5,10,20,30]

> head c
5

> tail c
[10,20,30]

> let d = tail (tail c)
> d
[20,30]
```



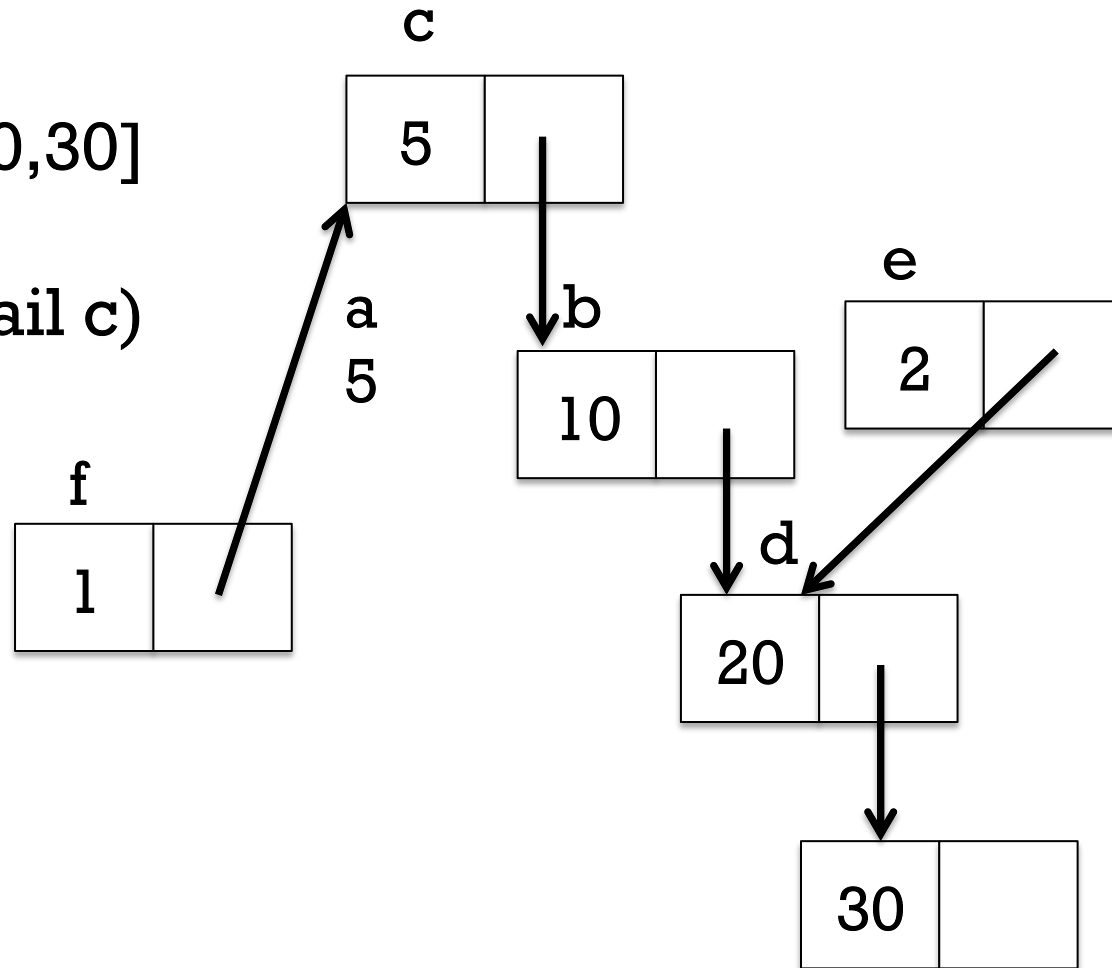
# "cons" lists, continued

A cons node can be referenced by multiple cons nodes.

```
> let a = 5
> let b = [10,20,30]
> let c = a:b
> let d = tail (tail c)
[20,30]

> let e=2:d
[2,20,30]

> let f=1:c
[1,5,10,20,30]
```



# "cons" lists, continued

What are the values of the following expressions?

```
> 1:[2,3]
[1,2,3]

> 1:2
...error...

> chr 97:chr 98:chr 99:[]
"abc"

> []:[]
[[]]

> [1,2]:[]
[[1,2]]

> []:[1]
...error...
```

cons is right associative

```
chr 97:(chr 98:(chr 99:[]))
```

Note: next set of slides!

# head and tail visually

It's important to understand that tail does not create a new list. Instead it simply returns an existing cons node.

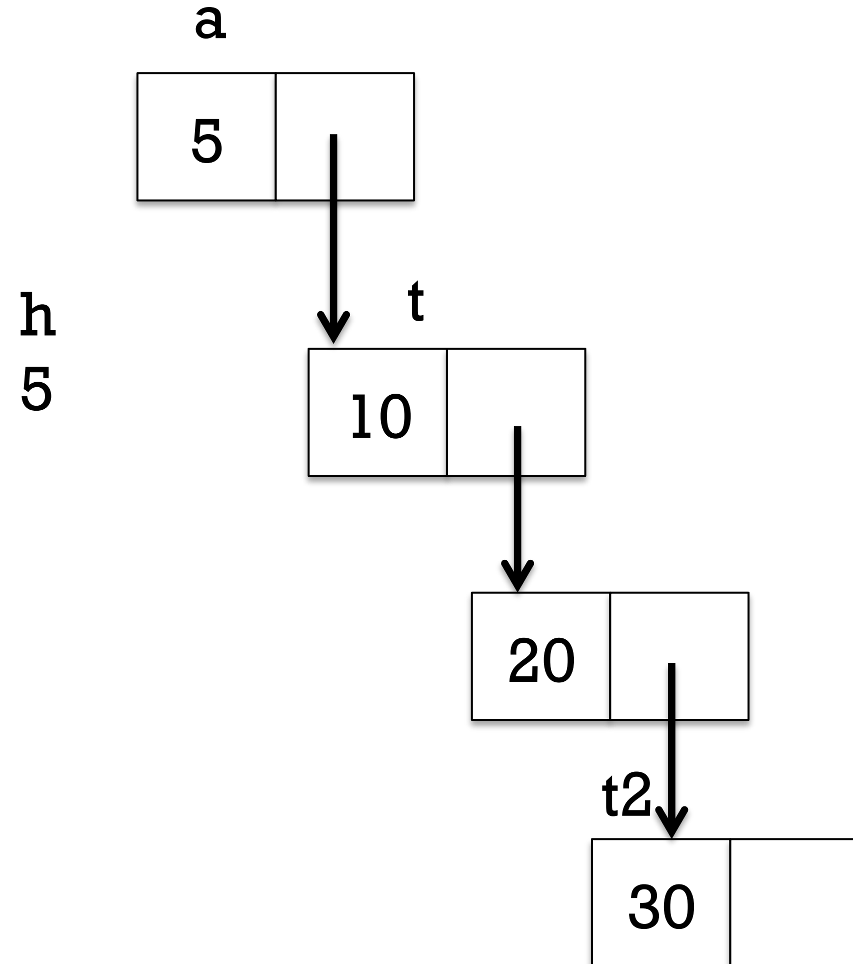
```
> let a = [5,10,20,30]

> let h = head a
> h
5

> let t = tail a
> t
[10,20,30]

> let t2 = tail (tail t)
> t2
[30]
```

h
5



# A little on performance

What operations are likely fast with cons lists?

- Get the head of a list
- Get the tail of a list
- Making a new list from a head and tail

What operations are likely slower?

- Get Nth element of a list
- Get length of a list

With cons lists, what does list concatenation involve?

```
> let m=[1..10000000]
> length (m++[0])
10000001
```

# True or false?

The head of a list is a one-element list.

False, unless...

...it's the head of a list of lists that starts with a one-element list

The tail of a list is a list.

True

The tail of an empty list is an empty list.

It's an error!

**length (tail (tail x)) == (length x) – 2**

True (assuming what?)

A cons list is essentially a singly-linked list.

True

A doubly-linked list might help performance in some cases.

Hmm...what's the backlink for a multiply-referenced node?

Changing an element in a list might affect the value of many lists.

Trick question! We can't change a list element. We can only "cons-up" new lists and reference existing lists.

# fromTo

Here's a function that produces a list with a range of integers:

```
> let fromTo first last = [first..last]

> fromTo 10 15
[10,11,12,13,14,15]
```

Problem: Write a recursive version of **fromTo** that uses the cons operator to build up its result.

# fromTo, continued

One solution:

```
fromTo first last
    | first > last = []
    | otherwise = first : fromTo (first+1) last
```

Evaluation of **fromTo 1 3** via substitution and rewriting:

```
fromTo 1 3
1 : fromTo (1+1) 3
1 : fromTo 2 3
1 : 2 : fromTo (2+1) 3
1 : 2 : fromTo 3  3
1 : 2 : 3 : fromTo (3+1) 3
1 : 2 : 3 : fromTo 4 3
1 : 2 : 3 : []
```

# fromTo, continued

Do **:set +s** to get timing and memory information, and make some lists. Try these:

```
fromTo 1 10
let f = fromTo          -- So we can type f instead of fromTo
f 1 1000
let f = fromTo 1        -- Note partial application
f 1000
let x = f 1000000
length x
take 5 (f 1000000)
```

# Excursion:
# A little bit with infinite lists and lazy evaluation

# Infinite lists

We can make an infinite list in Haskell! Here's one way:

```
> [1..]
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,2
1,22,23,24,25,26,27,28,29,30,31,32,^C
```

Any ideas on how to make use of an infinite list?

What does the following **let** create?

```
> let nthOdd = (!!) [1,3..]
nthOdd :: Int -> Integer
```

A function that produces the Nth odd number, zero-based.

Yes, we could say **let nthOdd n = (n*2)+1** but that wouldn't be nearly as much fun! (This *is* functional programming!)

# Infinite lists, continued

Consider the following **let**. Why does it complete?

```
> let fives=[5,10..]
fives :: [Integer]
```

A simplistic answer: Haskell uses *lazy evaluation*. It only computes as much of a value as it needs to.

(The deeper answer: Haskell uses *non-strict evaluation*. Conventional languages use *strict evaluation*.)

The function **take** produces the first N elements of a list.

```
> take 3 fives
[5,10,15]
```

Haskell computes only enough elements of **fives** to produce a result for **take 5**.

# Lazy evaluation

Here is an expression that is said to be *non-terminating:*

```
> length fives
...when tired of waiting...^C Interrupted.
```

But, we can bind a name to **length fives**:

```
> let numFives = length fives
numFives :: Int
```

That completes because Haskell hasn't yet needed to compute a value for **length fives**.

We can get another coffee break by asking Haskell to print the value of **numFives:**

```
> numFives
...after a while...^CInterrupted.
```

# Lazy evaluation, continued

We can use **:print** to explore lazy evaluation:

```
> let fives = [5,10..]

> :print fives
fives = (_t2::[Integer])

> take 3 fives
[5,10,15]
```

What do you think **:print fives** will now show?

```
> :print fives
fives = 5 : 10 : 15 : (_t3::[Integer])
```

# Lazy evaluation, continued

Speculate: Can infinite lists be concatenated?

```
> let values = [1..] ++ [5,10..] ++ [1,2,3]
> :t values
values :: [Integer]
```

How about this one?

```
> [1..] > [1,2,3,5]
False
```

False due to lexicographic comparison—4 < 5

Another one to consider:

```
> let fives = [5,10..]
> fives !! 100000000
500000005
```

# Experiment: How many **Char** values are there?

Here's one way to see how many distinct **Char** values exist:

```
> length ([minBound..maxBound]::[Char])
1114112
```

What does it mean?

**:info Char** shows **Char** is an instance of the **Bounded** type class.

Types that are instances of **Bounded** have **minBound** and **maxBound** defined.

Could we do it another way?

```
> length [(minBound::Char)..]
1114112
```

# Patterns

## *(redone!)* Motivation: Summing list elements

Imagine a function that computes the sum of a list's elements.

