Aliasing

Rasterization: Sampling a continuous curve (or region) at discrete points.

Pixels: Sample points.

Resolution: Sampling frequency.

Sampling is a well studied topic in signal processing.

Sampling & aliasing: Information loss due to low frequency sampling.

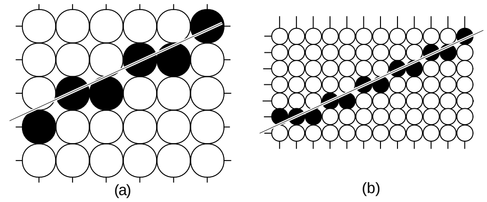
Nyquist sampling frequency:

- ☆ At least twice that of the highest frequency in the signal.
- ☆ No loss of information.

Slide 1

Antialiasing

Reduce aliasing by increasing sampling frequency

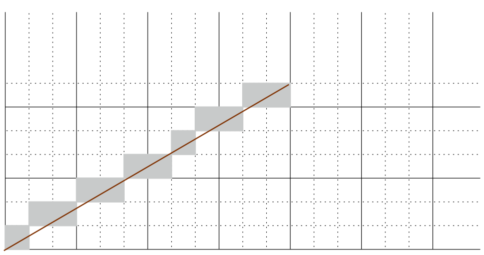


- ☆ Higher cost
- ☆ Efficiency: cannot render arbitrarily large frame at 30–60 frames/sec.

Modify the intensity of pixels to soften the images

Slide 2

Supersampling



- ☆ Imagine that the screen has higher resolution than the actual resolution.
  - Actual resolution: $s \times s$
  - Finer resolution: $\alpha s \times \alpha s$
- ☆ Each pixel can be viewed as an $\alpha \times \alpha$ grid of *subpixels*.
  α^2: # subpixels in each pixel
- ☆ Intensity of a pixel depends on the number of subpixels intersecting the line.

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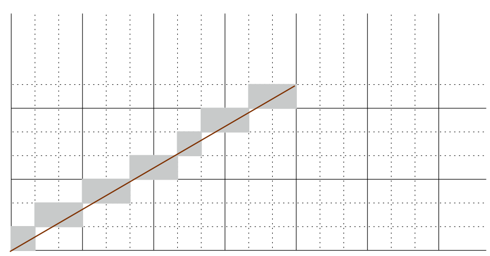
Unweighted supersampling

P: A pixel e: Segment α^2: # subpixels in P

$$\mathit{Intensity}(P) = \frac{\#\text{ subpixels intersecting } e}{\alpha^2}$$



- ☆ Assumed that pixels have finite area and lines have no width.
- ☆ Lines drawn on the screen have finite thickness
  GL_LINE_WIDTH specifies the line thickness in OpenGL.

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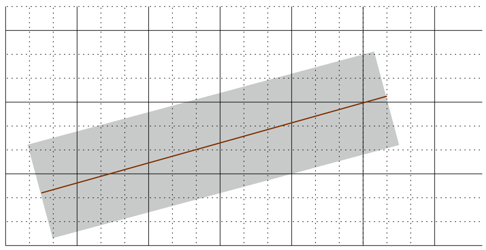
Unweighted supersampling



- ☆ Replace a line e by a rectangle $R(e)$ around e.
- ☆ Width of $R(e)$ depends on the line width.
- ☆ For each pixel P
  - A subpixel lies inside $R(e)$ if its *center* lies inside $R(e)$.
  - Count the number of subpixels of P lying inside $R(e)$.

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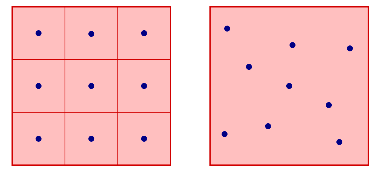
Unweighted Supersampling



Uniform sampling *Random sampling*

$$Intensity(P) = \frac{\#\text{ subpixels inside } R(e)}{\alpha^2}$$

Advantage:

- ☆ Intensity is distributed over more pixels.
- ☆ Softens the transition.

