Illumination Models

Z-buffer methods:

- ☆ Compute only direct lighting.
- ☆ Ignore secondary light sources.

Ray-tracing methods:

- ☆ Model specular reflection and refraction well.
- ☆ Still uses directionless ambient lighting.
- ☆ Not good for global lighting.

Radiosity methods:

- ☆ Introduced in 1984 by (Goral, Torrance, Greenberg, & Battaile).
- ☆ Use thermal radiation models to calculate global lighting.
- ☆ Good for ideal *diffuse* environments.
- ☆ Assumes conservation of light energy in a closed environment.
- ☆ Determines all light interactions in a view-independent way.

Slide 1

Radiosity

Radiosity: Power (Energy/unit time) leaving a surface at a given point, per unit area.

- ☆ Allows any surface to radiate power.
  - If light source, emits energy.
  - Otherwise, radiates a portion of the incident energy.
- ☆ Surfaces have finite area.
- ☆ Power leaving a surface:

  emitted power + reflected power.
- ☆ Need to *mesh* original surfaces to capture fine-scale illumination.

Slide 2

Radiosity

- ☆ $S_1, \ldots, S_n$: Set of surface patches.
- ☆ A_i: Area of S_i.
- ☆ ρ_i: Reflectance of S_i.
- ☆ B_i: Radiosity of S_i.
- ☆ E_i: Rate at which S_i emits power. (Energy per unit time per unit area.)

$$\begin{aligned} B_i A_i &= E_i A_i + \rho_i \cdot \text{total power incident to } S_i \\ &= E_i A_i + \rho_i \sum_{j=1}^{n} \text{Incident power from } S_j \\ &= E_i A_i + \rho_i \sum_{j=1}^{n} B_j A_j F_{ji} \end{aligned}$$

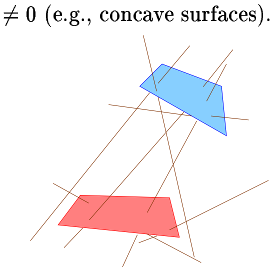
F_{ji}: Fraction of light leaving S_j that reaches S_i.

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ji} \frac{A_j}{A_i}.$$

Slide 3

Form Factors

- ☆ F_{ji}: Fraction of light leaving S_j that arrives at S_i
  - Depends on shape, orientation, & occlusion.
  - $F_{ii} \neq 0$ (e.g., concave surfaces).



- ☆ MA_i: Number of lines through S_i.
- ☆ MA_j: Number of lines through S_j.
- ☆ MA_jF_{ji}: # lines leaving S_j & reaching S_i.
- ☆ MA_iF_{ij}: # lines leaving S_i & reaching S_j.

$$A_i F_{ij} = A_j F_{ji}$$

$$F_{ij} = \frac{A_j}{A_i} F_{ji}.$$

Slide 4

Radiosity Equation

$$B_i = E_i + \rho_i \sum_{i=1}^{n} B_j F_{ij}$$

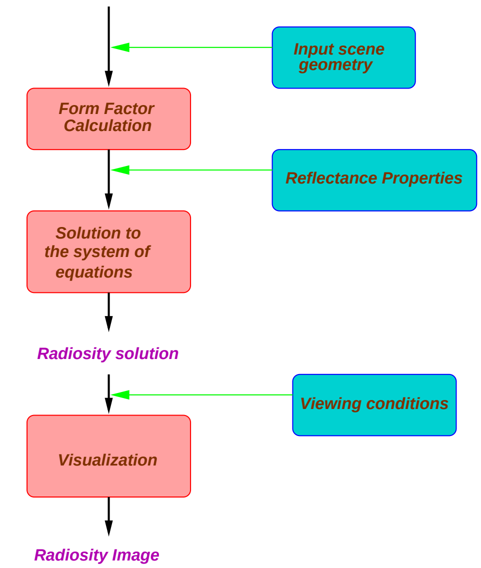
$$B_i - \rho_i \sum_{j=1}^{n} B_j F_{ij} = E_i$$

$$\underbrace{\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_1 F_{22} & \cdots & -\rho_2 F_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_2 F_{nn} \end{bmatrix}}_{\mathbf{M}} \cdot \underbrace{\begin{bmatrix} B_1 \\ B_2 \\ \cdots \\ B_n \end{bmatrix}}_{\mathbf{B}} = \underbrace{\begin{bmatrix} E_1 \\ E_2 \\ \cdots \\ E_n \end{bmatrix}}_{\mathbf{E}}$$

$$\mathbf{M} \cdot \mathbf{B} = \mathbf{E}$$

Slide 5

Radiosity Methods



Slide 6

Iterative Methods

- ☆ No closed form for the radiosity equation.
- ☆ Use numerical methods.
- ☆ Compute form factors F_{ij}, $1 \le i, j, \le n$.
- ☆ Set up initial conditions
  - $E_i > 0$ for light sources.
  - $E_i = 0$ for other surfaces.
  - Guess initial values of B_i, $1 \le i \le n$.