```
> sumElems  [1..10]
55

> :type sumElems
sumElems :: Num a => [a] -> a
```

Implementation:

```
sumElems list
    | null list = 0        -- null is function to test for empty list
    | otherwise = head list + sumElems (tail list)
```

It works but it's not idiomatic Haskell. We should use *patterns* instead!

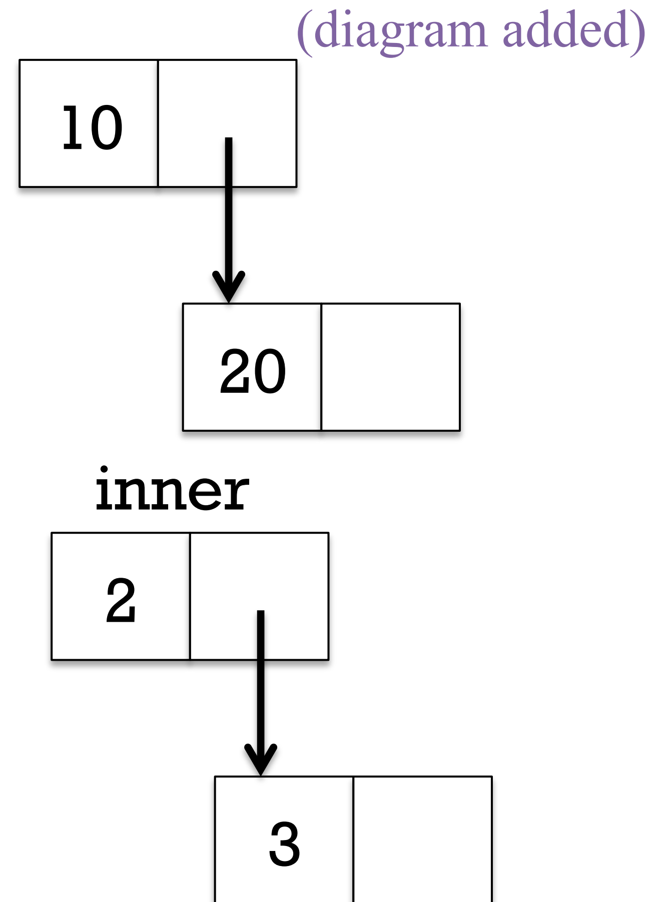
# Patterns

In Haskell we can use *patterns* to bind names to elements of data structures.

```
> let [x,y] = [10,20]
> x
10
> y
20

> let [inner] = [[2,3]]
> inner
[2,3]
```

x y
10 20




Speculate: Given a list like [10,20,30] how could we use a pattern to bind names to the head and tail of the list?

# Patterns, continued

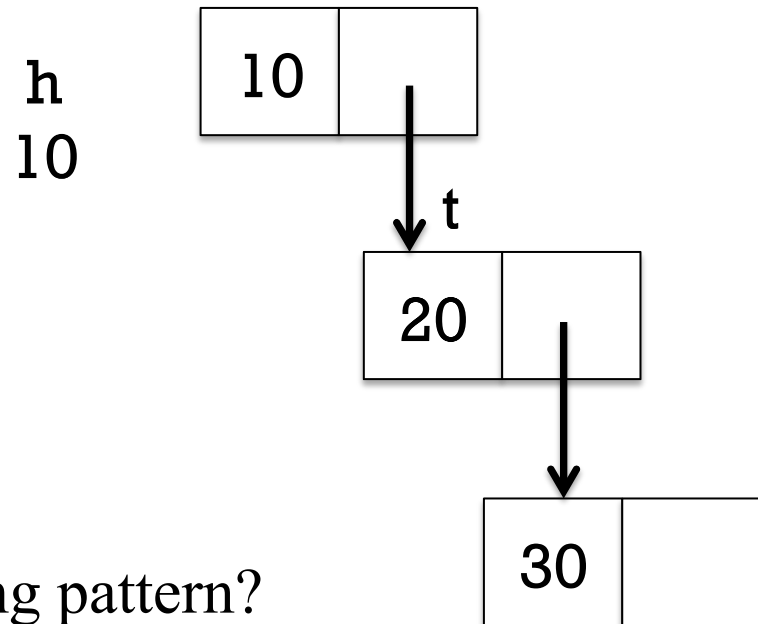
We can use the cons operator in a pattern.

```
> let h:t = [10,20,30]

> h
10

> t
[20,30]
```

(diagram added)



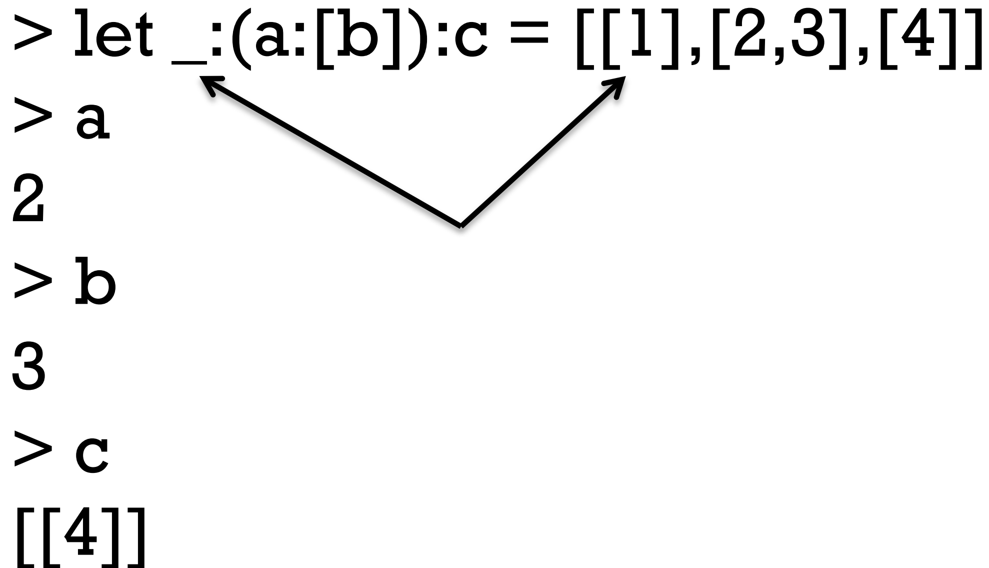

What values get bound by the following pattern?

```
> let a:b:c:d = [10,20,30]
> [c,b,a]          -- in a list so I could show them as a one-liner
[30,20,10]

> d                -- Why didn't I do [d,c,b,a] above?
[]
```

# Patterns, continued

If some part of a structure is not of interest, we indicate that with an underscore, known as the *wildcard pattern*.




No binding is done for the wildcard pattern.

The pattern mechanism is completely general—patterns can be arbitrarily complex.

# Patterns, continued

A name can only appear once in a pattern. This is invalid:

```
> let a:a:[] = [3,3]
<interactive>:25:5:
  Conflicting definitions for `a'
```

When using **let** as we are here, a failed pattern isn't manifested until we try to see what's bound to a name.

```
> let a:b:[] = [1]
> a
*** Exception: <interactive>:26:5-16: Irrefutable
pattern failed for pattern a : b : []
```

# Patterns in function definitions

Recall our non-idiomatic **sumElems**:

```
sumElems list
    | null list = 0
    | otherwise = head list + sumElems (tail list)
```

How could we redo it using patterns?

```
sumElems [] = 0
sumElems (h:t) = h + sumElems t
```

Note that **sumElems** appears on both lines and that there are no guards. **sumElems** has two *clauses*. (H10 4.4.3.1)

**The parentheses in (h:t) are required!!**

# Patterns in functions, continued

Here's a buggy version of **sumElems**:

```
buggySum [x] = x
buggySum (h:t) = h + buggySum t
```

What's the bug?

```
> buggySum [1..100]
5050
> buggySum []
*** Exception: slides.hs:(62,1)-(63,31): Non-exhaustive
patterns in function buggySum
```

If we use **ghci -fwarn-incomplete-patterns**, we'll get a warning when **:load**ing.

```
slides.hs:82:1: Warning:
    Pattern match(es) are non-exhaustive
    In an equation for `buggySum': Patterns not matched: []
```

# Practice

Describe in English what must be on the right hand side for a successful match.

let (a:b:c) = ...

A list containing at least two elements.

Does [[1,2]] match?

[2,3] ?

"abc" ?

let [x:xs] = ...

A list whose first element is a non-empty list.

Does words "a test" match?

[words "a test"] ?

[[]] ?

[[[]]] ?

# Recursive functions on lists

# Simple recursive list processing functions

Problem: Write **len x**, which returns the length of list **x**.

```
> len []
0

> len "testing"
7
```

Solution:

```
len [] = 0
len (_:t) = 1 + len t  -- since head isn't needed, use _
```

# Simple list functions, continued

Problem: Write **odds x**, which returns a list having only the odd numbers from the list **x**.

```
> odds [1..10]
[1,3,5,7,9]

 > take 10 (odds [1,4..])
[1,7,13,19,25,31,37,43,49,55]
```

Handy: odd :: Integral a => a -> Bool

Solution:

```
odds [] = []
odds (h:t)
    | odd h = h:odds t
    | otherwise = odds t
```

# Simple list functions, continued

Problem: write **isElem x vals**, like **elem** in the Prelude.

```
> isElem 5 [4,3,7]
False

> isElem 'n' "Bingo!"
True

> "quiz" `isElem` words "No quiz today!"
True
```

Solution:

```
isElem _ [] = False     -- Why a wildcard?
isElem x (h:t)
    | x == h = True
    | otherwise = x `isElem` t
```

# Simple list functions, continued

Problem: write a function that returns a list's maximum value.

```
> maxVal "maximum"
'x'

> maxVal [3,7,2]
7

> maxVal (words "i luv this stuff")
"this"
```

Solution:

```
maxVal [] = undefined
maxVal [x] = x
maxVal (x1:x2:xs)
   | x1 >= x2 = maxVal (x1:xs)
   | otherwise = maxVal (x2:xs)
```

# Sidebar: C and Python challenges

C programmers: Write **strlen** in C in a functional style. Do **strcmp** and **strchr**, too!

Python programmers: In a functional style write **size(x)**, which returns the number of elements in the string or list **x**. Restriction: You may not use **type()**.

Take a break?

# Tuples

# Tuples

A Haskell *tuple* is an ordered aggregation of two or more values of possibly differing types.

```
> (1, "two", 3.0)
(1,"two",3.0)
it :: (Integer, [Char], Double)

> (3 < 4, it)
(True,(1,"two",3.0))
it :: (Bool, (Integer, [Char], Double))
```

What's something we can represent with a tuple that we can't represent with a list?

# Tuples, continued

A function can return a tuple:

```
> let pair x y = (x,y)
```

What's the type of **pair?**

```
pair :: t -> t1 -> (t, t1)
-- why not a -> b -> (a,b)?
```

Let's play...

```
> pair 3 4
(3,4)

> pair (3,4)
<function>

> it 5
((3,4),5)
```

# Tuples, continued

The Prelude has two functions that operate on 2-tuples.

```
> let p = pair 30 "forty"
p :: (Integer, [Char])

> p
(30,"forty")

> fst p
30

> snd p
"forty"
```

# Tuples, continued

Recall: patterns used to bind names to list elements have the same syntax as expressions to create lists.

Patterns for tuples are like that, too.

Problem: Write **middle**, to extract a 3-tuple's second element.

```
> middle ("372", "CHVEZ 405", "Mitchell")
"CHVEZ 405"

> middle (1, [2], True)
[2]
```

# Tuples, continued

At hand:

```
> middle (1, [2], True)
[2]
```

Solution:

```
middle (_, m, _) = m
```

What's the type of **middle?**

```
middle :: (t, t1, t2) -> t1
```

Does the following call work?

```
> middle(1,[(2,3)],4)
[(2,3)]
```

# Tuples, continued

Here's the type of **zip** from the Prelude:

```
zip :: [a] -> [b] -> [(a, b)]
```

Speculate: What does **zip** do?

```
> zip ["one","two","three"] [10,20,30]
[("one",10),("two",20),("three",30)]

> zip ['a'..'z'] [1..]
[('a',1),('b',2),('c',3),('d',4),('e',5),('f',6),('g',7),('h',8),('i',
9),('j',10),...lots more... ('x',24),('y',25),('z',26)]
```

What's especially interesting about the second example?

# Tuples, continued

Problem: Write **elemPos**, which returns the zero-based position of a value in a list, or -1 if not found.

