## Lighting and Shading

- ☆ Flat shading
- ☆ Gouraud shading
- ☆ Phong shading
- ☆ Fast Phong shading
- ☆ Cook-Torrance model

**Slide 1**

## Constant-Intensity Shading

Also known as *flat shading*.

All pixels in a polygon are rendered with the same intensity.

Works reasonably well under the following assumptions:

- ☆ Object is a polyhedron; not an approximation of a curved surface.
- ☆ All light sources are far from the object; no distance attenuation.
- ☆ Viewing position is sufficiently far; $\mathbf{V} \cdot \mathbf{R}$ is constant.

`glShadeModel(GL_FLAT)`

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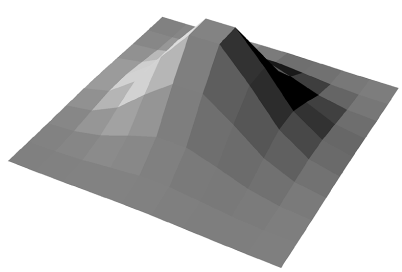
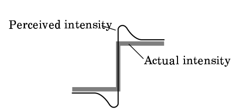
## Constant-Intensity Shading






- ☆ Lateral inhibition
- ☆ Mach bands

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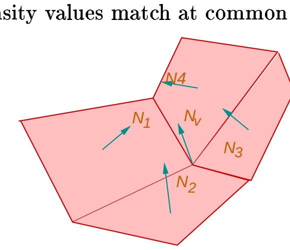
## Gouraud Shading

- ☆ Linear interpolation of intensity values at vertices of the polygon.
- ☆ Intensity values match at common edges.




- ☆ Determine the average normal at each vertex.

$$\mathbf{N}_v = \frac{\sum_{i=1}^{k} \mathbf{N}_k}{\left|\sum_{i=1}^{k} \mathbf{N}_k\right|}.$$

- ☆ Compute intensity at each vertex using the desired illumination model.
- ☆ Linearly interpolate the vertex intensities over the polygon.

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## Gouraud Shading

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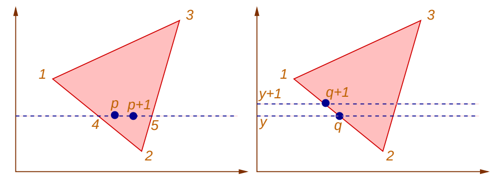
## Gouraud Shading




$$
\begin{aligned}
I_4 &= \frac{y_1 - y_4}{y_1 - y_2} I_2 + \frac{y_4 - y_2}{y_1 - y_2} I_1, \\
I_5 &= \frac{y_3 - y_5}{y_3 - y_2} I_2 + \frac{y_5 - y_2}{y_3 - y_2} I_3, \\
I_p &= \frac{x_5 - x_p}{x_5 - x_4} I_4 + \frac{x_p - x_4}{x_5 - x_4} I_5.
\end{aligned}
$$

$x_{p+1} = x_p + 1.$

$$
\begin{aligned}
I_{p+1} &= \frac{x_5 - x_p - 1}{x_5 - x_4} I_4 + \frac{x_p + 1 - x_4}{x_5 - x_4} I_5 \\
&= I_p + \frac{I_5 - I_4}{x_5 - x_4}.
\end{aligned}
$$

$$
I_{q+1} = I_q + \frac{I_1 - I_2}{y_1 - y_2}.
$$

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## Gouraud/Phong Shading

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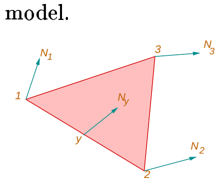
## Phong Shading

- ☆ Determine the unit normal at each vertex.
- ☆ Linearly interpolate the vertex normals over the surface of the polygon.
- ☆ Compute intensity at each pixel using the illumination model.




$$
\begin{aligned}
\mathbf{N}_p &= \frac{y - y_2}{y_1 - y_2} \mathbf{N}_1 + \frac{y_1 - y}{y_1 - y_2} \mathbf{N}_2 \\
\mathbf{N}_{p+1} &= \mathbf{N}_p + \frac{\mathbf{N}_1 - \mathbf{N}_2}{y_1 - y_2}.
\end{aligned}
$$

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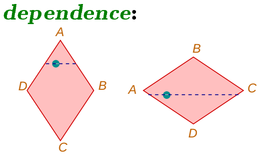
## Problems with Interpolated Shading

***Polygonal silhouette*:** Silhouette edges of curved surfaces are always polygonal, irrespective of approximation. Spurious creases

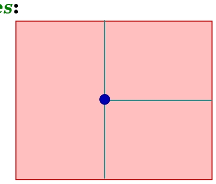
***Perspective distortion*:**

- ☆ Intensity is calculated in transformed coordinates
- ☆ $y_s = (y_1 + y_2)/2$, $I_s = (I_1 + I_2)/2$
- ☆ $z_s$ is not the midpoint

***Orientation dependence*:**




***Shared vertices*:**



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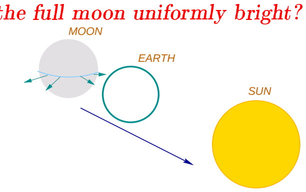
## Solar System

*Why is the full moon uniformly bright?*




- ☆ Sun can be approximated to a direction source.
- ☆ Moon can be approximated to a sphere.
- ☆ Less light should fall at points farther away from direction $D$.

