CSC 433/533 Computer Graphics

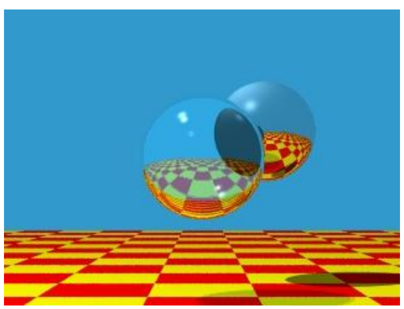
Alon Efrat
Thanks: Joshua Levine

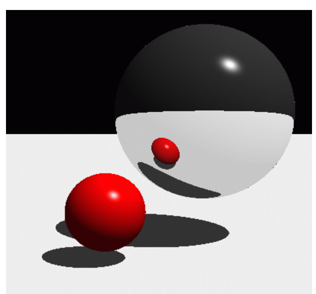
Lecture 15
Wrapping up distributed Ray Tracing

Triangle Meshes

Oct. 13, 2020

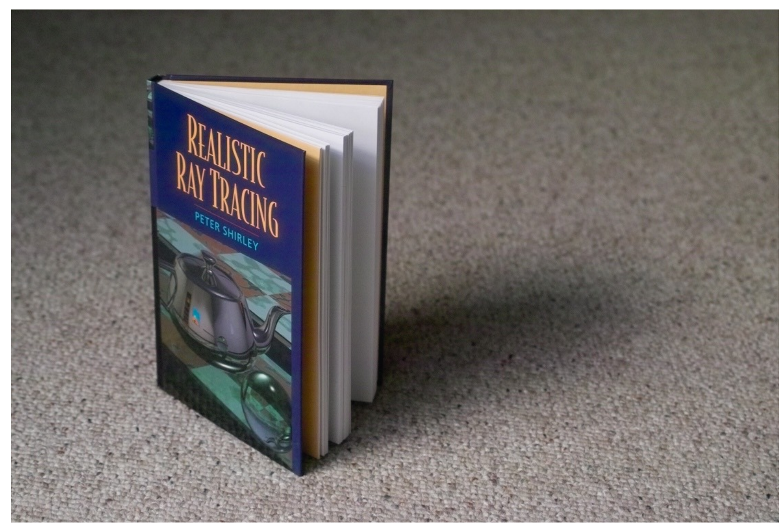
What's Wrong?



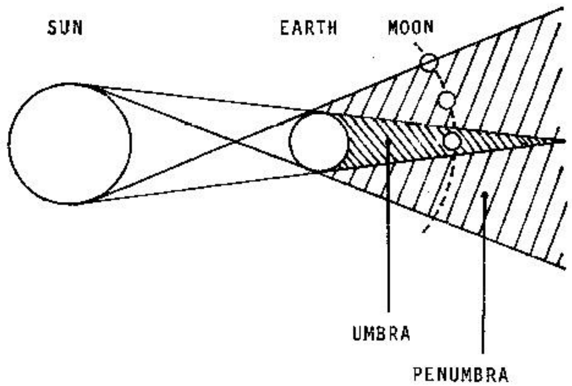


- No surface is a perfect mirror because surfaces rarely perfectly smooth

Soft Shadows



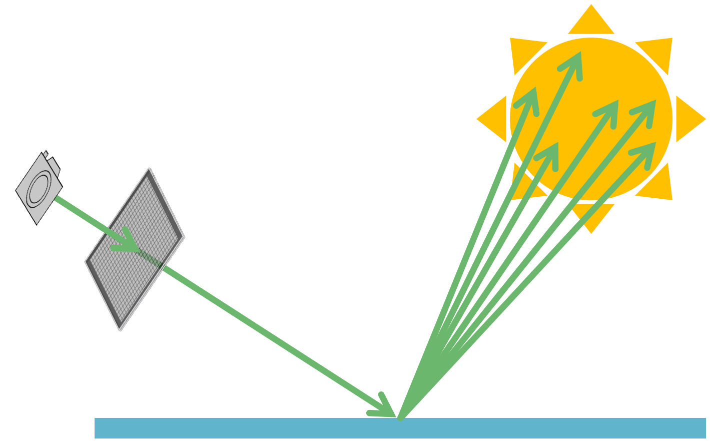
What Causes Soft Shadows



http://user.online.be/felixverbelen/lunecl.jpg

Lights aren't all point sources

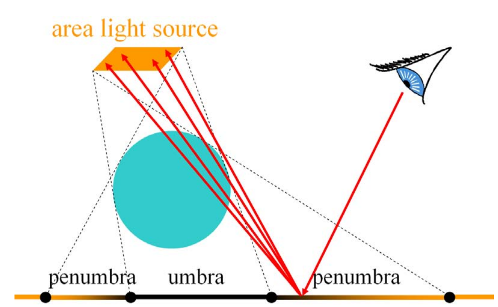
Distribution Soft Shadows

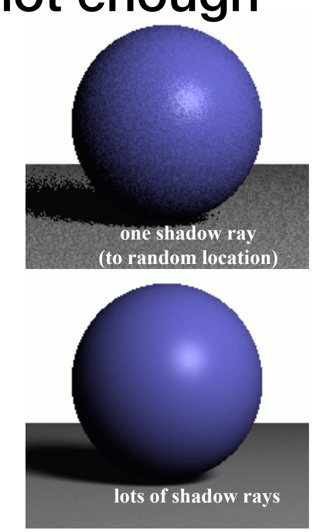


Randomly sample light rays

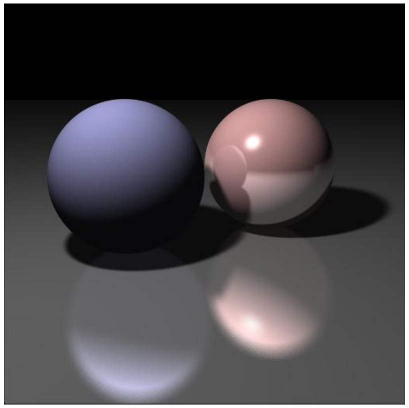
Computing Soft Shadows
One ray per pixel is not enough

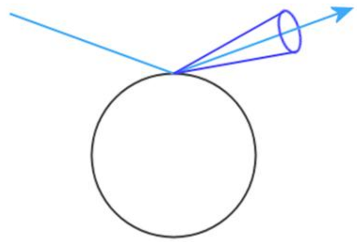
- Model light sources as spanning an area
- Sample random positions on area light source and average rays





Approach: Distribution Glossy Reflection by Randomly Sampling Rays

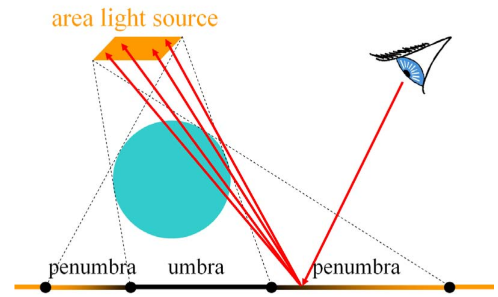


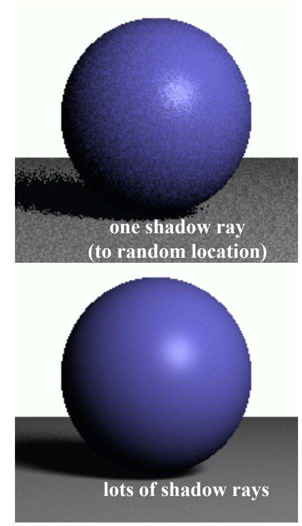


https://graphics.stanford.edu/wikis/cs148-11-fall/RaytracingResults
http://www.baylee-online.net/Projects/Raytracing/Algorithms/Glossy-Reflection-Transmission