```
> elemPos 'm' ['a'..'z']
12
```

Hint: Have a helper function do most of the work.

Solution:

```
elemPos x vals = elemPos' x (zip vals [0..])

elemPos' _ [] = -1
elemPos' x ((val,pos):vps)
    | x == val = pos
    | otherwise = elemPos' x vps
```

Note: next set of slides!

# Sidebar: To curry or not to curry?

Consider these two functions:

```
> let add_c x y = x + y     -- _c for curried arguments
add_c :: Num a => a -> a -> a

> let add_t (x,y) = x + y   -- _t for tuple argument
add_t :: Num a => (a, a) -> a
```

Usage:

```
> add_c 3 4
7

> add_t (3,4)
7
```

**Important:** Note the difference in types!

Which is better, **add_c** or **add_t**?

# The **Eq** type class and tuples

**:info Eq** shows many lines like this:

```
...
instance (Eq a, Eq b, Eq c, Eq d, Eq e) => Eq (a, b, c, d, e)
instance (Eq a, Eq b, Eq c, Eq d) => Eq (a, b, c, d)
instance (Eq a, Eq b, Eq c) => Eq (a, b, c)
instance (Eq a, Eq b) => Eq (a, b)
```

We haven't talked about **instance** declarations but let's speculate: What's being specified by the above?

**instance (Eq a, Eq b, Eq c) => Eq (a, b, c)**

If values of each of the three types **a**, **b**, and **c** can be tested for equality then 3-tuples of type **(a, b, c)** can be tested for equality.

The **Ord** and **Bounded** type classes have similar instance declarations.

# Lists vs. tuples

Type-wise, lists are homogeneous; tuples are heterogeneous.

We can write a function that handles a list of any length but a function that operates on a tuple specifies the arity of that tuple.

Example: we can't write an analog for **head**, to return the first element of an arbitrary tuple.

Even if values are homogeneous, using a tuple lets static type-checking ensure that an exact number of values is being aggregated.

Example: A 3D point could be represented with a 3-element list but using a 3-tuple guarantees points have three coordinates.

If there were *Head First Haskell* it would no doubt have an interview with List and Tuple, each arguing their own merit.

# More on patterns and functions

# Function bindings, refined

Earlier in the slides the general form of a function definition was shown as this: *name arg1 arg2 ... argN = expression*

This is more accurate:

*name pattern1 pattern2 ... patternN*
*guard1 = expression1*
*...*
*guardN = expression N*

For a given **name**, any number of clauses like the above may be specified. The set of clauses for a given name is the *binding* for that name. (See 4.4.3 in H10.)

If values in a call match the pattern(s) for a clause and a guard is true, the corresponding expression is evaluated.

# Literals in patterns

Literal values can be part or all of a pattern. Here's a 3-clause binding for **f**:

```
f 1 = 10
f 2 = 20
f n = n
```

For contrast, with guards:

```
f n
  | n == 1 = 10
  | n == 2 = 20
  | otherwise = n
```

Usage:

```
> f 1
10

> f 3
3
```

Remember: Patterns are tried in the order specified.

# Literals in patterns, continued

Here's **factorial** with guards:

```
factorial n
    | n == 0 = 1
    | otherwise = n * factorial (n - 1)
```

Here it is with a literal pattern:

```
factorial 0 = 1
factorial n = n * factorial (n - 1)
```

Which is better?

REPLACE!

```
parens1 c
    | c == '(' = "left"
    | c == ')' = "right"
    | otherwise = "neither'

parens2 '(' = "left"
parens2 ')' = "right"
parens2 _ = "neither"
```

# Literals in patterns, continued

**not** is a function:

```
> :type not
not :: Bool -> Bool

> not True
False
```

Problem: Using literals in patterns, define **not**.

Solution:

```
not True = False
not _ = True            -- Using wildcard avoids comparison
```

# Pattern construction

A pattern can be:

- A literal value such as 1, '`x`', or `True`
- An identifier (bound to a value if there's a match)
- An underscore (the wildcard pattern)
- A tuple composed of patterns
- A list of patterns in square brackets (fixed size list)
- A list of patterns constructed with `:` operators
- Other things we haven't seen yet

Note the recursion.

Patterns can be arbitrarily complicated.

3.17.1 in H10 shows the full syntax for patterns.

# The **where** clause for functions

Intermediate values and/or helper functions can be defined using an optional **where** clause for a function.

Here's an example to show the syntax; the computation is not meaningful.

```
f x
   | g x < 0 = g a + g b
   | a > b = g b
   | otherwise = g a * g b
  where {
    a = x * 5;
    b = a * 2 + x;
    g t = log t + a
    }
```

The names **a** and **b** are bound to expressions; **g** is a function binding.

The bindings in the **where** clause are done first (!), then the guards are evaluated in turn.

Like variables defined in a method or block in Java, **a**, **b**, and **g** are not visible outside the declaration.

## **where**, continued

Imagine a function that counts occurrences of even and odd numbers in a list.

```
> countEO [3,4,5]
(1,2)                    -- one even, two odds
```

Code:

```
countEO [] = (0,0)       -- no odds or evens in []
countEO (x:xs)
    | odd x = (evens, odds+1)
    | otherwise = (evens+1, odds)
  where {
    (evens, odds) = countEO xs    -- count tail first!
    }
```

Would it be awkward to write it without using **where**?

# where, continued

Imagine a function that returns every Nth value in a list:

```
> everyNth 2 [10,20,30,40,50]
[20,40]
> everyNth 3 ['a'..'z'] -- abcdefghijklmnopqrstuvwxyz
"cfilorux"
```

Can we write this without a helper function?

We could use **zip** to pair elements with positions to know that **30** is the third element, for example.

```
> let everyNth n xs = helper n (zip xs [1..])
```

To learn a different technique, let's not use **zip**.

# where, continued

helper function

Let's write a ~~version of everyNth~~ that has an extra parameter: the original one-based position of the head of the list:

161,163s/everyNthWithPos/helper/g

```
helper _ [] pos = []
helper n (x:xs) pos
    | (pos `rem` n == 0) = x : helper n xs (pos+1)
    | otherwise = helper n xs (pos+1)
```

We then write **everyNth**:

```
everyNth n xs = helper n xs 1
```

everyNth 2 [10,20,30,40,50] would lead to these calls:

```
helper 2 [10,20,30,40,50] 1
helper 2 [20,30,40,50] 2      -- 2 rem 2 == 0
helper 2 [30,40,50] 3
helper 2 [40,50] 4            -- 4 rem 2 == 0
helper 2 [50] 5
```

# **where**, continued

Let's rewrite using **where** to conceal **helper**:

```
everyNth n xs = helper n xs 1
   where {
      helper _ [] pos = [];
      helper n (x:xs) pos
         | pos `rem` n == 0 = x : helper n xs (pos+1)
         | otherwise = helper n xs (pos+1)
      }
```

Remember: DRY!

Just like a Java private method, **everyNth** can't be accessed outside the body of **helper**.

The code works, but it's repetitious! How can we improve it?

# **where**, continued

Repetitious version:

```
everyNth n xs = helper n xs 1
   where {
     helper _ [] pos = [];
     helper n (x:xs) pos
        | pos `rem` n == 0 = x : helper n xs (pos+1)
        | otherwise = helper n xs (pos+1) }
```

Let's use another **where** to bind **rest** to the recursive call's result.

```
everyNth n xs = helper n xs 1
   where {
      helper _ [] pos = [];
      helper n (x:xs) pos
         | pos `rem` n == 0 = x : rest
         | otherwise = rest
         where { rest = helper n xs (pos+1) }
      }
```

# The *layout rule* for **where** (and more)

This is a valid declaration with a **where** clause:

```
f x = a + b + g a where { a = 1; b = 2; g x = -x }
```

The where clause has three declarations enclosed in braces and separated by semicolons.

We can take advantage of the *layout rule* and write it like this instead:

```
f x = a + b + g a
    where
        a = 1
        b = 2
        g x = -x
```

Besides whitespace what's different about the second version?

# The layout rule, continued

At hand:

```
f x = a + b + g a
    where
        a = 1
        b = 2
        g x =
              -x
```

Another example:

```
f x = a + b + g a where a = 1
                        b = 2
                        g x =
                           -x
```

The <u>absence of a brace</u> after **where** activates the layout rule.

The column position of the <u>first token after **where**</u> establishes the column in which declarations of the **where** must start.

Note that the declaration of **g** is continued onto a second line; if the minus sign were at or left of the line, it would be an error.

# The layout rule, continued

Don't confuse the layout rule with indentation-based continuation of declarations! (See slides 75-76.)

The layout rule allows omission of braces and semicolons in **where**, **do**, **let**, and **of** blocks. (We'll see **do** and **let** later.)

Indentation-based continuation applies

1. outside of **where/do/let/of** blocks
2. inside **where/do/let/of** blocks when the layout rule is triggered by the absence of an opening brace.

The layout rule is also called the "off-side rule".

TAB characters are assumed to have a width of 8.

What other languages have rules of a similar nature?

# Larger examples

# travel

Imagine a robot that travels on an infinite grid of cells. Movement is directed by a series of one character commands: **n**, **e**, **s**, and **w**.

Let's write a function **travel** that moves the robot about the grid and determines if the robot ends up where it started (i.e., it got home) or elsewhere (it got lost).

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 1 | | | | |
| | | | | | 2 | |
| | | | | | | |
| | | | | | | |
| | | R | | | | |
| | | | | | | |

If the robot starts in square R the command string **nnnn** leaves the robot in the square marked 1.

The string **nenene** leaves the robot in the square marked 2.

**nnessw** and **news** move the robot in a round-trip that returns it to square R.

# **travel**, continued

Usage:

```
> travel "nnnn"        -- ends at 1
"Got lost"

> travel "nenene"    -- ends at 2
"Got lost"

> travel "nnessw"
"Got home"
```

| | | 1 | | | | |
|---|---|---|---|---|---|---|
| | | | | | 2 | |
| | | | | | | |
| | | | | | | |
| | | R | | | | |
| | | | | | | |

How can we approach this problem?

## **travel**, continued

One approach:

1. Map letters into integer 2-tuples representing X and Y displacements on a Cartesian plane.
2. Sum the X and Y displacements to yield a net displacement.

Example:

Argument value: "**nnee**"

Mapped to tuples: (0,1) (0,1) (1,0) (1,0)

Sum of tuples: (2,2)

Another:

Argument value: "**nnessw**"

Mapped to tuples: (0,1) (0,1) (1,0) (0,-1) (0,-1) (-1,0)

Sum of tuples: (0,0)

# **travel**, continued

Two helpers:

```
mapMove :: Char -> (Int, Int)
mapMove 'n' = (0,1)
mapMove 's' = (0,-1)
mapMove 'e' = (1,0)
mapMove 'w' = (-1,0)
mapMove c = error ("Unknown direction: " ++ [c])

sumTuples :: [(Int,Int)] -> (Int,Int)
sumTuples [] = (0,0)
sumTuples ((x,y):ts) = (x + sumX, y + sumY)
  where
    (sumX, sumY) = sumTuples ts
```

# **travel**, continued

**travel** itself:

```
travel s
    | disp == (0,0) = "Got home"
    | otherwise = "Got lost"
   where
     makeTuples [] = []
     makeTuples (c:cs) = mapMove c : makeTuples cs

     tuples = makeTuples s
     disp = sumTuples tuples
```

As is, **mapMove** and **sumTuples** (previous slide) are at the top level but **makeTuples** is hidden inside **travel**. How should they be arranged?

# Sidebar: top-level vs. hidden functions

```
travel s
   | disp == (0,0) = "Got home"
   | otherwise = "Got lost"
  where
    tuples = makeTuples s
    disp = sumTuples tuples

    makeTuples [] = []
    makeTuples (c:cs) =
        mapMove c:makeTuples cs

    mapMove 'n' = (0,1)
    mapMove 's' = (0,-1)
    mapMove 'e' = (1,0)
    mapMove 'w' = (-1,0)

    sumTuples [] = (0,0)
    sumTuples ((x,y):ts) = (x + sumX, y + sumY)
        where
          (sumX, sumY) = sumTuples ts
```

Top-level functions can be tested after code is loaded but functions inside a **where** block are not visible.

The functions at left are hidden in the **where** block but they can easily be changed to top-level using a shift or two with an editor.

New lines for **mapMove** and **sumTuples** not shown. (Lazy!)

# Real world problem: Planning a course

Here's an early question when planning a course:
"How many lectures will there be?"

How should we answer that question?

Write a Haskell program!

But maybe that's what only a maniac would do!

Should we Google for a course planning app instead?

# classdays

One approach:

```
> classdays ...arguments...
#1 H 1/15
#2 T 1/20
#3 H 1/22
#4 T 1/27
#5 H 1/29
...
```

What information do the arguments need to specify?

- First and last day
- Pattern, like M-W-F or T-H
- How about holidays?

# Arguments for **classdays**

Let's start with something simple:

```
> classdays (1,15) (5,6) [('H',5),('T',2)]
#1 H 1/15
#2 T 1/20
#3 H 1/22
#4 T 1/27
#5 H 1/29
...
```

The first and last days are represented with (*month*,*day*) tuples.

The third argument shows the pattern of class days: the first is a Thursday, and it's five days to the next class.

# Date handling

There's a **Data.Time.Calendar** module but writing two minimal date handling functions provides good practice.

```
> toOrdinal (12,31)
365    -- 12/31 is the last day of the year

> fromOrdinal 32
(2,1)  -- The 32nd day of the year is February 1.
```

What's a minimal data structure that could help us?