Slide 7

Iterative Methods

- ☆ Iterate the system until convergence.
- ☆ Computes a better approximation of B_i at each step.

$$\mathbf{M} \cdot \mathbf{B} = \mathbf{E} \qquad \mathbf{M} = [M_{ij}] \quad M_{ii} > 0$$

$$B_i = -\sum_{\substack{j=1 \\ j \ne i}}^{n} \frac{M_{ij}}{M_{ii}} B_j + \frac{E_i}{M_{ii}}.$$

Use any of the relaxation methods to compute the new value of B_i.

- ☆ Jacobian relaxation
- ☆ Gauss-Seidel relaxation

Slide 8

Iterative Methods

How do we compute $B_i^{(m)}$, value of B_i in the m-th iteration?

Jacobian relaxation:

Use values from the previous iteration for all B_j.

$$B_i^{(m)} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{M_{ij}}{M_{ii}} B_j^{(m-1)} + \frac{E_i}{M_{ii}}.$$

Gauss-Seidel relaxation:

Use values from the previous iteration for $j < i$ and from the current iteration for $j > i$.

$$B_i^{(m)} = -\sum_{j=1}^{i-1} \frac{M_{ij}}{M_{ii}} B_j^{(m)} - \sum_{j=i+1}^{n} \frac{M_{ij}}{M_{ii}} B_j^{(m-1)} + \frac{E_i}{M_{ii}}.$$

☆ In-place update of B_i's.
☆ Convergence rate is better.
☆ *Strictly diagonal dominant* matrices converge.

Slide 9

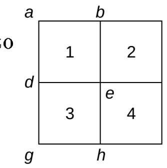
Continuous Shading

Decompose each surface into smaller pacthes
Radiosity within each patch is the same.

Interpolated Shading:

☆ Convert patch radiosity to vertex radiosity.

☆ Interpolate patch radiosity.



Vertex radiosity:

☆ *Interior vertex v:* Average of radiosity over adjacent patches

$$B_e = (B_1 + B_2 + B_3 + B_4)/2$$

Boundary vertex v_b: More complex procedure.

- Find a nearest interior vertex v_I.
- $f_1, \ldots, f_k$: faces adjacent to v_b.
- $(B_b + B_I)/2 = \sum_{i=1}^{k} B_i/k$.
- $(B_b + B_e)/2 = (B_1 + B_2)/2 \Rightarrow$
  $B_b = (3B_1 + 3B_2 - B_3 - B_4)/4$.

Slide 10

Form Factors

F_{ij}: What is the average number of lines leaving a point from S_i and reaching S_j?

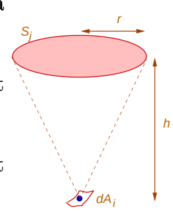
Example:

☆ Small patch dS_i with area dA_i.

☆ Parallel disk of radius r at distance h.

☆ F_{ij}: Solid angle from a point in dS_i to S_j.

☆ $F_{ij} = \dfrac{r^2}{h^2 + r^2}$



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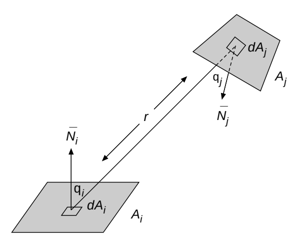
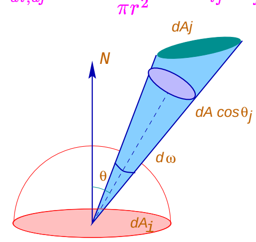
Form Factors



☆ dS_i, dS_j: Differential surfaces S_i, S_j
☆ dA_i, dA_j: Areas of differential surfaces dS_i, dS_j
☆ $F_{di,dj}$: Differential form factor from dS_i to dS_j.
☆ H_{ij}: 1 if dS_i visible from dS_j.

$$F_{di,dj} = \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j.$$



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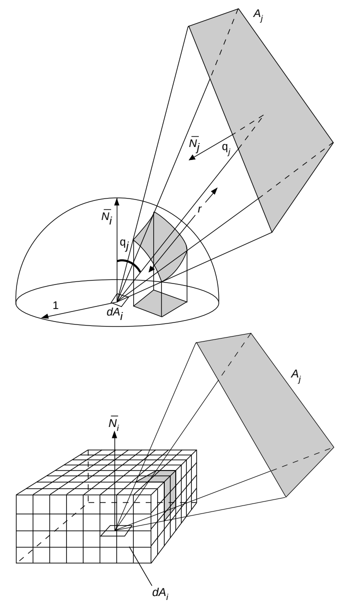
Form Factors

- ☆ $F_{di,j}$: Form factor from dS_i to S_j.
- ☆ F_{ij}: Form factor from S_i to S_j.

$$
\begin{aligned}
F_{di,j} &= \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j \\
F_{i,j} &= \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j dA_i
\end{aligned}
$$

Slide 13

Form Factors



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