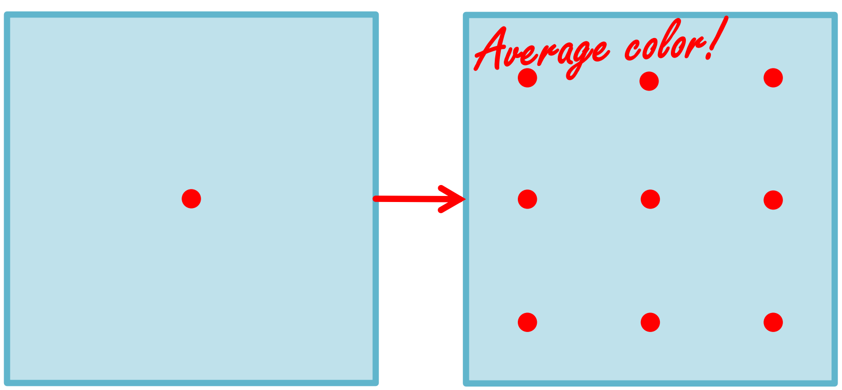
Computing Soft Shadows

- Model light sources as spanning an area
- Sample random positions on area light source and average rays
- Shoot several rays and calculate the average among them



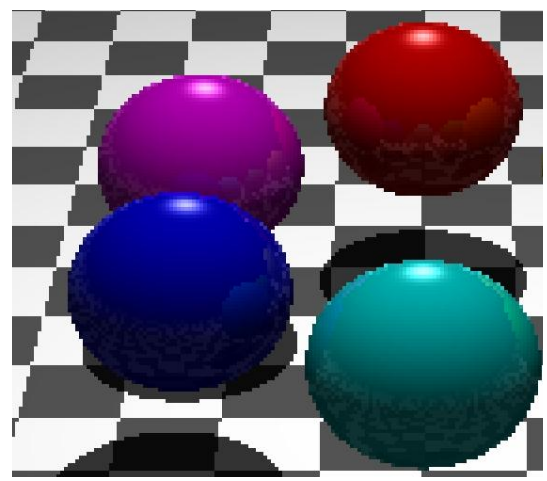


Distribution Antialiasing



Multiple rays per pixel

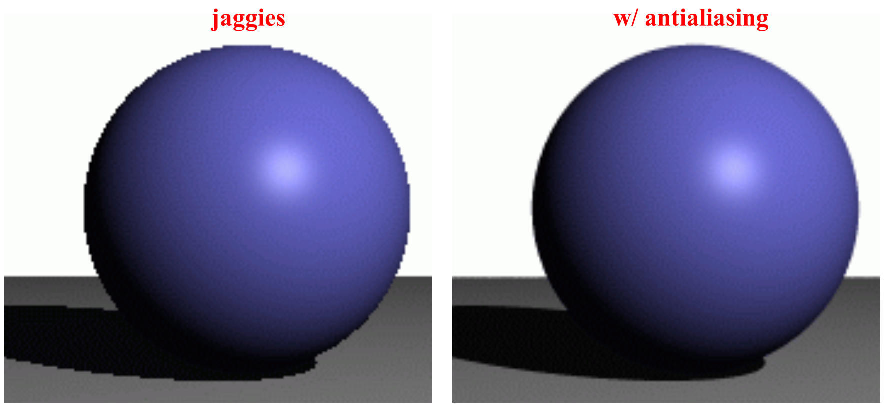
Problem: Aliasing



http://www.hackification.com/2008/08/31/experiments-in-ray-tracing-part-8-anti-aliasing/

Antialiasing w/ Supersampling

- Cast multiple rays per pixel, average result



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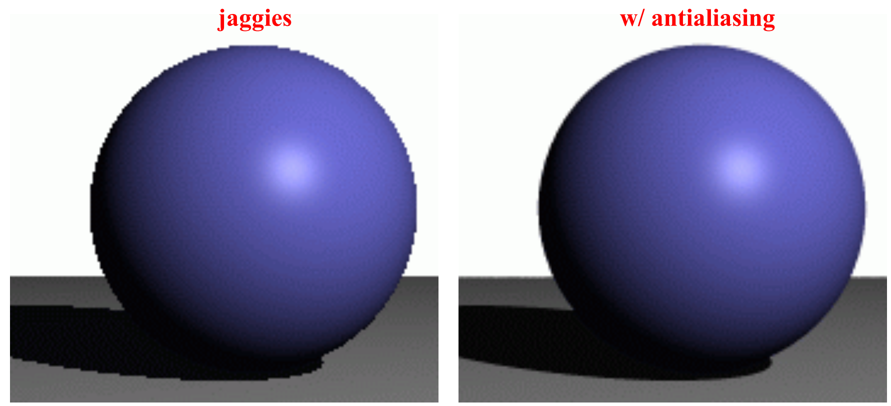
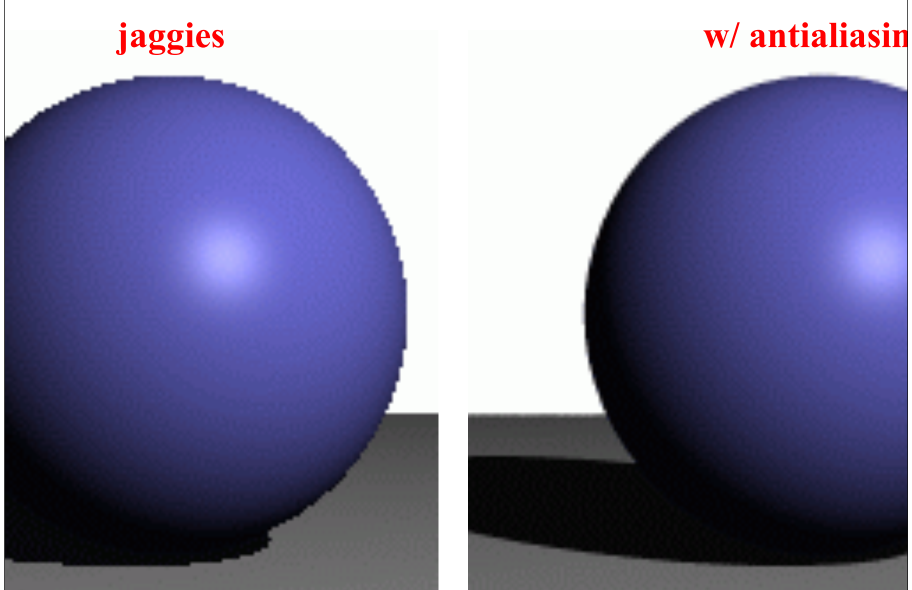
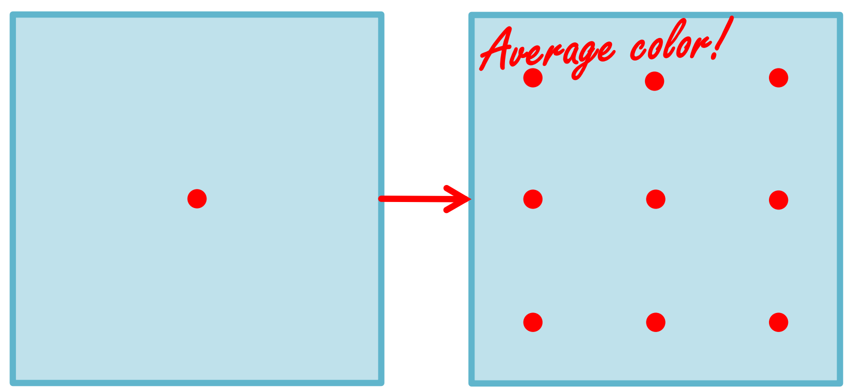


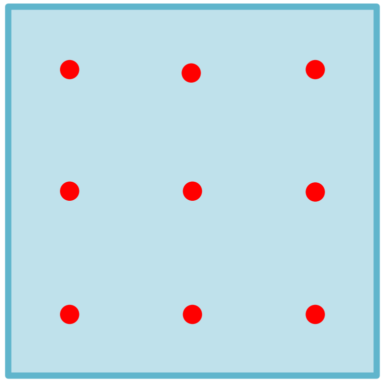


Distribution Ray Tracing: One ray per pixel is not enough



Multiple rays per pixel

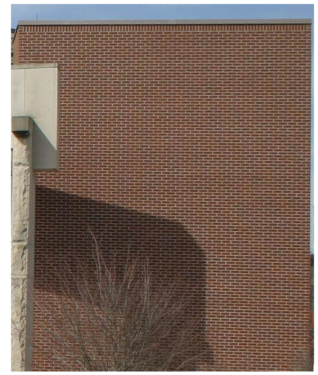
Distribution Antialiasing w/ Regular Sampling