```
[(0,0),(1,31),(2,59),(3,90),(4,120),(5,151),(6,181),
(7,212),(8,243),(9,273),(10,304),(11,334),(12,365)]
```

(1,31) *The last day in January is the 31st day of the year*

(7,212) *The last day in July is the 212th day of the year*

# toOrdinal and fromOrdinal

```
offsets = [(0,0),(1,31),(2,59),(3,90),(4,120),(5,151),(6,181),
(7,212),(8,243),(9,273),(10,304),(11,334),(12,365)]

toOrdinal (month, day) = days + day
  where
    (_,days) = offsets!!(month-1)

fromOrdinal ordDay =
    fromOrdinal' (reverse offsets) ordDay
  where
    fromOrdinal' ((month,lastDay):t) ordDay
      | ordDay > lastDay = (month + 1, ordDay - lastDay)
      | otherwise = fromOrdinal' t ordDay
```

```
> toOrdinal (12,31)
365
```

```
> fromOrdinal 32
(2,1)
```

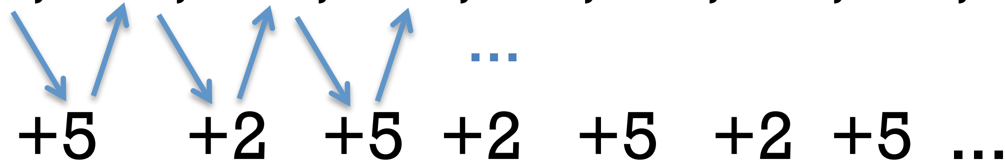
Recall:

```
> classdays (1,15) (5,6) [('H',5),('T',2)]
#1 H 1/15
#2 T 1/20
...
```

Ordinals for **(1,15)** and **(5,6)** are **15** and **126**, respectively.

With the Thursday-Tuesday pattern we'd see the dates progressing like this:

**15, 20, 22, 27, 29, 34, 36, 41, ...**



Imagine this series of calls to a helper, **classdays**':

```
classdays' 1 15  126 [('H',5),('T',2)]
classdays' 2  20 126 [('T',2),('H',5)]
classdays' 3  22 126 [('H',5),('T',2)]
classdays' 4  27  126 [('T',2),('H',5)]
...
classdays' 32 125 126 [('T',2),('H',5)]
classdays' 33 127 126 [('H',5),('T',2)]
```

Desired output:

```
#1 H 1/15
#2 T 1/20
#3 H 1/22
#4 T 1/27
...
#32 T 5/5
```

*(none!)*

What computations do we need to transform

```
classdays' 1 15  126 [('H',5),('T',2)]
```

into

"#1 H 1/15"?

We have: classdays' 1 15  126 [('H',5),('T',2)]

We want: "#1 H 1/15"

1 is lecture #1; 15 is 15th day of year

A handy function: show :: Show a => a -> String

```
> show 123
"123"
```

Let's write showOrdinal :: Integer -> [Char]

```
> showOrdinal 15
"1/15"
```

```
showOrdinal ordDay = show month ++ "/" ++ show day
   where
      (month,day) = fromOrdinal ordDay
```

We have: **classdays' 1 15 126 [('H',5),('T',2)]**
We want: "#1 H 1/15"
We wrote:

```
> showOrdinal 15
"1/15"
```

Now we're ready for a first version of **classdays'**:

```
classdays'
    lecNum first last ((dayOfWeek, daysToNext):_) =
      "#" ++ show lecNum ++ " " ++ [dayOfWeek] ++
      " " ++ showOrdinal first ++ "\n"
```

Usage:

```
> classdays' 1 15 126 [('H',5),('T',2)]
"#1 H 1/15\n"
> classdays' 32 125 126 [('T',2),('H',5)]
"#32 T 5/5\n"
```

Recall:

```
classdays'  1   15  126 [('H',5),('T',2)]
classdays'  2   20  126 [('T',2),('H',5)]
...
classdays' 32 125 126 [('T',2),('H',5)]
classdays' 33 127 126 [('H',5),('T',2)]
```

Desired output:

```
#1 H 1/15
#2 T 1/20
...
#32 T 5/5
```

*(none!)*

Let's "cons up" list out of the results of those calls...

```
> classdays' 1 15  126 [('H',5),('T',2)] :
    classdays' 2 20  126 [('T',2),('H',2)] :
        "...MORE..." : -- I literally typed  "...MORE..."
    classdays' 32 125 126 [('T',2),('H',5)] :
    classdays' 33 127 126 [('H',5),('T',2)] : []

["#1 H 1/15\n","#2 T 1/20\n","...MORE...","#32 T
5/5\n","#33 H 5/7\n"]
```

How close are the contents of that list to what we need?

At hand:

```
> classdays' 1 15  126 [('H',5),('T',2)] :
   classdays' 2 20  126 [('T',2),('H',5)] :
       "...MORE..." : -- I literally typed "...MORE..."
   classdays' 32 125 126 [('T',2),('H',5)] :
   classdays' 33 127 126 [('H',5),('T',2)] : []

["#1 H 1/15\n","#2 T 1/20\n","...MORE...","#32 T
5/5\n","#33 H 5/7\n"]
```

Now we're ready to write a recursive **classdays'**:

```
classdays'
 lecNum first last ((dayOfWeek, daysToNext):pairs)
   | first > last = []
   | otherwise = ("#" ++ show lecNum ++ " " ++
         [dayOfWeek] ++ " " ++ showOrdinal first ++ "\n")
     : classdays' (lecNum+1) (first+daysToNext) last pairs
```

At hand:

```
classdays' lecNum first last
                        ((dayOfWeek, daysToNext):pairs)
    | first > last = []
    | otherwise =
        ("#" ++ show lecNum ++ " " ++  [dayOfWeek]
         ++ " " ++ showOrdinal first ++ "\n")
      :
        classdays'
            (lecNum+1) (first+daysToNext) last pairs
```

Let's try it:

```
> classdays' 1 15  126 [('H',5),('T',2)]
["#1 H 1/15\n","#2 T 1/20\n"
*** Exception: Non-exhaustive patterns in function
classdays'
```

What's the problem?

```
> classdays' 1 15  126 [('H',5),('T',2)]
["#1 H 1/15\n","#2 T 1/20\n"
*** Exception: Non-exhaustive patterns ...

classdays' lecNum first last
                          ((dayOfWeek, daysToNext):pairs)
    | first > last = []
    | otherwise =
        (...format an entry like "#1 H 1/15"...)
      :  classdays'
            (lecNum+1) (first+daysToNext) last pairs
```

We ran out of pairs in [('H',5),('T',2)]! Ideas?

Just reverse [('H',5),('T',2)] each time instead of consuming it?

What about a MWF schedule? [('M',2),('W',2),('F',3)]

How about supplying more pairs?

```
> classdays' 1 15  126 [('H',5),('T',2),('H',5),('T',2)]
["#1 H 1/15\n","#2 T 1/20\n","#3 H 1/22\n",
"#4 T 1/27\n"
*** Exception: Non-exhaustive patterns
```

Would work if given enough pairs, but silly!  Ideas?

```
> :t cycle
cycle :: [a] -> [a]

> cycle [('H',5),('T',2)]
[('H',5),('T',2),('H',5),('T',2),('H',5),('T',2),('H',5),('T',2),('H',
5),('T',2),('H',5),('T',2),('H',5),('T',2),('H',5),('T',2),('H',5),('T',
2),('H',5),('T',2),('H',5),('T',2),('H',5),('T',2),('H',5),('T',2),('H',
5),('T',2),('H',5),...check Words with Friends...^C
```

**cycle** produces a supply of pairs that will <u>never</u> run out!

Let's replace the finite two-tuple list with a list of tuples that infinitely repeats!

```
> classdays' 1 15  126 (cycle [('H',5),('T',2)])
["#1 H 1/15\n","#2 T 1/20\n","#3 H 1/22\n",
    ...MORE...,
 "#30 T 4/28\n","#31 H 4/30\n","#32 T 5/5\n"]
```

Look!  A very practical use of an infinite list!

How would we handle it in Java?

# **classdays**—Final answer

```
classdays first last pattern = putStr (concat result)
  where
    result = classdays'
      1 (toOrdinal first) (toOrdinal last) (cycle pattern)
```

```
> classdays (1,15) (5,6) [('H',5),('T',2)]
#1 H 1/15
#2 T 1/20
#3 H 1/22
...
#31 H 4/30
#32 T 5/5
```

*(last line removed after copies)*

# tally

Consider a function **tally** that counts character occurrences in a string:

```
> tally "a bean bag"
a 3
b 2
  2
g 1
n 1
e 1
```

Note that the characters are shown in order of decreasing frequency.

How can this problem be approached?

In a nutshell: [('a',3),('b',2),(' ',2),('g',1),('n',1),('e',1)]

```
{-
    incEntry c tups

    tups is a list of (Char, Int) tuples that indicate how many
    times a character has been seen.

    incEntry produces a copy of tups with the count in the
    tuple containing the character c incremented by one.

    If no tuple with c exists, one is created with a count of 1.
-}

incEntry::Char -> [(Char,Int)] -> [(Char,Int)]
incEntry c [ ] = [(c, 1)]
incEntry c ((char, count):entries)
  | c == char = (char, count+1) : entries
  | otherwise = (char, count) : incEntry c entries
```

[('a',3),('b',2),(' ',2),('g',1),('n',1),('e',1)]

## **tally**, continued

Calls to **incEntry** with **'t', 'o', 'o':**

```
> incEntry 't' []
[('t',1)]

> incEntry 'o' it
[('t',1),('o',1)]

> incEntry 'o' it
[('t',1),('o',2)]
```

```
-- mkentries s calls incEntry for each character
--    in the string s

mkentries :: [Char] -> [(Char, Int)]
mkentries s = mkentries' s []
   where
      mkentries' [ ] entries = entries
      mkentries' (c:cs) entries =
         mkentries' cs (incEntry c entries)

> mkentries "tupple"
[('t',1),('u',1),('p',2),('l',1),('e',1)]

> mkentries "cocoon"
[('c',2),('o',3),('n',1)]
```

```
{- insert, isOrdered, and sort  provide an insertion sort -}
insert v [ ] = [v]
insert v (x:xs)
    | isOrdered (v,x) = v:x:xs
    | otherwise =  x:insert v xs

isOrdered ((_, v1), (_, v2)) = v1 > v2

sort [] = []
sort (x:xs) = insert x (sort xs)

> mkentries "cocoon"
[('c',2),('o',3),('n',1)]

> sort it
[('o',3),('c',2),('n',1)]
```

## tally, continued

```
{- fmt_entries prints (Char, Int) tuples one per line -}
fmt_entries [] = ""
fmt_entries ((c, count):es) =
    [c] ++ " " ++ (show count) ++ "\n" ++ fmt_entries es

{- grand finale -}
tally s = putStr (fmt_entries (sort (mkentries s)))
```

```
> tally "cocoon"
o 3
c 2
n 1
```

- How does this solution exemplify functional programming? (slide 23)
- How is it like imperative programming?
- How is it like procedural programming (s. 5)

# Running **tally** from the command line

Let's run it on lectura...

```
% code=/cs/www/classes/cs372/spring15/haskell

% cat $code/tally.hs
```

*... everything we've seen before and now a main:*

```
main = do
   bytes <- getContents  -- reads all of standard input
   tally bytes

% echo -n cocoon | runghc $code/tally.hs
o 3
c 2
n 1
```

# **tally** from the command line, continued

**$code/genchars N** generates N random letters:

```
% $code/genchars 20
KVQaVPEmClHRbgdkmMsQ
```

Lets tally a million characters:

```
% $code/genchars 1000000 |
        time runghc $code/tally.hs >out
21.79user 0.24system 0:22.06elapsed
% head -3 out
s  19553
V 19448
 J  19437
```

# **tally** from the command line, continued

Let's try a compiled executable.

```
% ghc --make -rtsopts tally.hs

% ls -l tally
-rwxrwxr-x 1 whm whm 1118828 Feb  1 22:41 tally

% $code/genchars 1000000 |
    time ./tally +RTS -K40000000 -RTS >out
7.44user 0.29system 0:07.82elapsed 98%CPU
```

Speculate: How fast would a Java version of **tally** run? C? Python? Ruby?

# Errors

# Syntax errors

What syntax errors do you see in the following file?

```
% cat synerrors.hs
let f x =
    | x < 0 == y + 10
    | x != 0 = y + 20
    otherwise = y + 30
  where
       g x:xs = x
       y =
       g [x] + 5
      g2 x = 10
```

# Syntax errors, continued

What syntax errors do you see in the following file?

```
% cat synerrors.hs
let f x =
    | x < 0 == y + 10
    | x != 0 = y + 20
    otherwise = y + 30
  where
      g x:xs = x
      y =
      g [x] + 5
     g2 x = 10
```

no **let** before functions in files

no = before guards

=, not == before result

use /= for inequality

missing | before **otherwise**

Needs parens: **(x:xs)**

continuation should be indented

violates layout rule (a.k.a. off-side rule)

# Syntax errors, continued

Line and column information is included in syntax errors.

```
% cat synerror2.hs
weather temp  | temp >= 80 = "Hot!"
              | temp >= 70   "Nice"
              | otherwise = "Cold!"

% ghci synerror2.hs
...
[1 of 1] Compiling Main ( synerror2.hs, interpreted )

synerror2.hs:3:14: parse error on input `|'
```

3:14 indicates an error has been detected at line 3, column 14.

What's the error?

# Type errors

If only concrete types are involved, type errors are typically easy to understand.