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Weighted supersampling

Assign higher weights to subpixels that lie near the center of the pixel.
W: Sum of weights of subpixels.

1	2	1
2	4	2
1	2	1

$$Intensity(P) = \frac{\sum \text{wt of subpixels lying in } R(e)}{W}$$

Unweighted supersampling: Assign weight 1 to each subpixel.

Weight matrix is called *mask*.

- ☆ Masks can be rather large.
- ☆ Mask may also include subpixels from other pixels.
  - A pixel may be drawn even if the rectangle does not intersect that pixel.
  - Reduces contrast between adjacent pixels.

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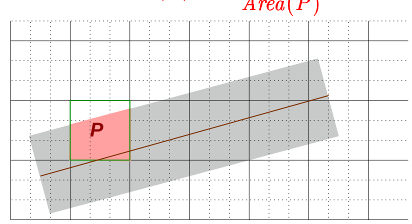
Area Sampling

$$Intensity(P) = \frac{Area(P \cap e)}{Area(P)}$$



- ☆ Supersampling approximates the area of $P \cap W(e)$.
- ☆ Can either compute the area exactly or use large supersampling to approximate the area.
- ☆ Lookup tables are used to expedite the algorithm.

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Weighted Area Sampling

Also known as *filtering techniques*.

- ☆ Assign a *weight distribution function* $W(x, y)$ to each pixel.
- ☆ $W(x, y)dA$: Weight of the small area at the point (x, y).
- ☆ $W(x, y)$ may be nonzero for points outside the pixel.

$$\int W(x, y)dA = 1.$$

$$Intensity(P) = \int_{R(e)} W(x, y)dx\, dy\ .$$

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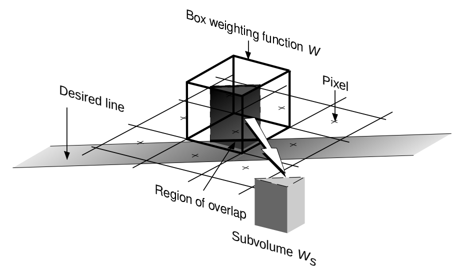
Box Filtering

Example: *Unweighted area sampling.*

A: Area of pixel P

$$W(x, y) = \begin{cases} 1/A & (x, y) \in P, \\ 0 & (x, y) \notin P \end{cases}$$



Intensity of P = Volume of the shaded prism.

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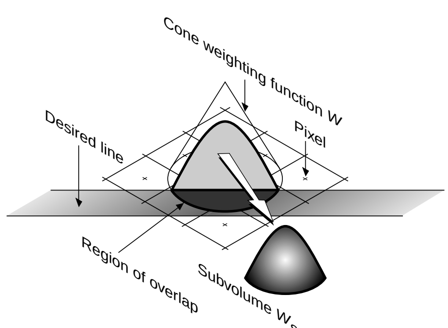
Cone Filter



Distance between the centers of pixels = 1.

c_P: Center of P.

$$W(q) = \begin{cases} (1 - d(c_P, q))/\pi & d(c_P, q) \le 1, \\ 0 & d(c_P, q) > 1. \end{cases}$$

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Cone Filter

- ☆ Radius of the cone is the distance between the centers of two adjacent pixels.
- ☆ Apex of the cone is at the center of the pixel.
- ☆ Weight decreases linearly with distance from the center.
- ☆ Rotation symmetry.
- ☆ Weight function spreads over adjacent pixels, so a pixel P is drawn even if $R(e)$ does not intersect P itself.
- ☆ Contrast between adjacent pixels is smaller.

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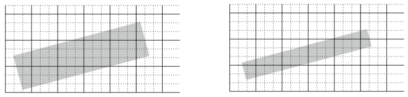
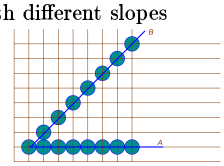
Other Issues

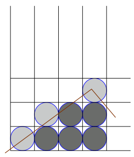
- ☆ Line width



- ☆ Lines with different slopes



- ☆ Polygon boundaries



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