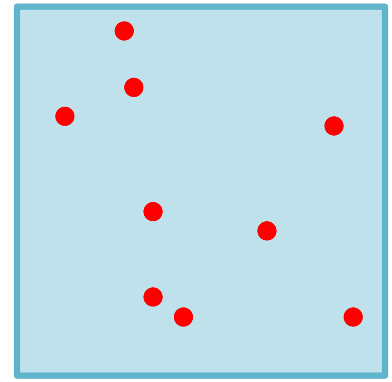


http://upload.wikimedia.org/wikipedia/commons/f/fb/Moire_pattern_of_bricks_small.jpg

Multiple rays per pixel

Even better: Distribution Antialiasing w/ Random Sampling

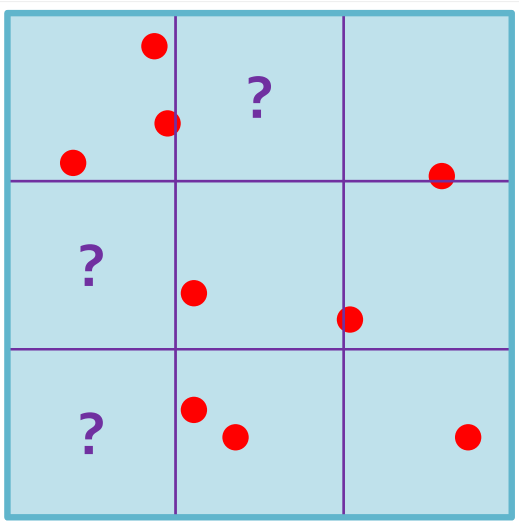




http://en.wikipedia.org/wiki/File:Moire_pattern_of_bricks.jpg

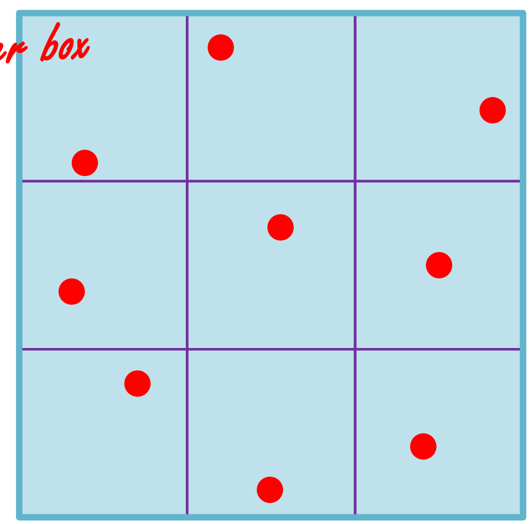
Remove Moiré patterns

Random Sampling Could Miss Regions Without Enough Sampling



Stratified (Jittered) Sampling

One ray per box



Problem: Focus
Real Lenses Have Depth of Field

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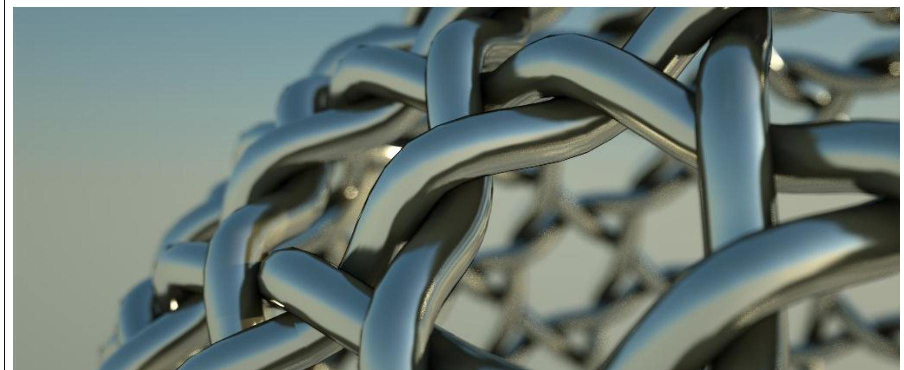
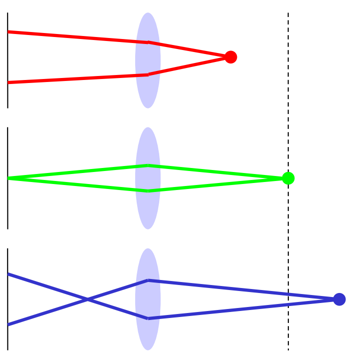
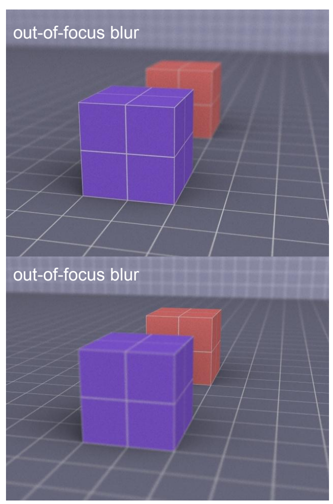


http://liam887.files.wordpress.com/2010/08/weaver.jpg

Depth of Field

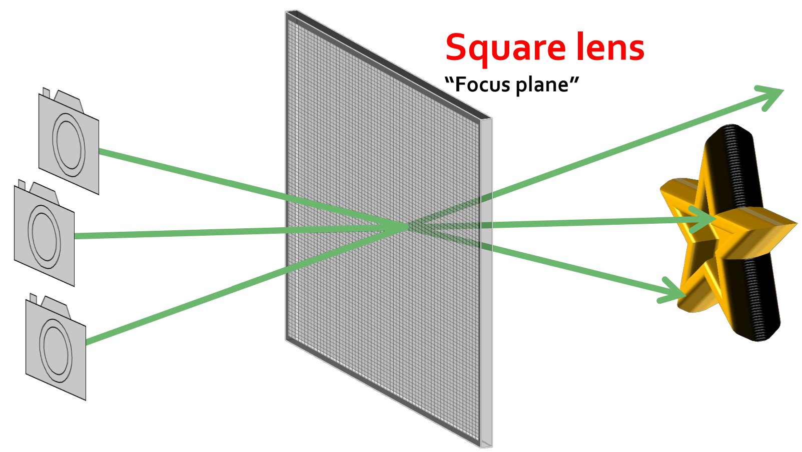
- Multiple rays per pixel, sample lens aperture





Justin Legakis

Distribution Depth of Field



Randomly sample eye positions

Problem: Exposure Time
Real Sensors Take Time to Acquire

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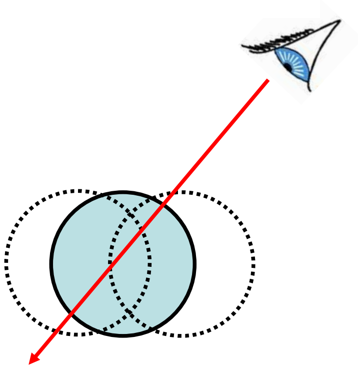
http://www.matkovic.com/anto/3dl-test-balls-01.jpg

Motion Blur

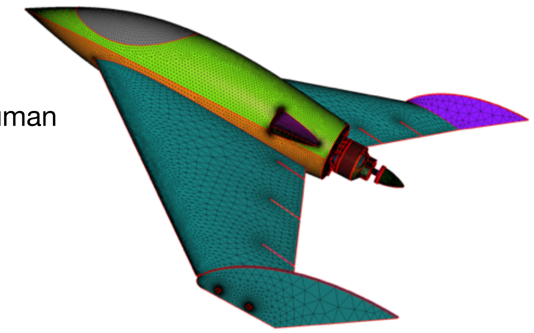
- Sample objects temporally over a time interval