```
> chr 'x'
<interactive>:9:5:
    Couldn't match expected type `Int' with actual
     type `Char'
    In the first argument of `chr', namely 'x'
    In the expression: chr 'x'
    In an equation for `it': it = chr 'x'

> :type chr
chr :: Int -> Char
```

# Type errors, continued

Code:

```
countEO (x:xs)
    | odd x = (evens, odds+1)
    | otherwise = (evens+1, odds)
   where (evens,odds) = countEO
```

What's the error?

```
Couldn't match expected type `(t3, t4)'
            with actual type `[t0] -> (t1, t2)'
   In the expression: countEO
   In a pattern binding: (evens, odds) = countEO
```

What's the problem?

It's expecting a tuple, (t3,t4) but it's getting a function, [t0] -> (t1, t2)

# Type errors, continued

Disregard! Fixed by **Text.Show.Functions!**

How about this one?

```
> length
  No instance for (Show ([a0] -> Int)) arising from a
    use of `print'
   Possible fix: add an instance declaration for
    (Show ([a0] -> Int))
   In a stmt of an interactive GHCi command: print it

> :type print
print :: Show a => a -> IO ()
```

Typing an expression at the **ghci** prompt causes it to be evaluated and **print** called with the result. The (trivial) result here is a function, and functions aren't in the **Show** type class.

# Type errors, continued

Code and error:

```
f x y
    | x == 0 = []
    | otherwise = f x
```

```
Couldn't match expected type `[a1]' with actual type
`t0 -> [a1]'
  In the return type of a call of `f'
  Probable cause: `f' is applied to too few arguments
  In the expression: f x
```

The error message is perfect in this case but in general note that an unexpected actual type that's a function suggests too few arguments are being supplied for some function.

# Type errors, continued

Is there an error in the following?

```
f [] = []
f [x] = x
f (x:xs) = x : f xs
```

```
Occurs check: cannot construct the infinite
  type: a0 = [a0]
 In the first argument of `(:)', namely `x'
 In the expression: x : f xs
 In an equation for `f': f (x : xs) = x : f xs
```

*("a0 is a list of a0s"--whm)*

Without the second pattern, it turns into an identity function on lists:
f [1,2,3] == [1,2,3]

What's the problem?

Technique: Comment out cases to find the troublemaker.

# Type errors, continued

What's happening here?

```
> :type ord
ord :: Char -> Int

> ord 5
<interactive>:2:5:
   No instance for (Num Char) arising from the
    literal `5'
   Possible fix: add an instance declaration for
    (Num Char)
```

Why does that error cite **(Num Char)**? It seems to be saying that if **Char** were in the **Num** type class the expression would be valid.

**Note: next set of slides!**

Take a break?

# Higher-order functions

# Functions as values

A fundamental characteristic of a functional language: functions are values that can be used as flexibly as values of other types.

This **let** creates a function value and binds the name **add** to it.

```
> let add x y = x + y
```

add

...code...

This **let** binds the name plus to the value of **add**, whatever it is.

```
> let plus = add
```

add, plus

...code...

Either of the names can be used to reference the function value:

```
> add 3 4
7
> plus 5 6
11
```

# Functions as values, continued

Can functions be compared?

```
> add == plus

<interactive>:25:5:
  No instance for (Eq (Integer -> Integer -> Integer))
    arising from a use of `=='
In the expression: add == plus
```

# Functions as values, continued

Line by line, what are the following expressions doing?

```
> let fs = [head, last]

> fs
[<function>,<function>]

> let ints = [1..10]

> head fs ints
1

> (fs!!1) ints
10
```

## Functions as values, continued

Is the following valid?

```
> [take, tail, init]
Couldn't match type `[a2]' with `Int'
  Expected type: Int -> [a0] -> [a0]
      Actual type: [a2] -> [a2]
  In the expression: init
```

What's the problem?

**take** does not have the same type as **tail** and **init**.

Puzzle: Make **[take, tail, init]** valid by adding two characters.

```
> [take 5, tail, init]
[<function>,<function>,<function>]
```

# A simple *higher-order function*

Definition: A *higher-order function* is a function that has one or more arguments that are functions.

**twice** is a higher-order function with two arguments: **f** and **x**

```
twice f x = f (f x)
```

What does it do?

```
> twice tail [1,2,3,4,5]
[3,4,5]

> tail (tail [1,2,3,4,5])
[3,4,5]
```

# twice, continued

At hand:

```
> let twice f x = f (f x)
> twice tail [1,2,3,4,5]
[3,4,5]
```

Let's make the precedence explicit:

```
> ((twice tail) [1,2,3,4,5])
[3,4,5]
```

Consider a partial application...

```
> let t2 = twice tail    -- like let t2 x = tail (tail x)
> t2
<function>
it :: [a] -> [a]
```

# **twice**, continued

At hand:

```
> let twice f x = f (f x)
> twice tail [1,2,3,4,5]
[3,4,5]
```

Let's **give** twice a partial application!

```
> twice (drop 2) [1..5]
[5]
```

Let's make a partial application with a partial application!

```
> twice (drop 5)
<function>
> it ['a'..'z']
"klmnopqrstuvwxyz"
```

Try these!

```
twice (twice (drop 3)) [1..20]
twice (twice (take 3)) [1..20]
```

# **twice**, continued

At hand:

```
twice f x = f (f x)
```

A *higher-order function* is a function that has one or more arguments that are functions.

What's the the type of **twice**?

```
> :t twice
twice :: (t -> t) -> t -> t
```

Parentheses added to show precedence:

```
twice :: (t -> t) -> (t -> t)
```

```
twice f  x  =  f (f x)
```

What's the correspondence between the elements of the clause and the elements of the type?

# The Prelude's **map** function

Recall **double x = x * 2**

**map** is a Prelude function that applies a function to each element of a list, producing a new list:

```
> map double [1..5]
[2,4,6,8,10]

> map length (words "a few words")
[1,3,5]

> map head (words "a few words")
"afw"
```

Is **map** a higher order function?

# map, continued

At hand:

```
> map double [1..5]
[2,4,6,8,10]
```

Write it!

```
map _ [] = []
map f (x:xs) = f x : map f xs
```

What is its type?

```
map :: (t -> a) -> [t] -> [a]
```

What's the relationship between the length of the input and output lists?

# map, continued

Mapping (via map) is applying a transformation (a function) to each of the values in a list, producing a new list of the same length.

```
> map chr [97,32,98,105,103,32,99,97,116]
"a big cat"

> map isLetter it
[True,False,True,True,True,False,True,True,True]

> map not it
[False,True,False,False,False,True,False,False,False]

> map head (map show it) -- Note: show True is "True"
"FTFFFTFFF"
```

## Sidebar: **map** can go parallel

Here's another map:

```
> map weather [85,55,75]
["Hot!","Cold!","Nice"]
```

This is equivalent:

```
> [weather 85, weather 55, weather 75]
["Hot!","Cold!","Nice"]
```

Because functions have no side effects, we can immediately turn a mapping into a parallel computation. We might start each function call on a separate processor and combine the values when all are done.

# map and partial applications

What's the result of these?

```
> map (add 5) [1..10]
[6,7,8,9,10,11,12,13,14,15]

> map (drop 1) (words "the knot was cold")
["he","not","as","old"]

> map (replicate 5) "abc"
["aaaaa","bbbbb","ccccc"]
```

# map and partial applications, cont.

What's going on here?

```
> let f = map double
> f [1..5]
[2,4,6,8,10]

> map f [[1..3],[10..15]]
[[2,4,6],[20,22,24,26,28,30]]
```

Here's the above in one step:

```
> map (map double) [[1..3],[10..15]]
[[2,4,6],[20,22,24,26,28,30]]
```

Here's one way to think about it:

[<u>(map double)</u> [1..3], <u>(map double)</u> [10..15]]

# Now that we're good at recursion...

Some of the problems on the next assignment will encourage working with higher-order functions by prohibiting recursion!

Think of it as isolating muscle groups when weight training.

Here's a simple way to avoid what's prohibited:

*Pretend that you no longer understand recursion!*

*What's a base case? Is it related to baseball?*

*Why would a function call itself? How's it stop?*

*Is a recursive plunge refreshing?*

If you were UNIX machines, I'd do **`chmod 0`** on an appropriate section of your brains.

## travel revisited

Recall our traveling robot: (slide 168)

```
> travel "nnee"
"Got lost"

> travel "nnss"
"Got home"
```

Recall our approach:

Argument value: "nnee"

Mapped to tuples: (0,1) (0,1) (1,0) (1,0)

Sum of tuples: (2,2)

How can we solve it non-recursively?

## **travel**, continued

Recall:

```
> :t mapMove
mapMove :: Char -> (Int, Int)

> mapMove 'n'
(0,1)
```

Now what?

```
> map mapMove "nneen"
[(0,1),(0,1),(1,0),(1,0),(0,1)]
```

Can we sum them with **map**?

# **travel**, continued

We have:

```
> let disps= map mapMove "nneen"
[(0,1),(0,1),(1,0),(1,0),(0,1)]
```

We want: (2,3)

Any ideas?

```
> :t fst
fst :: (a, b) -> a

> map fst disps
[0,0,1,1,0]

> map snd disps
[1,1,0,0,1]
```

# **travel**, revisited

We have:

```
> let disps= map mapMove "nneen"
[(0,1),(0,1),(1,0),(1,0),(0,1)]
> map fst disps
[0,0,1,1,0]
> map snd disps
[1,1,0,0,1]
```

We want: (2,3)

Ideas?

```
> :t sum
sum :: Num a => [a] -> a

> (sum (map fst disps), sum (map snd disps))
(2,3)
```

## **travel**—Final answer

```
travel :: [Char] -> [Char]
travel s
   | totalDisp == (0,0) = "Got home"
   | otherwise = "Got lost"
  where
    disps = map mapMove s
    totalDisp = (sum (map fst disps),
                 sum (map snd disps))
```

Did we have to understand recursion to write this?

A peek ahead:

```
> disps
[(0,1),(0,1),(1,0),(1,0),(0,1)]

> foldr (\(x,y) (ax,ay) -> (x+ax,y+ay)) (0,0) disps
(2,3)
```

## Sidebar: "sections"

Instead of using **map (add 5)** to add 5 to the values in a list, we should use a ***section*** instead: (it's the idiomatic way!)

```
> map (5+) [1,2,3]
[6,7,8]      -- [5+ 1, 5+ 2, 5+ 3]
```

More sections:

```
> map (10*) [1,2,3]
[10,20,30]

> map (++"*") (words "a few words")
["a*","few*","words*"]

> map ("*"++) (words "a few words")
["*a","*few","*words"]
```

# "sections", continued

Sections have one of two forms:

(*infix-operator value*)  Examples: (+5), (/10)

(*value infix-operator*)  Examples: (5*), ("x"++)

<u>Iff</u> the operator is commutative, the two forms are equivalent.

```
> map (3<=) [1..4]
[False,False,True,True]
```

[3 <= 1, 3 <= 2, 3 <= 3, 3 <= 4]

```
> map (<=3) [1..4]
[True,True,True,False]
```

[1 <= 3, 2 <= 3, 3 <= 3, 4 <= 4]

Sections aren't just for **map**; they're a general mechanism.

```
> twice (+5) 3
13
```

# Filtering

Another higher order function in the Prelude is **filter**:

```
> filter odd [1..10]
[1,3,5,7,9]

> filter isDigit "(800) 555-1212"
"8005551212"
```

What's **filter** doing?

What is the type of **filter**?

```
filter :: (a -> Bool) -> [a] -> [a]
```

# filter, continued

More...

```
>  filter (<= 5) (filter odd [1..10])
[1,3,5]

> map (filter isDigit) ["br549", "24/7"]
["549","247"]

> filter (`elem` "aeiou") "some words here"
"oeoee"
```

*Note that (`elem` ...) is a section!*

```
elem :: Eq a => a -> [a] -> Bool
```

# filter, continued

At hand:

```
> filter odd [1..10]
[1,3,5,7,9]

> :t filter
filter :: (a -> Bool) -> [a] -> [a]
```

Let's write **filter**!

```
myfilter _ [] = []
myfilter f (x:xs)
   | f x = x : filteredTail
   | otherwise = filteredTail
  where
   filteredTail = myfilter f xs
```

## **filter** uses a *predicate*

**filter**'s first argument (a function) is called a *predicate* because inclusion of each value is predicated on the result of calling that function with that value.

Several Prelude functions use predicates. Here are two:

```
all :: (a -> Bool) -> [a] -> Bool
> all even [2,4,6,8]
True
> all even [2,4,6,7]
False

dropWhile :: (a -> Bool) -> [a] -> [a]
> dropWhile isSpace "  testing  "
"testing  "
> dropWhile isLetter it
"  "
```

# map vs. filter

For reference:

```
> map double [1..10]
[2,4,6,8,10,12,14,16,18,20]

> filter odd [1..10]
[1,3,5,7,9]
```

map:

transforms values

length *input* == length *output*

filter:

selects values

0 <= length *output* <= length *input*

# Anonymous functions

We can map a section to double the numbers in a list:

```
> map (*2) [1..5]
[2,4,6,8,10]
```

Alternatively we could use an *anonymous function*:

```
> map (\x -> x * 2) [1..5]
[2,4,6,8,10]
```

What are things we can do with an anonymous function that we can't do with a section?

