Achromatic Light

Attributes:

Luminance: Physical intensity
Brightness: Perceived intensity

Intensity Display:

★ b: # bits to encode $2^b$ levels of intensity.
★ How should levels be distributed?
  - Black: 0
  - White: 1
★ Linear scale: Uniformly distributed
  $I_k = k/2^b, 0 \leq k < 2^b$

Intensity Perception:

★ Eyes perceive ratio of intensities.
★ Intensity 0.11 is to 0.1, perceptually the same
  as 0.55 is to 0.5.

Logarithmic Display

Ideally: Each level should increase the intensity by
a constant factor.

$n = 2^b - 1, I_0$: minimum intensity, $I_n = 1$.

$I_1 = rI_0, I_2 = rI_1 = r^2I_0, I_3 = rI_2 = r^3I_0$.

$$I_n = r^nI_0 = 1.0 \Rightarrow r = \left(\frac{1}{I_0}\right)^{1/n}$$

$$I_j = I_0^{1-j/n}$$

Example: $I_0 = 1/128, b = 3, n = 7$.

$I = \frac{1}{128} \cdot \frac{1}{128} \cdot \frac{1}{128} \cdot \frac{1}{128} \cdot \frac{1}{128} \cdot \frac{1}{128} \cdot \frac{1}{128}$.

★ Dynamic range: $1/I_0$.
★ # Intensities: $n = \log (1/I_0)$
★ Typical value of $r$: 1.01.

## Typical values

<table>
<thead>
<tr>
<th>Display Media</th>
<th>Dynamic Range</th>
<th># Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRT</td>
<td>50 - 200</td>
<td>100</td>
</tr>
<tr>
<td>Photograph prints</td>
<td>400 - 300</td>
<td>465</td>
</tr>
<tr>
<td>Photograph slides</td>
<td>300</td>
<td>700</td>
</tr>
<tr>
<td>Coated B/W paper</td>
<td>200</td>
<td>465</td>
</tr>
<tr>
<td>Coated colored paper</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>Newspaper print</td>
<td>10</td>
<td>234</td>
</tr>
</tbody>
</table>
Gamma Correction

- $V$: Input voltage.
- $N$: # electrons in the beam.
  \[ N \propto V \]
  \[ I = k'N^\gamma = kV^\gamma \quad 1.8 \leq \gamma \leq 2.5. \]
  \[ V = \left( \frac{I}{k'} \right)^{1/\gamma}. \]

How do we determine the voltage for a perceived intensity?

- Compute the $I_j$ closest to $I$.
  \[ j = \text{Round}(\log_2 \frac{I}{k}); \quad I_j = I_0^{-j/n}. \]
- $V_j = \text{Round}\left(\left( \frac{I_j}{k} \right)^{1/\gamma}\right)$. 

Halftone Approximation

- More intensity levels than allowed by the device.
- Integration of intensity over small areas.
- A small area is printed with black disks.
- Black area is proportional to $1 - I$.
- Newspaper: 60–80 halftones per inch.
  Magazine: 110–200 halftones per inch.

Example: 2 x 2 pixel area of bilevel display

- 0
- 1
- 2
- 3
- 4

$n \times n$ pixel matrix generates $n^2 + 1$ levels.

3 x 3 pixel matrix values:

\[
\begin{bmatrix}
6 & 8 & 4 \\
1 & 0 & 3 \\
5 & 2 & 7
\end{bmatrix}
\]
**Color Specification**

**Hue:** Distinguishes among colors.
Example: red, green, blue, purple.

**Saturation:** Distance from equal-intensity gray.
Example: red vs pink, royal blue vs sky blue.

**Lightness:** Perceived intensity of reflected light.

**Brightness:** Perceived intensity of self-luminous light.

**Artist's view:**
- **Tint:** Saturation decreases with white pigments.
- **Shade:** Lightness decreases with black pigments.
- **Tone:** Combination of above.

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**Colorimetry**

**Dominant wavelength:** Hue.
Wavelength of the color we see.

**Excitation purity:** Saturation.
Ratio of pure light of the dominant wavelength and the white light present in the color.

**Luminance:** Lightness, brightness.
Amount of light energy.

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**Human Visual System**

Retina has four main layers:
- **Epithelium**
- **Rods and cones**
- **Bipolar cells**
- **Ganglion cells**

- Many more rods (≈ 120M) than cones (≈ 6M).
- Rods are more sensitive.
- Rods are responsible for achromatic light.
- Cones detect colors.
- Cones are concentrated near fovea: ≈ 147K cones/mm.

**Trichromatic Theory:**
- First proposed by Thomas Young, 1801.
- Refined by Hermann von Helmholtz, 1861.
- Three types of cones: blue, green, red.
- Explains color blindness.
  - Protanope (red blindness)
  - Deutanope (green blindness)
  - Tritanope (blue blindness); very rare.
Trichromatic Theory

Spectral Response of three types of cones.

Luminous efficiency.  
Conjecture: Sum of the response of three cones.

Conjecture: Colors can be specified by the positive weighted sum of red, blue, and green.