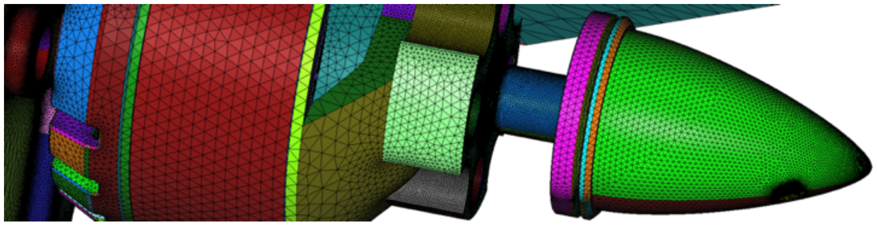
Rob Cook

Next: Triangle Meshes and other data structures

- FOCG, Ch. 12
- Check out recommended reading for some additional references



- Patrick Laug & Houman Borouchaki 2013
- 1,844,460 triangles



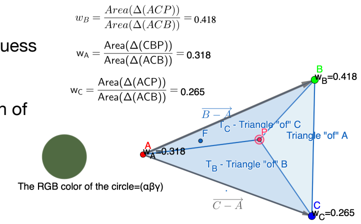
Interpolation and Barycentric coordinates

https://www.geogebra.org/m/gfau2ksn

Input a triangle given by 3 points, and attribute (say color) at each point

Also given - a point P. What is the reasonable guess about the attribute at P?

Need to interpolate using a convex combination of weights w_A, w_B, w_C , all positive and sum to 1.



Interpolation and Barycentric coordinates

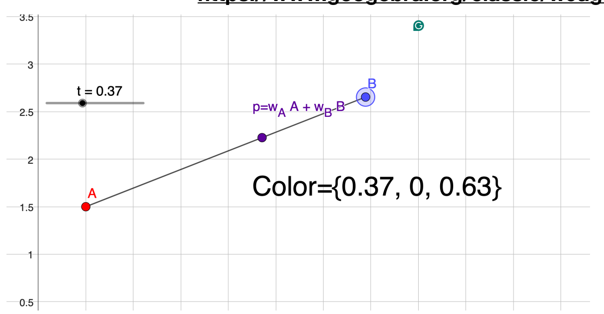
Input a triangle given by 2 points A,B , and attribute (say color) at each point

Also given - a point P. What is the reasonable guess about the attribute at P?

Need to interpolate using a convex combination of weights w_A, w_B , all positive and sum to 1.

If not all positive or not sum to 1 - then p is not on this segment.

https://www.geogebra.org/classic/w9agsjve



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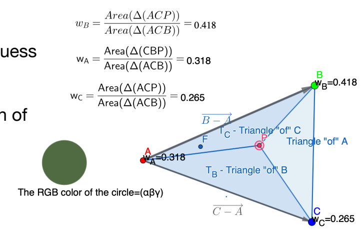


- For a pixel P inside a triangle ΔABC, the Barycentric coordinates w_A, w_B, w_C
- Specify how much weight show we give A, B, C to create P

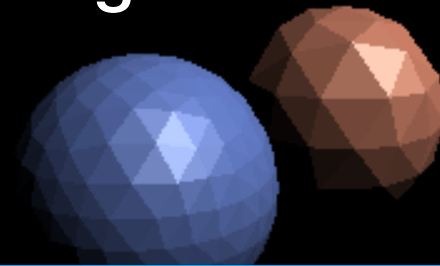
$$P = w_A \cdot A + w_B \cdot B + w_A \cdot C =$$

- Specifically $$(w_A + w_B + w_C)A + w_B\overrightarrow{(B-A)} + w_C\overrightarrow{(C-A)} = =$$ $$A + w_B\overrightarrow{(B-A)} + w_C\overrightarrow{(C-A)} =$$
- If A,B,C specifies locations, then P is on the triangle they defines
- If A,B,C are colors, then the same linear combination specifies how to interpolates the colors.
- $w_A = \frac{Area(\Delta CBP)}{Area(\Delta ABC)}$ - note = the triangle of A is the triangle that does NOT include A.
- This is also used to check if P is inside ΔABC - just check if $w_A + w_B + w_c == 1$

Shading on surfaces

- In practice, we have colors given either to each pixel (texture), or color for each vertex. The discussion below is only about shading
- For simplicity, assume surface has uniform color
- Problem: How could we produce the shading ? Shedding requires normal for each pixels
- If we are happy with a polyhedra surface - just compute for each face the normal.
- If on the other hand, the surface interpolates a smooth surface (e.g. a sphere), we should think about other alternative

Shading on Surfaces



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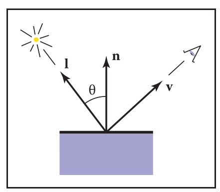
Remember Diffuse Shading

- Simple model: amount of energy from a light source depends on the direction at which the light ray hits the surface
- Results in shading that is *view independent*

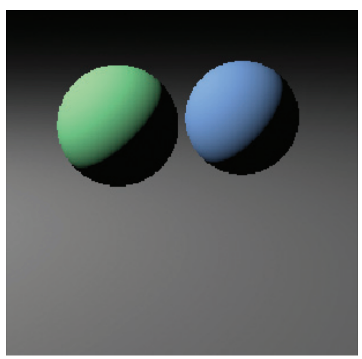


$$L_d = k_d I \max(0, \mathbf{n} \cdot \mathbf{l})$$

diffuse coefficient

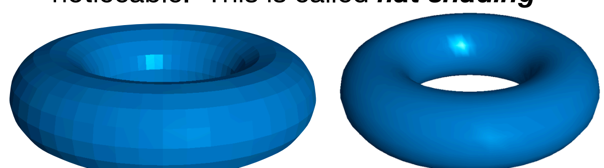
intensity/color of light

cos θ



Diffuse and specula shading on triangle meshes

- The shading of each triangle is determined by its normal (same normal for all points in the triangle). Edges of triangles are very noticeable. This is called ***flat shading***

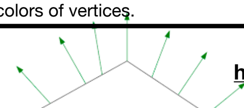


Diffuse shading formula

$$L_d = k_d \cdot I(\vec{\mathbf{n}} \cdot \vec{l})$$

$\vec{l}$ *-direction to light*

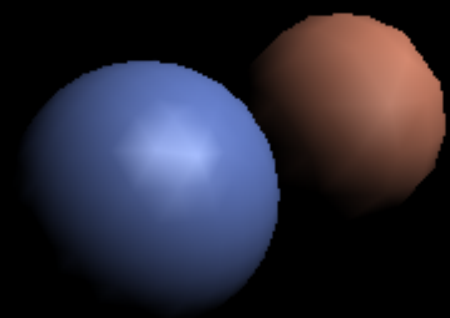
First Improvement, called **Gouraud shading**
- Compute normal at each triangle
- Approximate the normal at each vertex (sum normals of adjacent triangles, divide by their number and re-normalized)
- Compute shading at each **vertex** (using both Diffuse and specular shading)
- For each interval vertex, interpolate colors of vertices.



https://www.geogebra.org/m/vfw9bpxu

https://www.geogebra.org/m/tdstyjrx

Results of Gouraud Shading Pipeline



- Compute approx normal at vertices, compute their color and interpolate colors.