```
> map (\n -> n * 3 + 7) [1..5]
[10,13,16,19,22]

> filter (\x -> head x == last x) (words "pop top suds")
["pop","suds"]
```

# Anonymous functions, continued

The general form:

```
\ pattern1 ... patternN -> expression
```

Simple syntax suggestion: enclose the whole works in parentheses.

```
map (\x -> x * 2) [1..5]
```

The typical use case for an anonymous function is a single instance of supplying a higher order function with a computation that can't be expressed with a section or partial application.

Anonymous functions are also called *lambdas*, *lambda expressions*, and *lambda abstractions*.

The \ character was chosen due to its similarity to λ, used in Lambda calculus, another system for expressing computation.

Take a break?

# Example: longest line(s) in a file

Imagine a program to print the longest line(s) in a file, along with their line numbers:

```
% runghc longest.hs /usr/share/dict/web2
72632:formaldehydesulphoxylate
140339:pathologicopsychological
175108:scientificophilosophical
200796:tetraiodophenolphthalein
203042:thyroparathyroidectomize
```

What are some ways in which we could approach it?

# **longest**, continued

Let's work with a shorter file for development testing:

```
% cat longest.1
data
to
test
```

**readFile** in the Prelude returns the full contents of a file as a string:

```
> readFile "longest.1"
"data\nto\ntest\n"
```

To avoid wading into I/O yet, let's focus on a function that operates on a string of characters (the full contents of a file):

```
> longest "data\nto\ntest\n"
"1:data\n3:test\n"
```

# **longest**, continued

Let's work through a series of transformations of the data:

```
> let bytes = "data\nto\ntest\n"

> let lns = lines bytes
["data","to","test"]
```

Note: To save space, values of **let** bindings are being shown immediately after each **let**. E.g., **> lns** is not shown above.

Let's use **zip3** and **map length** to create (length, line-number, line) triples:

```
> let triples = zip3 (map length lns) [1..] lns
[(4,1,"data"),(2,2,"to"),(4,3,"test")]
```

# **longest**, continued

We have (length, line-number, line) triples at hand:

```
> triples
[(4,1,"data"),(2,2,"to"),(4,3,"test")]
```

Let's use **sort :: Ord a => [a] -> [a]** on them:

```
> let sortedTriples = reverse (sort triples)
[(4,3,"test"),(4,1,"data"),(2,2,"to")]
```

Note that by having the line length first, triples are sorted first by line length, with ties resolved by line number.

We use **reverse** to get a descending order.

If line length weren't first, we'd instead use

```
Data.List.sortBy :: (a -> a -> Ordering) -> [a] -> [a]
```

and supply a function that returns an **Ordering**.

# **longest**, continued

At hand:

```
> sortedTriples
[(4,3,"test"),(4,1,"data"),(2,2,"to")]
```

We'll handle ties by using **takeWhile** to get all the triples with lines of the maximum length.

Let's use a helper function to get the first element of a 3-tuple:

```
> let first (len, _, _) = len
> let maxLength = first (head sortedTriples)
4
```

**first** will be used in another place but were it not for that we might have used a pattern:

```
let (maxLength,_,_) = head sortedTriples
```

# **longest**, continued

At hand:

```
> sortedTriples
[(4,3,"test"),(4,1,"data"),(2,2,"to")]

> maxLength
4
```

Let's use **takeWhile :: (a -> Bool) -> [a] -> [a]** to get the triples having the maximum length:

```
> let maxTriples = takeWhile
    (\triple -> first triple  == maxLength) sortedTriples
[(4,3,"test"),(4,1,"data")]
```

anonymous function for takeWhile

# longest, continued

At hand:

```
> maxTriples
[(4,3,"test"),(4,1,"data")]
```

Let's map an anonymous function to turn the triples into lines prefixed with their line number:

```
> let linesWithNums =
      map (\(_,num,line) -> show num ++ ":" ++ line)
            maxTriples
["3:test","1:data"]
```

We can now produce a ready-to-print result:

```
> let result = unlines (reverse linesWithNums)
> result
"1:data\n3:test\n"
```

# **longest**, continued

Let's package up our work into a function:

```
longest bytes = result
   where
      lns = lines bytes
      triples = zip3 (map length lns) [1..] lns
      sortedTriples = reverse (sort triples)
      maxLength = first (head sortedTriples)
      maxTriples = takeWhile
          (\triple -> first triple  == maxLength) sortedTriples
      linesWithNums =
           map (\(_,num,line) -> show num ++ ":" ++ line)
           maxTriples
      result = unlines (reverse linesWithNums)

      first (x,_,_) = x
```

# **longest**, continued

At hand:

```
> longest "data\nto\ntest\n"
"1:data\n3:test\n"
```

Let's add a **main** that handles command-line args and does I/O:

```
% cat longest.hs
import System.Environment (getArgs)
import Data.List (sort)

longest bytes = ...from previous slide...

main = do
    args <- getArgs  -- Get command line args as list
    bytes <- readFile (head args)
    putStr (longest bytes)
```

Execution:

```
$ runghc longest.hs /usr/share/dict/words
39886:electroencephalograph's
```

# Function composition

Given two functions **f** and **g**, the composition of **f** and **g** is a function **c** that for all values of **x**, **(c x)** equals **(f (g x))**

Here is a primitive **compose** function that applies two functions in turn:

```
> let compose f g x = f (g x)
```

Its type: (How many arguments?)

```
(b -> c) -> (a -> b) -> a -> c

> compose init tail [1..5]
[2,3,4]

> compose signum negate 3
-1
```

# Composition, continued

Haskell has a function composition operator. It is a dot (.)

```
> :t (.)
(.) :: (b -> c) -> (a -> b) -> a -> c
```

Its two operands are functions, and its result is a function.

```
> let numwords = length  .  words

> numwords "just testing this"
3
```

# Composition, continued

Problem: Using composition create a function that returns the next-to-last element in a list:

```
> ntl [1..5]
4

> ntl "abc"
'b'
```

Solution:

```
> let ntl = head . tail . reverse
```

Another?

```
> let ntl = head . reverse . init
```

# Composition, continued

Problem: Create a function to remove the digits from a string:

```
> rmdigits "Thu Feb  6 19:13:34 MST 2014"
"Thu Feb   :: MST "
```

Solution:

```
> let rmdigits = filter (not . isDigit)
```

Given the following, describe **f**:

```
> let f = (*2) . (+3)

> map f [1..5]
[8,10,12,14,16]
```

Would an anonymous function be a better choice?

# Composition, continued

Given the following, what's the type of **numwords**?

```
> :type words
words :: String -> [String]

> :type length
length :: [a] -> Int

> let numwords = length . words
```

Type:

```
numwords :: String -> Int
```

<u>Assuming a composition is valid, the type is based only on the input of the rightmost function and the output of the leftmost function.</u>

```
(.) :: (b -> c) -> (a -> b) -> a -> c
```

# Point-free style

Recall **rmdigits**:

```
> rmdigits "Thu Feb  6 19:13:34 MST 2014"
"Thu Feb   :: MST "
```

What the difference between these two bindings for **rmdigits**?

```
rmdigits s = filter (not . isDigit) s

rmdigits  = filter (not . isDigit)
```

The latter declaration is in *point-free style.* (Look, no **s**s!)

A point-free binding of a function **f** has NO parameters!

Is the following a point-free function binding or a partial application?

```
t5 = take 5
```

# Point-free style, continued

Problem: Using point-free style, bind **len** to a function that works like the Prelude's **length**.

Hint:

```
> :t const
const :: a -> b -> a

> const 10 20
10

> const [1] "foo"
[1]
```

Solution:

```
len = sum . map (const 1)
```

See also: *Tacit programming* on Wikipedia

# Hocus pocus with higher-order functions

# Mystery function

What's this function doing?

```
f  a = g
   where
      g  b = a + b
```

Type?

```
f :: Num a => a -> a -> a
```

Interaction:

```
> let f ' = f 10
> f ' 20
30

> f 3 4
7
```

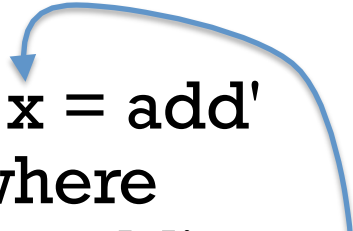
# DIY Currying

Consider this claim:

A function definition in curried form, which is idiomatic in Haskell, is really just syntactic sugar.

Compare these two <u>completely equivalent</u> declarations for **add**:

```
add x y = x + y
```



```
add x = add'
    where
        add' y = x + y
```

A language construct that makes something easier to express but doesn't add a new capability is called *syntactic sugar*.

The result of the call **add 5** is essentially this function:

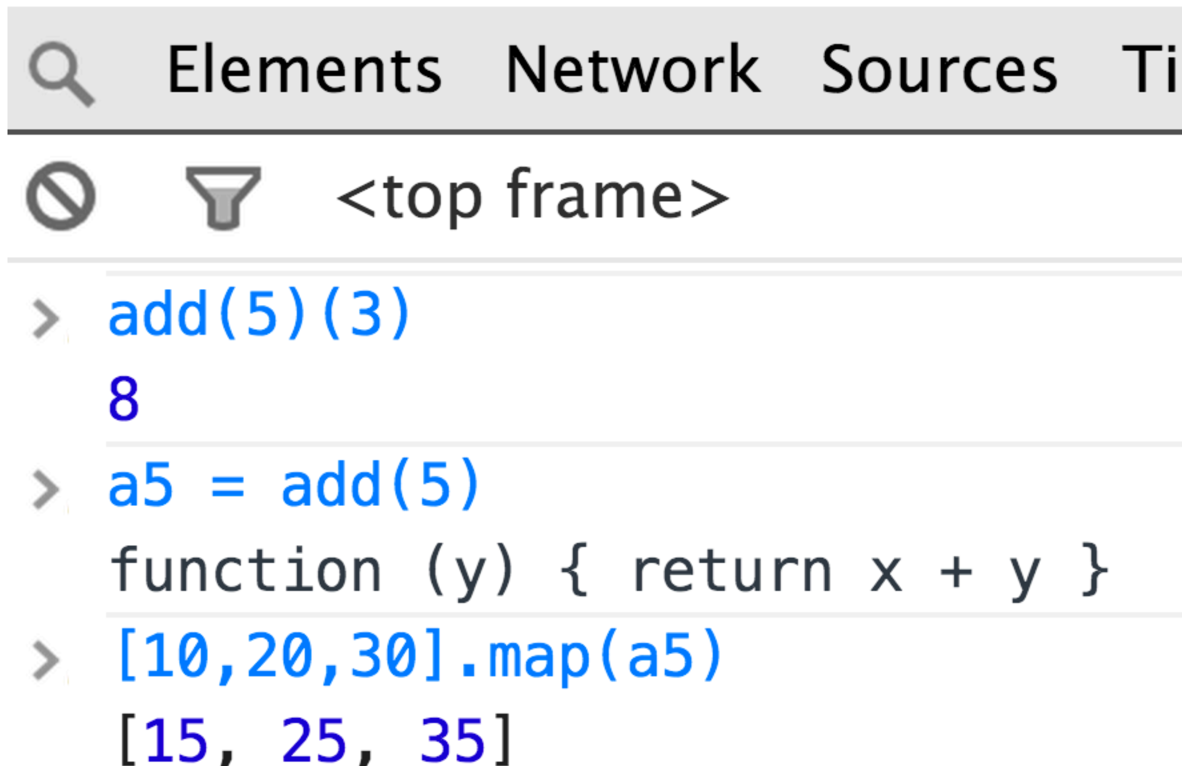
```
add' y = 5 + y
```

The combination of the code for **add'** and the binding for **x** is known as a *closure*. It contains what's needed for execution.

# DIY currying in JavaScript

JavaScript doesn't provide the syntactic sugar of curried function definitions but we can do this:

```
function add(x) {
    return function (y) { return x + y }
}
```




Try it in Chrome!

View>Developer>JavaScript Console brings up the console.

Type in the code for add on one line.

# DIY currying in Python

```
>>> def add(x):
...         return lambda y: x + y
...

>>> f = add(5)

>>> type(f)
<type 'function'>

>>> map(f, [10,20,30])
[15, 25, 35]
```

# Another mystery function

Here's another mystery function:

```
> let m f x y = f y x

> :type m
m :: (t1 -> t2 -> t) -> t2 -> t1 -> t
```

Can you devise a call to `m`?

