**Illumination Models**

*Z-buffer methods:*
- Compute only direct lighting.
- Ignore secondary light sources.

*Ray-tracing methods:*
- Model specular reflection and refraction well.
- Still uses direct and ambient lighting.
- Not good for global lighting.

**Radiosity methods:*
- Introduced in 1984 by (Goral, Terrance, Greenberg, & Battaile).
- Use thermal radiation models to calculate global lighting.
- Good for ideal diffuse environments.
- Assumes conservation of light energy in a closed environment.
- Determines all light interactions in a view-independent way.

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**Radiosity**

*Power (Energy/unit time) leaving a surface at a given point, per unit area.*

- Allows any surface to radiate power.
  - If light source, emits energy.
  - Otherwise, radiates a portion of the incident energy.
- Surfaces have finite area.
- Power leaving a surface:
  \[ \text{emitted power} + \text{reflected power} \]
- Need to mesh original surfaces to capture fine-scale illumination.

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**Form Factors**

*\( F_{ji} \): Fraction of light leaving \( S_j \) that arrives at \( S_i \)*

- Depends on shape, orientation, & occlusion.
- \( F_{ii} \neq 0 \) (e.g., concave surfaces).

*\( M_A \): Number of lines through \( S_i \).*
*\( M_A \): Number of lines through \( S_j \).*
*\( M_A F_{ji} \): \# lines leaving \( S_j \) & reaching \( S_i \).*
*\( M_A F_{ij} \): \# lines leaving \( S_i \) & reaching \( S_j \).*

\[ A_i F_{ij} = A_j F_{ji} \]
\[ F_{ij} = \frac{A_j}{A_i} F_{ji} \]
Radiosity Equation

\[
B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ij} - \rho_i B_i \sum_{j=1}^{n} F_{ij}
\]

\[
\begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix} = \begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
\]

\[
M = B - I + \rho F
\]

Slide 5

Radiosity Methods

Input scene geometry
Form Factor Calculation
Reflectance Properties
Solution to the system of equations
Radiosity solution
Viewing conditions
Visualization
Radiosity Image

Slide 6

Iterative Methods

* No closed form for the radiosity equation.
* Use numerical methods.
* Compute form factors \( F_{ij} \), \( 1 \leq i, j \leq n \).
* Set up initial conditions
  * \( E_i > 0 \) for light sources.
  * \( E_i = 0 \) for other surfaces.
  * Guess initial values of \( B_i \), \( 1 \leq i \leq n \).

Slide 7

Iterative Methods

* Iterate the system until convergence.
* Computes a better approximation of \( B_i \) at each step.

\[
M \cdot B = E \quad M = [M_{ij}] \quad M_{ii} > 0
\]

\[
B_i = - \sum_{j \neq i}^{n} \frac{M_{ij} B_j}{M_{ii}} + \frac{E_i}{M_{ii}}
\]

Use any of the relaxation methods to compute the new value of \( B_i \).

* Jacobian relaxation
* Gauss-Seidel relaxation

Slide 8
Iterative Methods

How do we compute $B^{(m)}_i$, value of $B_i$ in the $m$-th iteration?

**Jacobian relaxation:**
Use values from the previous iteration for all $B_j$:

$$B^{(m)}_i = -\sum_{j=1}^{n} \frac{M_{ij}}{M_{ii}} B^{(m-1)}_j + \frac{E_i}{M_{ii}}$$

**Gauss-Seidel relaxation:**
Use values from the previous iteration for $j < i$ and from the current iteration for $j > i$.

$$B^{(m)}_i = -\sum_{j=1}^{i-1} \frac{M_{ij}}{M_{ii}} B^{(m)}_j - \sum_{j=i+1}^{n} \frac{M_{ij}}{M_{ii}} B^{(m-1)}_j + \frac{E_i}{M_{ii}}$$

- In-place update of $B_i$'s.
- Convergence rate is better.
- Strictly diagonal dominant matrices converge.

Continuous Shading

Decompose each surface into smaller patches. Radiosity within each patch is the same.

Interpolated Shading:

- Convert patch radiosity to vertex radiosity.
- Interpolate patch radiosity.

**Vertex radiosity:**

- **Interior vertex $v$:** Average of radiosity over adjacent patches
  $$B_v = (B_1 + B_2 + B_3 + B_4)/4$$
- **Boundary vertex $v_b$:** More complex procedure.
  - Find a nearest interior vertex $v_i$.
  - $f_1, \ldots, f_k$: faces adjacent to $v_b$.
  - $(B_{f_1} + B_{v_i})/2 \sum_{k=1}^{k} B_{f_k} / k$.
  - $(B_{f_2} + B_{v_i})/2 = (B_{f_1} + B_{v_i})/2 \Rightarrow B_{f_1} = (3B_1 + 3B_2 - B_3 - B_4)/4$

Form Factors

$F_{ij}$: What is the average number of lines leaving a point from $S_i$ and reaching $S_j$?

**Example:**

- Small patch $dS_i$ with area $dA_i$.
- Parallel disk of radius $r$ at distance $h$.
- $F_{ij}$: Solid angle from a point in $dS_i$ to $S_j$.
- $F_{ij} = \frac{r^2}{h^2 + r^2}$

Form Factors

$$F_{di,j} = \frac{\cos \theta_i \cos \theta_j}{\pi r^2} H_{ij} dA_j$$

$\partial S_i, \partial S_j$: Differential surfaces $S_i, S_j$
$\partial A_i, \partial A_j$: Areas of differential surfaces $dS_i, dS_j$
$F_{di,j}$: Differential form factor from $dS_i$ to $dS_j$.
$H_{ij}$: 1 if $dS_i$ visible from $dS_j$. 

$\theta_i, \theta_j$: Directions of $dS_i, dS_j$.
Form Factors

\[ F_{d,i,j} \]: Form factor from \( dS_i \) to \( S_j \).
\[ F_{i,j} \]: Form factor from \( S_i \) to \( S_j \).

\[
F_{d,i,j} = \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} H_{ij} \, dA_j
\]
\[
F_{i,j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} H_{ij} \, dA_j \, dA_i
\]