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## Cook-Torrance Model

- ☆ Extends Torrance-Sparrow, 1967; Blinn 1977.
- ☆ Based on a consideration of incident energy rather than intensity.
- ☆ Specular reflection is based on a physical microfacet model.
  - originally aimed to render polished metallic surfaces.
  - Phong model is good for colored plastic surfaces.
  - Phong model is also inaccurate for illumination at low angles of incidence.
  - Specular bump depends on the angle of incidence.
- ☆ Color change within the highlight is based on Fresnel's law and properties of material.

Three main components:

- ☆ Microfacet model of the surface.
- ☆ Fresnel's formula for reflection and refraction.
- ☆ Roughness of surfaces.

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## Cook-Torrance Model

$$R = k_d R_d + k_s R_s, \qquad k_d + k_s \le 1$$

$$I_r = R_a I_a + \sum_{j=1}^{n} I_{L_j} (\mathbf{N} \cdot \mathbf{L}_j) d\omega_{i_j} (k_d R_d + k_s R_s).$$

$$R_d = \frac{1}{\pi}$$

$$R_s = \frac{1}{\pi} \frac{F \cdot D \cdot G}{(\mathbf{N} \cdot \mathbf{L})(\mathbf{N} \cdot \mathbf{V})}.$$

- ☆ F: Fresnel term. Depends on wavelength, incidence angle, and material.
- ☆ D: Distribution function of microfacets.
- ☆ G: Geometric attenuation term.
- ☆ $\mathbf{N} \cdot \mathbf{L}$: Normalization for the surface area that the light sees per unit area
- ☆ $\mathbf{N} \cdot \mathbf{V}$: Similar normalization for viewpoint
- ☆ Polarization was incorporated;
  *(Wolf & Kurlander, 1990)*
  Use Jones matrices to describe polarization.

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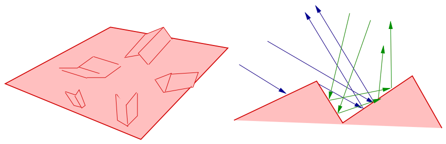
## Microfacets

- ☆ Surface consists of many ∨-shaped grooves (wedges).
- ☆ Each groove is lined with flat mirrors.
- ☆ Direction of each groove is random.



- ☆ Reflection off one groove causes *specular* reflection.
- ☆ Light bouncing off multiple grooves and interacting with substrate causes *diffuse reflection*.
- ☆ *Geometry of microfacets:* microfacets block incident as well as reflected light.

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## Roughness of Surface

- ☆ Characterizes the distribution of slopes of the grooves.
- ☆ Different distribution of slopes cause different pattern of reflection.

$$D = \frac{\exp\left[-((\tan\alpha)/m)^2\right]}{m^2\cos^4\alpha}$$

- ☆ **H**: Halfway vector between **L** and **V**.
- ☆ $\alpha = \cos^{-1}(\mathbf{N}\cdot\mathbf{H})$
- ☆ $m$: Root-mean-square slope of microfacets
  - Small $m$ ($\approx 0.2$): Smooth surface; sharp reflection.
  - Large $m$ ($\approx 0.8$): Rough surface; spreads reflection.

Can use a linear combination of distributions

$$D = \sum_k w_k D(m_k).$$

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## Fresnel's Formula

- ☆ Expresses reflectance of a perfectly smooth mirror.
- ☆ Describes the amount of light reflected and refracted at the interface of two medium.
- ☆ Depends on wavelength of the incident light, geometry of surface, and the angle of incidence.
- ☆ Derived from Maxwell's equation.

For unpolarized light:

$$F = \frac{1}{2}\frac{\sin^2(\theta_i-\theta_t)}{\sin^2(\theta_i+\theta_t)}\left(1+\frac{cos^2(\theta_i+\theta_t)}{\cos^2(\theta_i-\theta_t)}\right).$$

- ☆ $\theta_i$: Angle of incidence; $\cos^{-1}(\mathbf{L}\cdot\mathbf{H})$.
- ☆ $\theta_t$: Angle of refraction; $\sin\theta_t = (\eta_{i\lambda}/\eta_{t\lambda})\sin\theta_i$
- ☆ $\eta_{i\lambda}, \eta_{t\lambda}$ are functions of $\lambda$.

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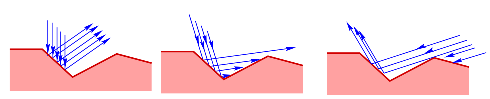
## Geometric Attenuation



- ☆ Attenuation factor due to the effect of shadowing by the microfacets.
- ☆ *Shadowing:* Incident light blocked by microfacets.
  $$G_s = \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{L})}{\mathbf{V}\cdot\mathbf{H}}.$$
- ☆ *masking:* Reflected light blocked by microfacets.
  $$G_m = \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{V})}{\mathbf{V}\cdot\mathbf{H}}.$$

$$G = \min\{1, G_s, G_m\}.$$

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