```
> m add 3 4
7

> m (++) "a" "b"
"ba"
```

What is `m` doing? What could `m` be useful for?

# flip

At hand:

```
m f x y = f y x
```

m is actually a Prelude function named **flip**:

```
> :t flip
flip :: (a -> b -> c) -> b -> a -> c

> flip take [1..10] 3
[1,2,3]

> let ftake = flip take
> ftake [1..10] 3
[1,2,3]
```

Any ideas on how to use it?

# flip, continued

At hand:

```
flip f x y = f y x

> map (flip take "Haskell") [1..7]
["H","Ha","Has","Hask","Haske","Haskel","Haskell"]
```

Problem: write a function that behaves like this:

```
> f 'a'
["a","aa","aaa","aaaa","aaaaa",...
```

Solution:

```
> let f x = map (flip replicate x) [1..]
```

# **flip**, continued

From assignment 1:

```
> splits "abcd"
[("a","bcd"),("ab","cd"),("abc","d")]
```

Many students have noticed the Prelude's **splitAt**:

```
> splitAt 2 [10,20,30,40]
([10,20],[30,40])
```

Problem: Write **splits** using higher order functions but no explicit recursion.

Solution:

```
splits list = map (flip splitAt list) [1..(length list - 1)]
```

# The $ operator

$ is the "application operator". Note what :info shows:

```
> :info ($)
($) :: (a -> b) -> a -> b
infixr 0 $      -- right associative infix operator with very
                -- low precedence
```

The following declaration of $ uses an infix syntax:

```
f $ x  =  f x        -- Equivalent: ($) f x = f x
```

Usage:

```
> negate $ 3 + 4
-7
```

What's the point of it?

# The `$` operator, continued

`$` is a low precedence, right associative operator that applies a function to a value:

```
f $ x  =  f x
```

Because `+` has higher precedence than `$` the expression

```
negate $ 3 + 4
```

groups like this:

```
negate $ (3 + 4)
```

How does the following expression group?

```
filter (>3) $ map length $ words "up and down"

filter (>3) (map length (words "up and down"))
```

# Currying the uncurried

Problem: We're given a function whose argument is a two-tuple but we wish it were curried so we could use a partial application of it.

```
g :: (Int, Int) -> Int
g (x,y) = x^2 + 3*x*y + 2*y^2

> g (3,4)
77
```

Solution: Curry it with **curry** from the Prelude!

```
> map (curry g 3) [1..10]
[20,35,54,77,104,135,170,209,252,299]
```

Your problem: Write **curry**!

# Currying the uncurried, continued

At hand:

```
> g (3,4)
77
> map (curry g 3) [1..10]
[20,35,54,77,104,135,170,209,252,299]
```

Here's **curry**, and use of it:

```
curry :: ((a, b) -> c) -> (a -> b -> c)
```
*(latter parens added to help)*

```
curry f x y = f (x,y)

> let cg = curry g
> :type cg
cg :: Int -> Int -> Int

> cg 3 4
77
```

# Currying the uncurried, continued

At hand:

```
curry :: ((a, b) -> c) -> (a -> b -> c)
```
*(parentheses added)*

```
curry f x y = f (x, y)

> map (curry g 3) [1..10]
[20,35,54,77,104,135,170,209,252,299]
```

The key: **(curry g 3)** is a partial application of **curry**!

Call: **curry g 3**

Dcl: **curry f x y = f (x, y)**

**= g (3, y)**

# Currying the uncurried, continued

At hand:

**curry :: ((a, b) -> c) -> (a -> b -> c)** *(parentheses added)*
**curry f x y = f (x, y)**

```
> map (curry g 3) [1..10]
[20,35,54,77,104,135,170,209,252,299]
```

Let's get **flip** into the game!

```
> map (flip (curry g) 4) [1..10]
[45,60,77,96,117,140,165,192,221,252]
```

The counterpart of **curry** is **uncurry**:

> ~~uncurry (+) $ (3,4)~~ uncurry (+) (3,4)
7

# A **curry** function for JavaScript

```
function curry(f) {
    return function(x) {
        return function (y) { return f(x,y) }
        }
    }
```

```
Elements  Network  Sources  Timeline  Profiles  Resourc
<top frame>
> function add(x,y) {return x + y}
  undefined
> c_add = curry(add)
  function (x) { return function (y) { return f(x,y) } }
> add_5 = c_add(5)
  function (y) { return f(x,y) }
> [10,20,30].map(add_5)
  [15, 25, 35]
```

**Note: next set of slides!**

QUIZ!

# Folding

# Reduction

We can *reduce* a list by a binary operator by inserting that operator between the elements in the list:

[1,2,3,4] reduced by + is 1 + 2 + 3 + 4

["a","bc", "def"] reduced by ++ is "a" ++ "bc" ++ "def"

Imagine a function **reduce** that does reduction by an operator.

```
> reduce (+) [1,2,3,4]
10

> reduce (++) ["a","bc","def"]
"abcdef"

> reduce max [10,2,4]        -- think of 10 `max` 2 `max` 4
10

> map (reduce max) (permutations [10,2,4])
[10,10,10,10,10,10]      -- permutations is from Data.List
```

# Reduction, continued

At hand:

```
> reduce (+) [1,2,3,4]
10
```

An implementation of **reduce**:

```
reduce _ [] = undefined
reduce _ [x] = x
reduce op (x:xs) = x `op` reduce op xs
```

Does **reduce + [1,2,3,4]** do

```
((1 + 2) + 3) + 4
```

or

```
1 + (2 + (3 + 4))
```

?

In general, when would the grouping matter?

# foldl1 and foldr1

In the Prelude there's no reduce but there is foldl1 and foldr1.

```
> foldl1 (/) [1,2,3]
0.16666666666666666      -- left associative: (1 / 2) / 3

> foldr1 (/) [1,2,3]       -- right associative: 1 / (2 / 3)
1.5
```

Here's the type of foldr1:

```
foldr1 :: (a -> a -> a) -> [a] -> a
```

Here's the type of a related function, foldr (no "1"):

```
foldr :: (a -> b -> b) -> b -> [a] -> b
```

What are the differences between the two?

# foldr1 vs. foldr

For reference:

```
foldr1 :: (a -> a -> a) -> [a] -> a
foldr :: (a -> b -> b) -> b -> [a] -> b
```

Use:

```
> foldr1 (+) [1..4]
10

> foldr (+) 0 [1..4]
10

> foldr (+) 0 []      -- Empty list is exception with foldr1
0
```

# foldr1 vs. foldr, continued

For reference:

```
foldr1 :: (a -> a -> a) -> [a] -> a  -- reduction, like 1+2+3+4
foldr :: (a -> b -> b) -> b -> [a] -> b  -- something different...
```

To aid understanding, here's a folding function written with the names **elem** (element) and **acm** (accumulated value).

```
> foldr (\elem acm -> acm + elem) 0 [1..4]
10
```

initial value for acm

Here's the BIG DEAL with **foldr**: it can fold a list of values into a <u>different</u> type!

```
> foldr (\elem acm -> show elem ++ "." ++ acm) "<" [1..4]
"1.2.3.4.<"   -- IMPORTANT: Numbers in; [Char] out!
```

Another way to think about it: `` 1 `f` (2 `f` (3 `f` (4 `f` "<"))) ``

The folding function

# Folding

Fill in the blank, creating a folding function that can be used to compute the length of a list:

```
> foldr (\ _______________ ) 0 [10,20,30]
3
```

Solution:

```
> let len = foldr (\elem acm -> acm + 1) 0
> len ['a'..'z']
26
```

Problem: Define map in terms of foldr.

```
> let mp f = foldr (\elem acm -> f elem : acm) []
> mp toUpper "test"
"TEST"
```

# Folding, continued

Recall our even/odd counter

```
> countEO [3,4,7,9]
(1,3)
```

Define it terms of foldr!

```
> let eo = foldr (\val (e,o) ->
                  if even val then (e+1,o) else (e,o+1)) (0,0)
> eo [3,4,7,9]
(1,3)

> eo []
(0,0)
```

Strictly FYI: Instead of if/else we could have used Haskell's case:

```
> let eo = myfoldr (\val (e,o) ->
    case even val of {True -> (e+1,o); False -> (e,o+1)}) (0,0)
```

# Folding, continued

Here's a definition for foldr. We're using a type specification with multicharacter type variables to help know which is which:

```
foldr :: (val -> acm -> acm) -> acm -> [val] -> acm
foldr f acm [] = acm
foldr f acm (val:vals) = f val ( foldr f acm vals )
```

When loaded, we see this:

```
> :t foldr
foldr :: (val -> acm -> acm) -> acm -> [val] -> acm

> foldr (\val acm -> acm ++ val) "?" ( words "a test here" )
"?heretesta"
```

IMPORTANT: There's NO connection between the type variable names and the names in functions. We might have done this instead: `foldr (\v a -> a ++ v) ...`

# Folding, continued

Problem: Write reverse in terms of a foldr.

A solution, but with an issue:

```
rv1 = foldr (\val acm -> acm ++ [val]) []
```

The issue: ++ is relatively expensive wrt. cons.

By definition, foldr operates like this:

```
foldr f zero [x1, x2, ..., xn] == x1 `f` (x2 `f` ... (xn `f` zero)...)
```

The first application of f is with the last element and the "zero" value, but the first cons would need to be with the first element of the list.

# Folding, continued

The counterpart of foldr is foldl. Compare their meanings:

```
foldr f zero [x1, x2, ..., xn] == x1 `f` (x2 `f` ... (xn `f` zero)...)

foldl f zero [x1, x2, ..., xn] == (...((zero `f` x1) `f` x2) `f`...)`f` xn
```

Note the "zeros"

Their types, with long type variables:

```
foldr :: (val -> acm -> acm) -> acm -> [val] -> acm

foldl :: (acm -> val -> acm) -> acm -> [val] -> acm
```

Problem: Write reverse in terms of foldl.

```
> let rev = foldl (\acm val -> val:acm) []
> rev "testing"
"gnitset"
```

# Folding, continued

Recall **paired** from assignment 2:

```
> paired "((())())"
True
```

Can we implement **paired** with a fold?

```
counter (-1) _ = -1
counter total '(' = total + 1
counter total ')' = total - 1
counter total _ = total

paired s = foldl counter 0 s == 0
```

Point-free:

```
paired = (0==) . foldl counter 0
```

# Folding, continued

**Data.List.partition** partitions a list based on a predicate:

```
> partition isLetter "Thu Feb 13 16:59:03 MST 2014"
("ThuFebMST","  13 16:59:03  2014")

> partition odd [1..10]
([1,3,5,7,9],[2,4,6,8,10])
```

Write it using a fold!

```
sorter f val (pass, fail) =
   if f val then (val:pass, fail)   -- ML escapee from 2006!
            else (pass, val:fail)

partition f = foldr (sorter f) ([],[])
```

# map vs. filter vs. folding

map:

transforms values

length *input* == length *output*

filter:

selects values

0 <= length *output* <= length *input*

folding

Input: A list of values and an initial value for accumulator

Output: A value of any type and complexity

True or false?

Any operation that processes a list can be expressed in a terms of a fold, perhaps with a simple wrapper.

# We can fold a list of anythings into anything!

Far-fetched folding:

Refrigerators in Gould-Simpson to
((grams fat, grams protein, grams carbs), calories)

Keyboards in Gould-Simpson to
[("a", #), ("b", #), ..., ("@2", #), ("CMD", #)]

[Backpack] to
(# pens, pounds of paper,
[(title, author, [page #s with the word "computer")])

[Furniture]
to a structure of 3D vertices representing a *convex hull* that could hold any single piece of furniture.

**Replacement for slide "Scans"!**

# The challenge of folding

The challenge: Write a function such that **f val acm** can do the work for you. Think about the "zero" value. Imagine a series of calls.

**foldr** does the rest! (For **foldl**, it's **f acm val**.)