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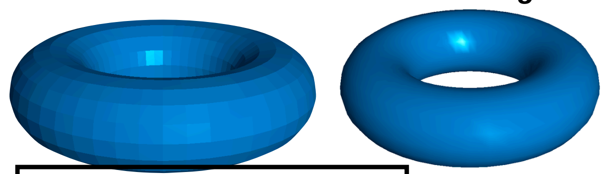

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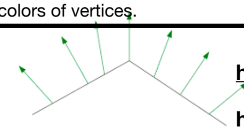
Second Improvement, called **Phong shading**
- Compute approx normal at vertex (same as Gouraud)
- Approximate the normal at pixel by interpolating the normals of its vertices.
- For each vertex, computing shading using the approximated normal



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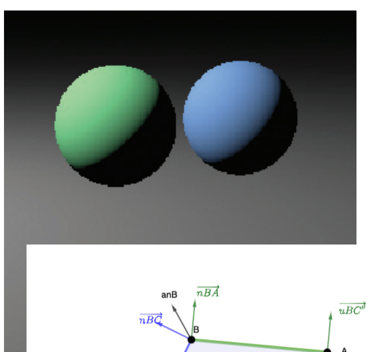

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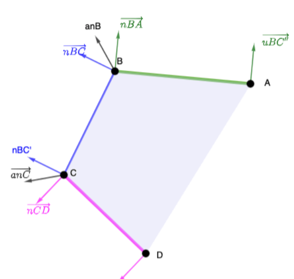
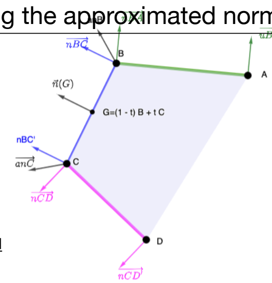




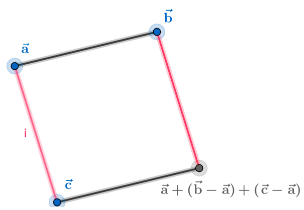
https://www.geogebra.org/m/vfw9bpxu

https://www.geogebra.org/m/tdstyjrx

Modeling Complex Shapes

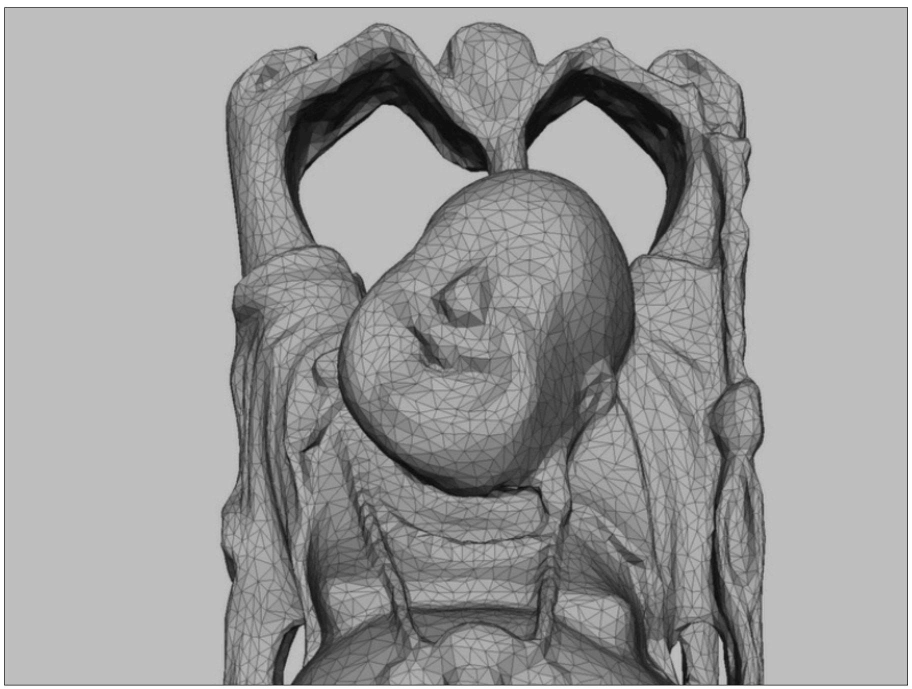
Recall: Shape Models That We Have So Far

- Implicit Shapes ($f(\mathbf{p}) = 0$ for all $\mathbf{p}$ on shape):
  - Sphere: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$
  - Plane: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{a}) \cdot \mathbf{n} = 0$
- Parametric Shapes ($\mathbf{p}(t)$ is a point on shape for all t):
  - Rays: $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$
  - Triangles:
    $p = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}, \quad 0 \leq \alpha, \beta, \gamma$ and $\alpha + \beta + \gamma = 1$
  - Triangle (second form)
    $p = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}), \quad 0 \leq \beta, \gamma$ and $\beta + \gamma \leq 1$
  - Parallelogon $p = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}) \qquad 0 \leq \beta, \gamma \leq 1$



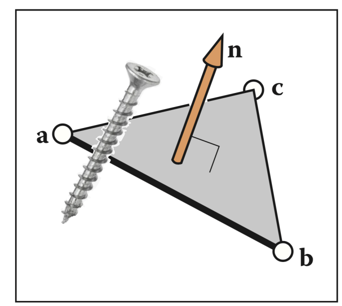
Triangle Meshes

- Are used in a huge number of applications
- Can be used to represent complex shapes by breaking them into simple (perhaps the simplest) two-dimensional elements

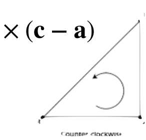


Definition of Triangles

- 3 **vertices** (points $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ in 3D space)
- The normal of the triangle is a vector, $\mathbf{n}$, that points to its front side
- Convention: vertices listed in counter-clockwise order from the "front" of the triangle

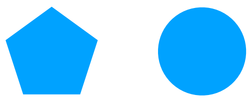


$$\mathbf{n} = (\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$$



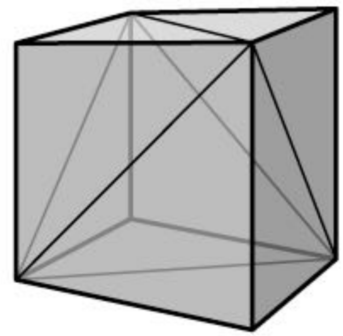
Definition of Triangle Meshes

- In short, a collection of triangles in 3D space that are connected to form a surface
  - Terminology: vertices, edges, triangles
  - Surface is piecewise planar, except where two triangle meet which forms a crease and their shared edge
  - Meshes are often a **piecewise** approximation of a smooth surface. We will study how graphics can hide the artifacts, creates the illusion of a **smooth** surface without increasing their comolexity.



A Simple Mesh

- How many vertices? How many triangles?



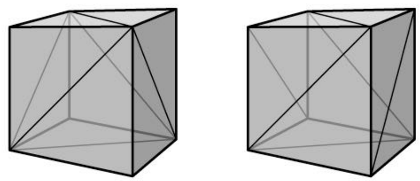
Mesh Topology

Two Considerations for Meshes

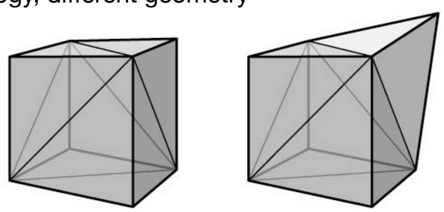
- We typically care about the mesh being a good approximation to a surface:
  - This leads to questions of **mesh geometry**, e.g.: How many triangles? where to place their vertices?
- We also care about how these triangles are connected
  - This leads to questions of **mesh topology**, e.g.: Are there holes in the mesh? How do triangles intersect?
  - Mesh topology can affect assumptions on algorithms that process meshes

Topology vs. Geometry

- Same geometry, different topology



- Same topology, different geometry

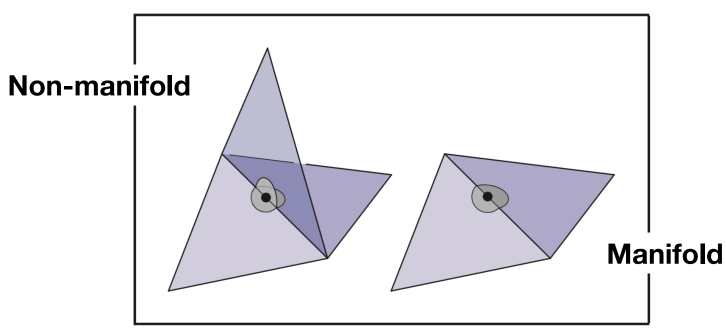


Topological Validity

- Meshes that approximate surfaces should be manifolds
- Definition: A (2-dimensional) **manifold** is a space where every point locally appears to be 2-dimensional space
  - 3 cases: points that are on edges, points that are vertices, and points that are interior to triangles.