Slide 13

R, G, B values needed to match the color luminance for each dominant wavelength.
Some values are negative!
Commission Internationale de l’Éclairage (CIE), 1931!
Defined three primary colors: X, Y, Z.

Slide 14

Trichromatic Theory

☆ Y: Same as luminance-efficiency curve.
☆ Each color C is specified as a positive weighted sum of X, Y, Z.
☆ \( C = XX + YY + ZZ \).
☆ \( P(\lambda) \): spectral distribution of C.
\( X = k \int P(\lambda)x_\lambda d\lambda, \quad Y = k \int P(\lambda)y_\lambda d\lambda, \quad Z = k \int P(\lambda)z_\lambda d\lambda. \)
☆ \((X, Y, Z)\) specify a color; called XYZ-space.

Slide 15

Chromaticity

Chromaticity:
☆ depends on dominant wavelength & saturation.
☆ Independent of luminance.
\[ x = \frac{X}{X + Y + Z}, \quad y = \frac{Y}{X + Y + Z}, \quad z = \frac{Z}{X + Y + Z} \]
Chromaticity is encoded in the cross-section of the cone with the plane \( X + Y + Z = 1. \)

Slide 16

☆ Interior and boundary contain all visible chromaticities.
☆ Spectral pure colors lie along the boundary.
White light: Center dot C, near \( x = y = z = 1/3 \)
Power distribution that is close to daylight at a correlated temp. 6774K.
Color adding:

- Combination of two colors $C_1, C_2$ lies on the segment $C_1C_2$.
  - $A$ is mix of white $C$ and $B$.
- **Dominant wavelength** of $A$ is $B$.
- **Excitation purity** of $A = |AC|/|BC|$.

Complementary colors:

- Mixing complementary colors $C_1, C_2$ produces white $C$.
  - $C$ is the midpoint of the segment $C_1C_2$.
- $D$ and $E$ are complementary colors.

Non spectral colors:

- Cannot be defined by a dominant wavelength (e.g. $F$).
- Define dominant wavelength as:
  - Dominant wavelength of its complementary color.
- Excitation purity remains the same.
  - Excitation purity of $F$: $|CF|/|CG|$.

**Color Gamut**

- Mixture of $I, J, K$ produces colors in $\triangle IJK$.
- $\triangle IJK$: **color gamut** of $I, J, K$.

**RGB Model**

- Additive color model.
- Uses Cartesian coordinate system.
- Origin is black.
- Grays: principal diagonal.

Linear relationship between $RGB$ and $XYZ$ colors.

$$
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
X_r & X_g & X_b \\
Y_r & Y_g & Y_b \\
Z_r & Z_g & Z_b
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
$$
CMY Model

- Subtractive primitives.
- Cartesian coordinate system.
- Start with white; color is specified by what is removed.

Example: RGB color printer.
- Cyan ink absorbs red light.
- Magenta ink absorbs green light.
- Yellow ink absorbs blue light.

\[
\begin{bmatrix}
  C \\
  M \\
  Y
\end{bmatrix} = \begin{bmatrix}
  1 \\
  1 \\
  1
\end{bmatrix} - \begin{bmatrix}
  R \\
  G \\
  B
\end{bmatrix}
\]

\[C + M + Y = \text{Black}\]

CMYK Model: Use black ink (K: Carbon)
\[K = \min(C, M, Y).\]
\[C = C - K, M = M - K, Y = Y - K.\]

YIQ Model

- Used for TV broadcasting.
- Compatibility with black and white TV.
- \(Y\): Primary color \(Y\): luminance efficiency.
- \(Y\) component is used by black and white TV.
- Chromaticity is encoded by \(I\) and \(Q\).

\[
\begin{bmatrix}
  Y \\
  I \\
  Q
\end{bmatrix} = \begin{bmatrix}
  0.299 & 0.587 & 0.114 \\
  0.596 & -0.275 & -0.321 \\
  0.212 & -0.523 & 0.311
\end{bmatrix} \begin{bmatrix}
  R \\
  G \\
  B
\end{bmatrix}
\]

\[R \times 0.67 = 0.21, 0.14\]
\[y = 0.33, 0.71, 0.09\]

White: \(x_w = 0.31, y_w = 0.316\), and \(Y_w = 100.00\)

YIQ Model

- Human eyes are more sensitive to luminance than to chromaticity.
- Assigns more bandwidth to \(Y\)
  - 4MHz to \(Y\)
  - 1.5 MHz to \(I\)
  - 0.6 MHz to \(Q\)
- Use \(Y\) values to disambiguate colors when converting to black and white TV.
- Objects lying in a narrow field of view:
  - Color is not so important.
  - Use only one color(1).

HSV Model

- Top of hexcone:
  - \(V = 1\): bright colors.
  - \(H\): Angle around the vertical axis:
    - \(R = 0^\circ, G = 120^\circ, B = 240^\circ\).
- Complementary colors are 180° apart.
- \(S \in [0, 1]\): Fractional distance from the vertical axis.
- Apex: Black (\(V = 0\)), white: \(S = 0, V = 1\), gray \(V < 1, S = 0\).