```
> foldr (\val acm ->
     if  val `elem` "aeiou" then acm+1 else acm) 0 "ate"
2

> foldr (\val acm@(n, vows) ->
     if  val `elem` "aeiou" then (n+1, val:vows) else acm) (0,[]) "ate"
(2,"ae")

vowelPositions s = reverse result
  where (_,result,_) =
          foldl (\acm@(n, vows,pos) val ->   -- NOTE: now foldl!
             if  val `elem` "aeiou" then (n, (val,pos):vows,pos+1)
                                     else (n,vows,pos+1)) (0,[],0) s

> vowelPositions "Down to Rubyville!"
[('o',1),('o',6),('u',9),('i',13),('e',16)]
```

*scans* are similar to folds but all intermediate values are produced:

```
> scanl (+) 0 [1..5]
[0,1,3,6,10,15]

> let scanEO = scanl (\(e,o) val ->
          if even val then (e+1,o) else (e,o+1)) (0,0)

> scanEO [1,3,5,6,7,9]
[(0,0),(0,1),(0,2),(0,3),(1,3),(1,4),(1,5)]
```

# User-defined types

# A **Shape** type

A new type can be created with a **data** declaration.

Here's a simple **Shape** type whose instances represent circles or rectangles:

```
data Shape =
    Circle Double |            -- just a radius
    Rect Double Double         -- width and height
         deriving Show
```

The shapes have dimensions but no position.

**Circle** and **Rect** are *data constructors*.

"**deriving Show**" declares **Shape** to be an instance of the **Show** type class, so that values can be shown using some simple, default rules.

**Shape** is called an *algebraic type* because instances of **Shape** are built using other types.

# Shape, continued

Instances of Shape are created by calling the data constructors:

```
> let r1 = Rect 3 4
> r1
Rect 3.0 4.0

> let r2 = Rect 5 3

> let c1 = Circle 2

> let shapes = [r1, r2, c1]

> shapes
[Rect 3.0 4.0,Rect 5.0 3.0,Circle 2.0]
```

```
data Shape =
   Circle Double |
   Rect Double Double
      deriving Show
```

Lists must be homogeneous—why are both Rects and Circles allowed in the same list?

# Shape, continued

The data constructors are just functions—we can use all our function-fu with them!

```
data Shape =
   Circle Double |
   Rect Double Double
     deriving Show
```

```
> :t Circle
Circle :: Double -> Shape

> :t Rect
Rect :: Double -> Double -> Shape

> map Circle [2,3] ++ map (Rect 3) [10,20]
[Circle 2.0,Circle 3.0,Rect 3.0 10.0,Rect 3.0 20.0]
```

# Shape, continued

Functions that operate on algebraic types use patterns based on their data constructors.

```
area (Circle r) = r ** 2 * pi
area (Rect w h) = w * h
```

```
data Shape =
   Circle Double |
   Rect Double Double
      deriving Show
```

Usage:

```
> r1
Rect 3.0 4.0

> area r1
12.0

> shapes
[Rect 3.0 4.0,Rect 5.0 3.0,Circle 2.0]

> map area shapes
[12.0,15.0,12.566370614359172]

> sum $ map area shapes
39.56637061435917
```

# **Shape**, continued

Let's make the **Shape** type an instance of the **Eq** type class.

What does **Eq** require?

```
> :info Eq
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool
```

Default definitions from **Eq**:

```
(==) a b = not $ a /= b
(/=)  a b = not $ a == b
```

Let's say that two shapes are equal if their areas are equal. (Iffy!)

```
instance Eq Shape where
   (==) r1 r2 = area r1 == area r2
```

Usage:

```
> Rect 3 4 == Rect 6 2
True

> Rect 3 4 == Circle 2
False
```

## **Shape**, continued

Let's see if we can find the biggest shape:

```
> maximum shapes
 No instance for (Ord Shape) arising from a use of
`maximum'
   Possible fix: add an instance declaration for (Ord
Shape)
```

What's in **Ord**?

```
> :info Ord
class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<) :: a -> a -> Bool
  (>=) :: a -> a -> Bool
  (>) :: a -> a -> Bool
  (<=) :: a -> a -> Bool
  max :: a -> a -> a
  min :: a -> a -> a
```

**Eq a => Ord a** requires would-be **Ord** classes to be instances of **Eq**. (Done!)

Like **==** and **/=** with **Eq**, the operators are implemented in terms of each other.

# Shape, continued

Let's make **Shape** an instance of the **Ord** type class:

```
instance Ord Shape where
    (<) r1 r2 = area r1 < area r2          -- < and <= are sufficient
    (<=) r1 r2 = area r1 <= area r2
```

Usage:

```
> shapes
[Rect 3.0 4.0,Rect 5.0 3.0,Circle 2.0]

> map area shapes
[12.0,15.0,12.566370614359172]

> maximum shapes
Rect 5.0 3.0

> Data.List.sort shapes
[Rect 3.0 4.0,Circle 2.0,Rect 5.0 3.0]
```

Note that we didn't need to write functions like **sumOfAreas** or **largestShape**—we can express those in terms of existing operations

# Shape all in one place

Here's all the **Shape** code: (in **shape.hs**)

```
data Shape =
    Circle Double |
    Rect Double Double
        deriving Show

area (Circle r) = r ** 2 * pi
area (Rect w h) = w * h

instance Eq Shape where
    (==) r1 r2 = area r1 == area r2

instance Ord Shape where
    (<) r1 r2 = area r1 < area r2
    (<=) r1 r2 = area r1 <= area r2
```

What would be needed to add a **Figure8** shape and a **perimeter** function?

How does this compare to a **Shape/Circle/Rect** hierarchy in Java?

# Two simple algebraic types

Let's look at the **compare** function:

```
> :t compare
compare :: Ord a => a -> a -> Ordering
```

**Ordering** is a simple algebraic type, with only three values:

```
> :info Ordering
data Ordering = LT | EQ | GT

> [r1,r2]
[Rect 3.0 4.0,Rect 5.0 3.0]

> compare r1 r2
LT

> compare r2 r1
GT
```

What do you suppose **Bool** really is?

```
> :info Bool
data Bool = False | True
```

# A binary tree

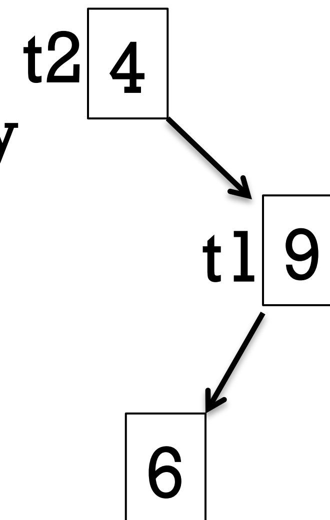
Here's an algebraic type for a binary tree:

```
data Tree a = Node a (Tree a) (Tree a)   -- tree.hs
            | Empty
              deriving Show
```

The **a** is a type variable. Our **Shape** type used **Double** values but **Tree** can hold values of any type!

```
> let t1 = Node 9 (Node 6 Empty Empty) Empty
> t1
Node 9 (Node 6 Empty Empty) Empty

> let t2 = Node 4 Empty t1
> t2
Node 4 Empty (Node 9 (Node 6 Empty Empty) Empty)
```



## Tree, continued

Here's a function that inserts values, maintaining an ordered tree:

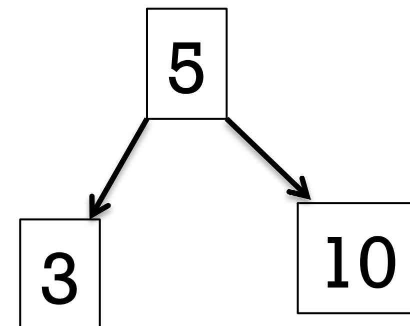
```
insert Empty v = Node v Empty Empty
insert (Node x left right) value
    | value <= x = (Node x (insert left value) right)
    | otherwise = (Node x left (insert right value))
```

Let's insert some values...

```
> let t = Empty
> insert t 5
Node 5 Empty Empty

> insert it 10
Node 5 Empty (Node 10 Empty Empty)

> insert it 3
Node 5 (Node 3 Empty Empty) (Node 10 Empty Empty)
```




How many **Node**s are constructed by each of the insertions?

# **Tree**, continued

Here's an in-order traversal that produces a list of values:

```
inOrder Empty = []
inOrder (Node val left right) =
    inOrder left ++ [val] ++ inOrder right
```

What's an easy way to insert a bunch of values?

```
> let t = foldl insert Empty [3,1,9,5,20,17,4,12]
> inOrder t
[1,3,4,5,9,12,17,20]

> inOrder $ foldl insert Empty "tim korb"
" bikmort"

> inOrder $ foldl insert Empty [Rect 3 4, Circle 1, Rect 1 2]
[Rect 1.0 2.0,Circle 1.0,Rect 3.0 4.0]
```

# Maybe

Here's an interesting type:

```
> :info Maybe
data Maybe a = Nothing | Just a
```

Speculate: What's the point of it?

Here's a function that uses it:

```
> :t Data.List.find
Data.List.find :: (a -> Bool) -> [a] -> Maybe a
```

How could we use it?

```
> find even [3,5,6,8,9]
Just 6

> find even [3,5,9]
Nothing

> case (find even [3,5,9]) of { Just _ -> "got one"; _ -> "oops!"}
"oops!"
```

# A little I/O

# Sequencing

Consider this function declaration

```
f2 x = a + b + c
   where
     a = f x
     b = g x
     c = h x
```

```
a = f x
c = h x
b = g x
```

```
c = h x
b = g x
a = f x
```

Haskell guarantees that the order of the **where** clause bindings is inconsequential—those three lines can be in any order.

What enables that guarantee?

(Pure) Haskell functions depend only on the argument value. For a given value of **x**, **f x** always produces the same result.

You can shuffle the bindings of any function's **where** clause without changing the function's behavior! (Try it with **longest**, slide 233.)

# I/O and sequencing

Imagine a **getInt** function, which reads an integer from standard input (e.g., the keyboard).

Can the **where** clause bindings in the following function be done in any order?

```
f x = r
   where
      a = getInt
      b = getInt
      r = a * 2 + b + x
```

The following is not valid syntax but ignoring that, is it reorderable?

```
greet name = ""
   where
      putStr "Hello, "
      putStr name
      putStr "!\n"
```

# I/O and sequencing, continued

One way we can specify that operations are to be performed in a specific sequence is to use a **do**:

```
% cat io2.hs
main = do
   putStrLn "Who goes there?"
   name <- getLine
   let greeting = "Hello, " ++ name ++ "!"
   putStrLn greeting
```

Interaction:

```
% runghc io2.hs
Who goes there?
whm (typed)
Hello, whm!
```

# Actions

Here's the type of **putStrLn**:

**putStrLn :: String -> IO ()** *("unit",* **()***, is the no-value value)*

The type **IO x** represents an interaction with the outside world that produces a value of type **x**. Instances of **IO x** are called *actions*.

When an action is evaluated the corresponding outside-world activity is performed.

```
> let hello = putStrLn "hello!"
hello :: IO ()
```

*(Note: no output here!)*

*(Type of* **hello** *is an action.)*

```
> hello
hello!
it :: ()
```

*(Evaluating* **hello***, an action, caused output.)*

# Actions, continued

The value of **getLine** is an action that reads a line:

```
getLine :: IO String
```

We can evaluate the action, causing the line to be read, and bind a name to the string produced:

```
> s <- getLine
testing

> s
"testing"
```

Note that **getLine** is not a function!

# Actions, continued

Recall **io2.hs**:

```
main = do
   putStrLn "Who goes there?"
   name <- getLine
   let greeting = "Hello, " ++ name ++ "!"
   putStrLn greeting
```

Note the type: **main :: IO ()**. We can say that **main** is an action. Evaluating **main** causes interaction with the outside world.

```
> main
Who goes there?
hello?
Hello, hello?!
```

# Is it pure?

A pure function (1) always produces the same result for a given argument value, and (2) has no side effects.

Is this a pure function?

```
twice :: String -> IO ()
twice s = do
   putStr s
   putStr s
```

`twice "abc"` will always produce the same value, an action that if evaluated will cause `"abcabc"` to be output.

# The Haskell solution for I/O

We want to use pure functions whenever possible but we want to be able to do I/O, too.

In general, evaluating an action produces side effects.

Here's the Haskell solution for I/O in a nutshell:

Actions can evaluate other actions and pure functions but pure functions don't evaluate actions.

Recall **longest.hs** from 233-234:

```
longest bytes = result where ...lots...
main = do
   args <- getArgs -- gets command line arguments
   bytes <- readFile (head args)
   putStr (longest bytes)
```

# In conclusion...

# If we had a whole semester...

If we had a whole semester to study functional programming, here's what might be next:

- Infinite data structures (like **`let x = 1:x`**)
- How lazy/non-strict evaluation works
- Implications and benefits of referential transparency (which means that the value of a given expression is always the same).
- Functors (structures that can be mapped over)
- Monoids (a set of things with a binary operation over them)
- Monads (for representing sequential computations)
- Zippers (a structure for traversing and updating another structure)
- And more!

Jeremiah Nelson and Matt Gautreau are great local resources for Haskell!

# Even if you never use Haskell again...

Recursion and techniques with higher-order functions can be used in most languages. Some examples:

JavaScript, Python, PHP, all flavors of Lisp, and lots of others:
Functions are "first-class" values; anonymous functions are supported.

C
Pass a function pointer to a recursive function that traverses a data structure.

C#
Excellent support for functional programming with the language itself, and LINQ, too.

Java 8
Lambda expressions are in!

OCaml
"an industrial strength programming language supporting functional, imperative and object-oriented styles" – **OCaml.org**
**http://www.ffconsultancy.com/languages/ray_tracer/comparison.html**

# Killer Quiz!