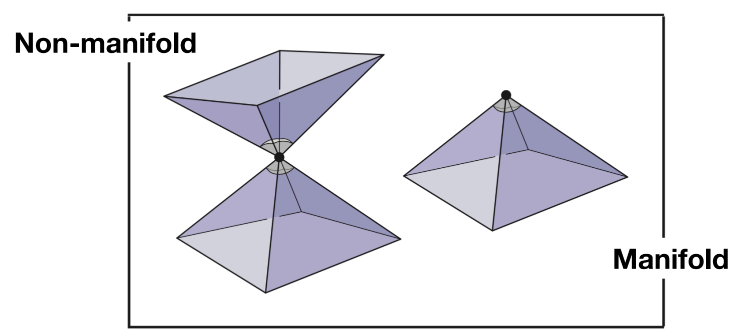
When is a Mesh a Manifold?

- Definition: A (2-dimensional) manifold is a space where every point locally appears to be 2-dimensional space
- Implication: Every edge is shared by exactly two triangles



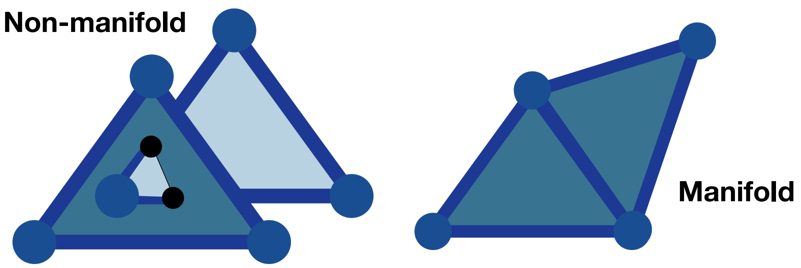
When is a Mesh a Manifold?

- Definition: A (2-dimensional) manifold is a space where every point locally appears to be 2-dimensional space
- Implication: Every vertex has a single, complete loop of triangles around it



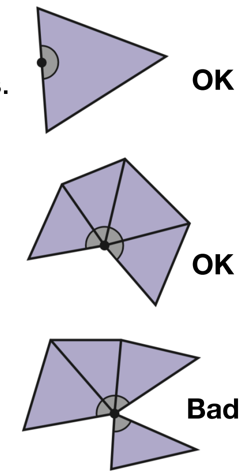
When is a Mesh a Manifold?

- Definition: A (2-dimensional) manifold is a space where every point locally appears to be 2-dimensional space
- Implication: Triangles only intersect at vertices and edges



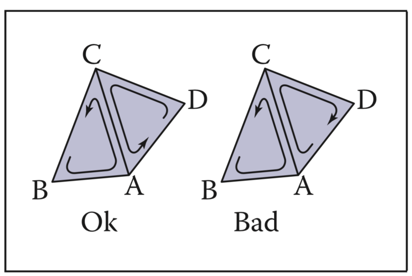
Manifolds with Boundary

- Sometimes, we relax the manifold condition to allow meshes with boundaries.
- Every point on a **manifold with boundary** either locally appears to be 2-dimensional space or 2-dimensional half-space
  - Every edge is used by either one or two triangles
  - Every vertex connects to a single edge-connected set of triangles

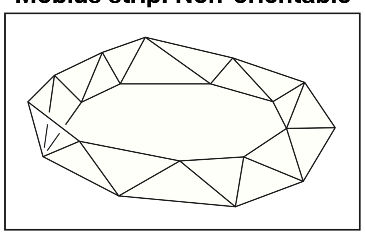


Consistent Orientation

- In many applications, all triangles facing the same way is important
  - Can be used to distinguish inside from outside.
- If consistent: neighboring triangles will appear to disagree on the order of vertices on their shared edge



Möbius strip: Non-orientable



Simple Representations of Triangle Meshes

Important Concerns w/ Representing Triangle Meshes

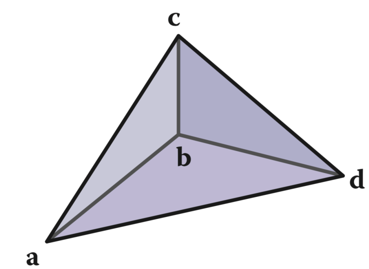
- Efficiency of storage size
  - Many representations store redundant information
- Efficiency of access
  - How quickly can we get the information we need for rendering?
  - How quickly can we get neighborhood information, for mesh modification?

Using Separate Triangles

- Use a simple structure to store each triangle:

```
Triangle {
  vertexPositions[3];  //Vec3
};
```

- Store a triangle mesh using an array of `Triangle`
- Problems: The coordinates and other properties (colors) of a vertex are stored multiple times: Could be bad because of
  1. Wasteful (large numbers)
  2. concurrency issues
  3. In certain scenarios, the very same vertex might appear with different locations. For example, start from a vertex at $x = 1/3$. Resize by scaling by 3. Is the vertex at $x = 0.99999$ or at $x = 1$?



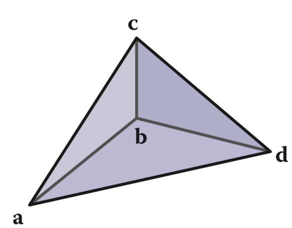
#	*Vertex* 0	*Vertex* 1	*Vertex* 2
0	(a_x, a_y, a_z)	(b_x, b_y, b_z)	(c_x, c_y, c_z)
1	(b_x, b_y, b_z)	(d_x, d_y, d_z)	(c_x, c_y, c_z)
2	(a_x, a_y, a_z)	(d_x, d_y, d_z)	(b_x, b_y, b_z)

Using Indexed Meshes

- Triangles share a common list of vertices, storing only references/pointers:
- A vertex (and its related information (RGB etc) is stored only once.

```
Triangle {
  vertices[3]; //object reference or int
};

Vertex {
  position;    //Vec3
};
```

- Store a triangle mesh using two arrays, one of `Vertex` and the other of `Triangle`
- `We will study data structures that could expedite some operations (not today)`



Triangles

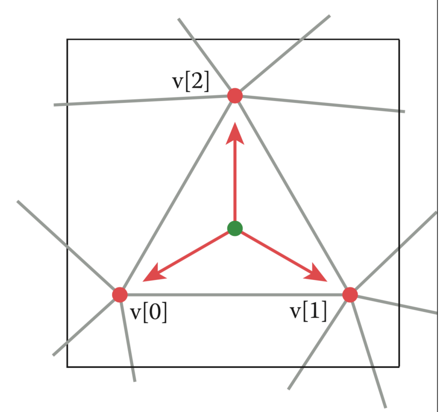
#	*Vertices*
0	(0, 1, 2)
1	(1, 3, 2)
2	(0, 3, 1)

Vertices

#	*Position*
0	(a_x, a_y, a_z)
1	(b_x, b_y, b_z)
2	(c_x, c_y, c_z)
3	(d_x, d_y, d_z)

Using Indexed Meshes

- Each triangle thus tracks references to the vertices associated with it

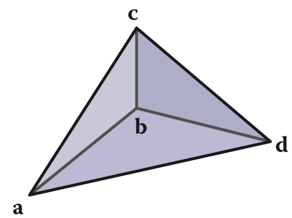


Using Indexed Meshes

- Alternatively one can store using array indices directly:

```
IndexedMesh {
  vertices[num_verts];   //Vec3
  triIndices[num_tris];  //int
};
```

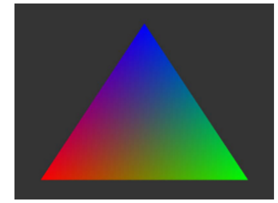
- Plus, it is easy (or at least easier) to see which two triangles share an edge.

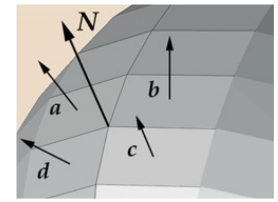


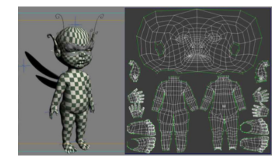
Triangles #	Triangles Vertices	Vertices #	Vertices Position
0	(0, 1, 2)	0	(a_x, a_y, a_z)
1	(1, 3, 2)	1	(b_x, b_y, b_z)
2	(0, 3, 1)	2	(c_x, c_y, c_z)
		3	(d_x, d_y, d_z)

Data on Meshes

- Typically, we store a variety of data on meshes as well
- Can store this on vertices, triangles, or even edges
- Examples:
  - Colors stored on vertices
  - Normals stored on faces
  - Texture coordinates stored on vertices
- Information stored on vertices is typically interpolated with **barycentric coordinates**

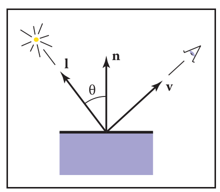






Remember Diffuse Shading

- Simple model: amount of energy from a light source depends on the direction at which the light ray hits the surface
- Results in shading that is *view independent*

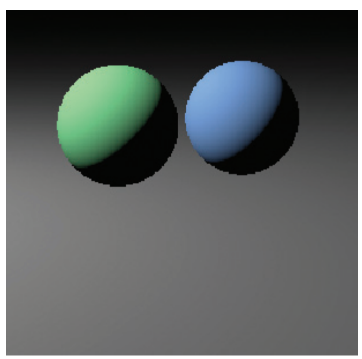


$$L_d = k_d I \max(0, \mathbf{n} \cdot \mathbf{l})$$

diffuse coefficient → k_d

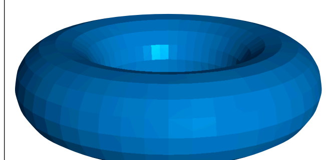
intensity/color of light → I

cos θ → $\mathbf{n} \cdot \mathbf{l}$



Diffuse and specula shading on triangle meshes

- The shading of each triangle is determined by its normal (same normal for all points in the triangle). Edges of triangles are very noticeable. This is called ***flat shading***

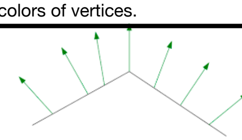


Diffuse shading formula

$$L_d = k_d \cdot I(\vec{\mathbf{n}} \cdot \vec{l})$$

$\vec{l}$ *-direction to light*

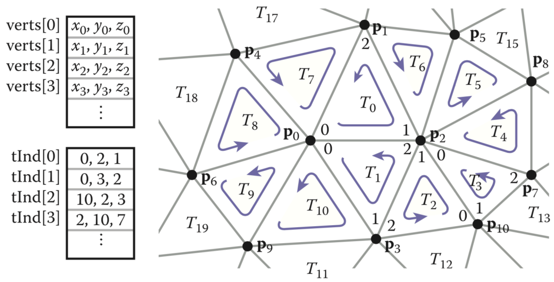
First Improvement, called **Gouraud shading**
- Compute normal at each triangle
- Approximate the normal at each vertex (average the normals of the adjacent triangles)
- Compute shading at each **vertex** (using both Diffuse and specular shading)
- For each interval vertex, interpolate colors of vertices.



Second Improvement, called **Phong shading**
- Compute approx normal at vertex (same as Gouraud)
- Approximate the normal at pixel by interpolating the normals of its vertices.
- For each vertex, computing shading using the approximated normal

Mesh File Formats: *.obj

- Widely used format for indexed meshes
- Supports additional data stored on vertices and polygons



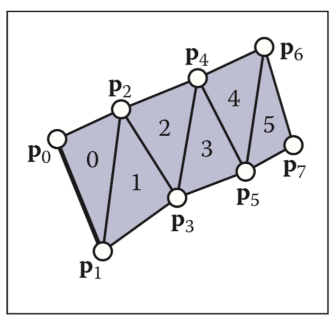

```
#sample .obj file

v 0.000 0.000 0.000
v 0.500 0.809 0.309
v 1.000 0.000 -0.309
v 0.583 -0.720 0.225
v -0.630 0.750 0.025
...

f 1 3 2
f 1 4 3
f 11 3 4
f 3 11 7
...
```

We will get back to geometric data structures in the future.

Triangle Meshes More Efficient Representations

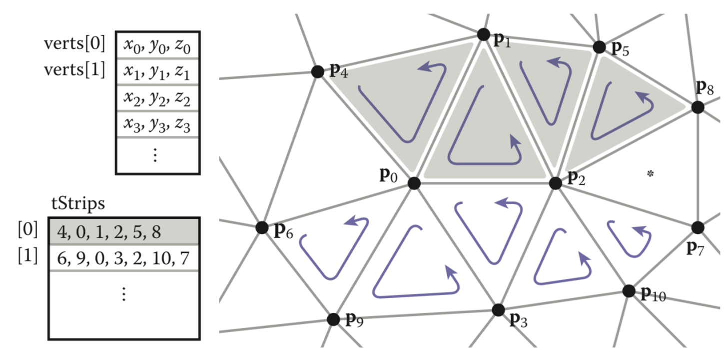
Triangle Strips

- Idea: Rely on the mesh property and group triangles that share common vertices
- Create a new triangle by reusing the last two vertices in the strip
- [0,1,2,3,4,5,6,7] specifies the sequence on the right with triangles (0,1,2), (1,2,3), (2,3,4) …
- Have to invert every other for consistent orientation



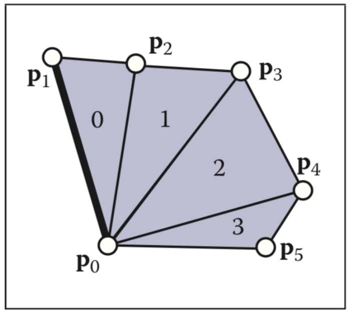
Triangle Strips

- Complex meshes store list of strips
- How long of a strip to use?



Triangle Fans

- Same idea as triangle strips, but keep the earliest vertex in the list instead of the last two
- [0,1,2,3,4,5] specifies the sequence on the right with triangles (0,1,2), (0,2,3), (0,3,4), …



Mesh Data Structures and Queries

Queries on Meshes

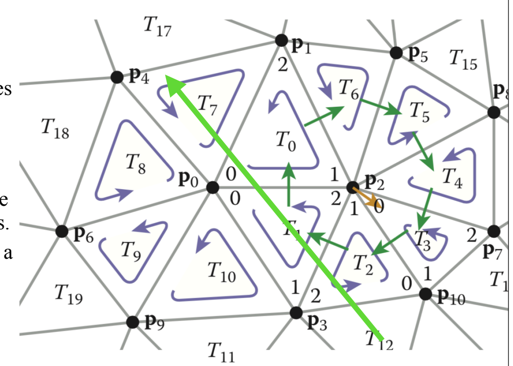
- For face, find all:
  - Vertices
  - Edges
  - Adjacent faces
- For vertex, find all:
  - Incident edges
  - Incident triangles
  - Neighboring vertices
- For edge, find:
  - Two adjacent faces
  - Two adjacent vertices

```
Triangle {
  v[3];    //Vertex
  e[3];    //Edge
  adj[3]; //Triangle
}

Vertex {
  t[];     //Triangle
  e[];     //Edge
  adj[];   //Vertex
}

Edge {
  v[2];    //Vertex
  t[2];    //Triangle
}
```

Can we do better?

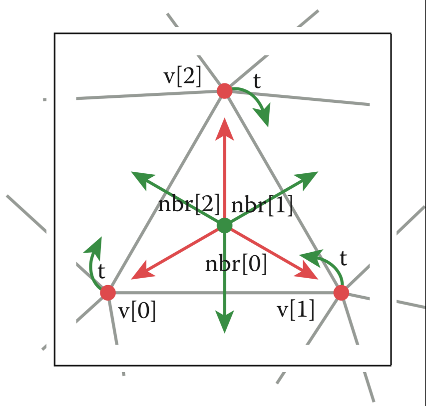
Typical Operations on the Data Structure

- Once a triangle T_0 is given which triangles are neighboring T_0 ?
- Given a ray r, which triangles intersect r?
- (Older material) is a point q=(x,y,z) inside T_0 ? (solved with barycentric coordinates.
- Who are the triangles that are adjacent to a vertex p_0 ?



Triangle-Neighbor Structure

- Let's try first extending the indexed mesh structure for sharing vertices
- Add pointers, `nbr[]`, to 3 neighboring triangles
- Add a single pointer, `t`, for each vertex to one of its adjacent triangles
- Can now enumerate triangles adjacent to vertices

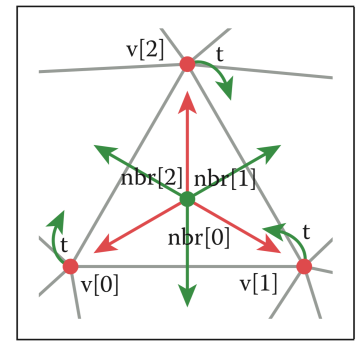


Triangle-Neighbor Structure

```
Triangle {
  v[3];     //Vertex
  nbr[3];   //Triangle
}

Vertex {
  ...
  t;        //Triangle
}
```

…or…

```
IndexedMesh {
  ...
  tInd[num_tris];   //int[3]
  tNbr[num_tris];   //int[3]
  vTri[num_verts];  //int
};
```



Triangle-Neighbor Structure

vTri

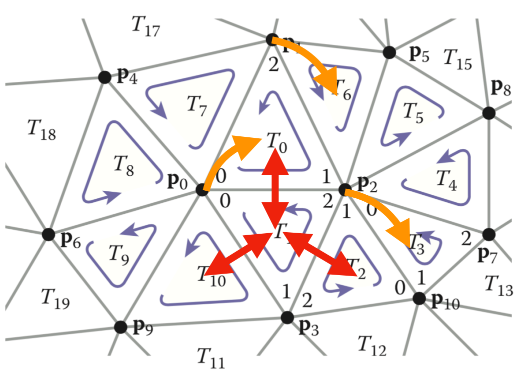
[0]	0
[1]	6
[2]	3
[3]	1
	⋮

tNbr

[0]	1, 6, 7
[1]	10, 2, 0
[2]	3, 1, 12
[3]	2, 13, 4
	⋮

tInd

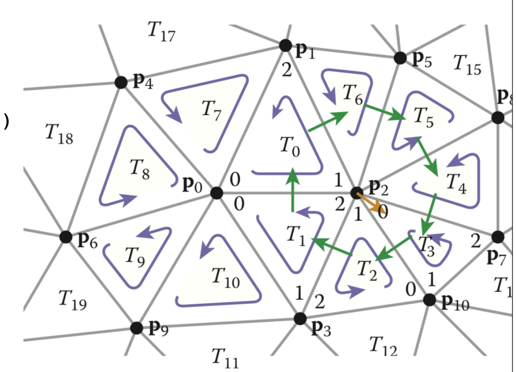
[0]	0, 2, 1
[1]	0, 3, 2
[2]	10, 2, 3
[3]	2,10,7
	⋮



Triangle-Neighbor Structure

```
TrianglesOfVertex(v) {
  t = v.t
  do {
    find i where (t.v[i] == v)
    t = t.nbr[i]
  } while (t != v.t);
}
```

- Can optimize by storing pointers to neighboring edges

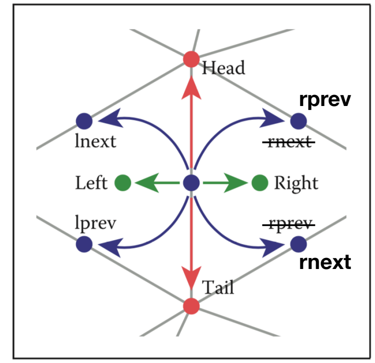


Triangle-Neighbor Structure

- Recall that indexed meshes needed $36*n_v$ bytes and $n_t \approx 2n_v$
- We added an array of triples of indices (per triangle)
  - This increases storage by $3*4*n_t$ or $24*n_v$ bytes
- We also added an array of representative triangle per vertex
  - This increases storage by $4*n_v$ bytes
- Total storage: 36 + 24 + 4 = 64 bytes per vertex
  - Still not as much as separate triangles

Winged-Edge Structure

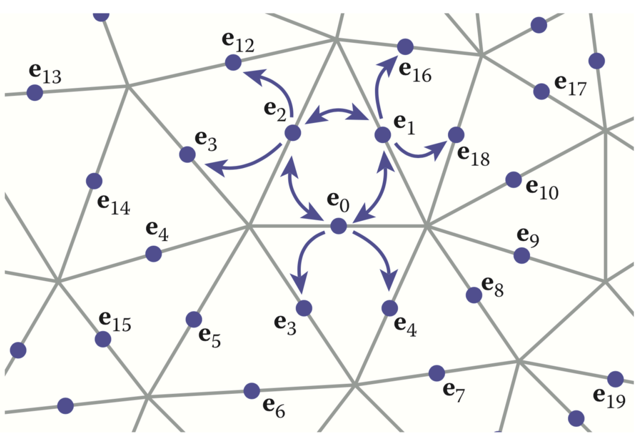
- Widely used mesh structure that focuses on edges instead of triangles
- Edges store pointers to:
  - Head/Tail vertices
  - Left/Right triangles
  - Left/Right "next" edges
  - Left/Right "previous" edges
- Each vertex/triangle stores one pointer to some edge



Winged-Edge Structure

Winged-edge table

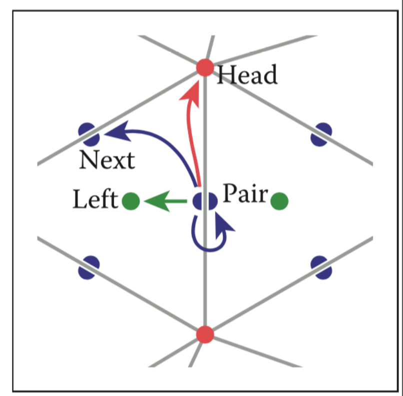
	ln	rp	lp	rn
[0]	1	4	2	3
[1]	18	0	16	2
[2]	12	1	3	0
	⋮			



Half-Edge Structure

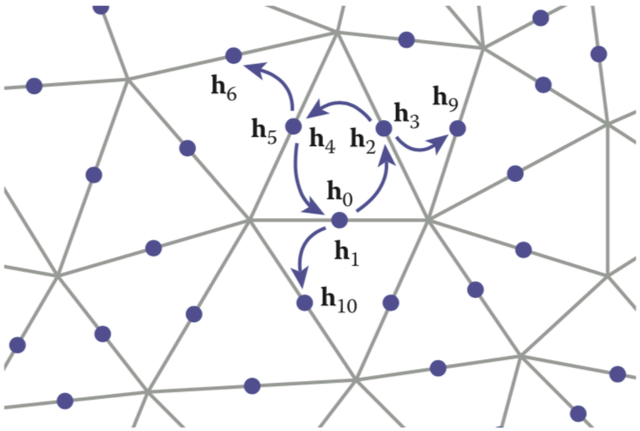
(sometimes called Doubly connected Edge List -DCEL)

- Simplifies winged-edge, removes awkwardness of checking which way edges are oriented
- Each **half-edge** store pointers to:
  - Head vertex
  - Left triangle
  - Left "next" edge
  - The opposite "pair" half-edge (the twin edge)
- Each vertex/triangle stores one pointer to a half-edge



Half-Edge Structure

	Pair	Next
hedge[0]	1	2
hedge[1]	0	10
hedge[2]	3	4
hedge[3]	2	9
hedge[4]	5	0
hedge[5]	4	6
	⋮	



Half-Edge Structure

```
HEdge {
  pair, next;  //HEdge
  v;           //Vertex
  f;           //Face
};

EdgesOfVertex(v) {
  h = v.h;
  do {
    h = h.next.pair;
  } while (h != v.h);
}
```

```
EdgesOfVertex(v) {
  e = v.e;
  do {
    if (e.tail == v) {
      e = e.lprev;
    } else {
      e = e.rprev;
    }
  } while (e != v.e);
}
```

Winged-Edge Implementation

Half-Edge Storage Requirements

- Vertex data: 3 floats for position, 1 int for edge reference
  - 4*4 = $16n_v$ bytes
- Face data: 1 int for edge reference
  - 4*1 = $4*n_t$ = $8n_v$ bytes.
- Edge data, 4 ints for references, but store a pair of half edges for each edge
  - $n_h \approx 6n_v$
  - 8*4*6 = $96n_v$ bytes.
- In total, $120n_v$ bytes.