Animation

Jungle Book (1967)

Pencil Test

Computer Animation
Keyframing

Keyframe Animation

• Idea: Draw a subset of important frames (called **key frames**) and fill in the rest with in-betweens
• In hand-drawn animation, the head animator would draw the poses and the assistants would do the rest
• In computer animation, the artist draws the keys and the computer does the in-betweening
  • Interpolation is used to fill in the rest!

Double Buffering

• If you draw directly to video buffer, the user will see the drawing happen
• Particularly noticeable artifacts when doing animation

Controlling geometry conveniently

• Manually place every control point at every keyframe?
  – labor intensive
  – hard to get smooth, consistent motion
• Animate using smaller set of meaningful degrees of freedom
  – modeling DOFs are inappropriate for animation
e.g. “move one square inch of left forearm”
  – animation DOFs need to be higher level
e.g. “bend the elbow”
Controlling shape for animation

- Start with modeling DOFs (control points)
- Deformations control those DOFs at a higher level
  - Example: move first joint of second finger on left hand
- Animation controls control those DOFs at a higher level
  - Example: open/close left hand
- Both cases can be handled by the same kinds of deformers

Character with DOFs

- Surface is deformed by a set of bones
- Bones are in turn controlled by a smaller set of controls
- The controls are useful, intuitive DOFs for an animator to use

Interpolating Rotations
**The most basic animation control**

- Affine transformations position things in modeling
- Time-varying affine transformations move things around in animation
- A hierarchy of time-varying transformations is the main workhorse of animation
  - and the basic framework within which all the more sophisticated techniques are built

**Interpolating transformations**

- Move a set of points by applying an affine transformation
- How to animate the transformation over time?
  - interpolate the matrix entries from keyframe to keyframe?
    - *this is fine for translations but bad for rotations*

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**Interpolating Rotations**

\[
\frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
\]

90° CW  \hspace{1cm} 90° CCW

**Not a rotation matrix!**

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**Interpolating transformations**

- Linear interpolation of matrices is not effective
  - leads to shrinkage when interpolating rotations
- One approach: always keep transformations in a canonical form (e.g. translate-rotate-scale)
  - then the pieces can be interpolated separately
  - rotations stay rotations, scales stay scales, all is good

Issues occur when the source and target angles are not close to each other
Could Instead Decompose Rotation by Euler Angles

Parameterizing rotations

- Euler angles
  - rotate around x, then y, then z
  - nice and simple
  \[
  R(\theta_x, \theta_y, \theta_z) = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)
  \]

Gimbal Lock
**Quaternions Representation and their properties**

- Representing each rotation as a 4 values
- Encapsulate a rotation axis, and amount of rotation
- (if rotation axis is X,Y,Z, then we are back to Eulear Coordinates)
- Corresponds to points in the 4D unit sphere. Yet lets stick to the 3D unit sphere
- Represent rotations by source and destination on unit sphere, with the understanding that rotation is along a geodesic (shortest path).
- No Gimble lock
- Could be represented as $4 \times 4$ matrices, so could be concatenated easily (matrix multiplication)

Rotation from $q_1 \rightarrow q_2$ could be specified by the axis of rotation $(\alpha - q_1) \times (\alpha - q_2)$ and the length (in radians) of this arc

This is a good start. This solves the Gimble Lock issue, but fail to address

1) Rotation around its own axis (the missing degree of freedom)
2) Concatenations of rotations

**Interpolating between quaternions**

- Why not linear interpolation?
  - Need to be normalized
  - Does not have constant rate of rotation

$$\beta = \psi$$

$$v(t) = w_0v_0 + w_1v_1$$

$$\sin \alpha = \sin \beta = \sin(\pi - \psi) = \sin \psi$$

$$w_0 = \sin \beta / \sin \psi$$

$$w_1 = \sin \alpha / \sin \psi$$

$$v = \cos^{-1}(v_0 \cdot v_1)$$

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$$\alpha + \beta = \pi$$

$$\alpha = \pi - \beta$$

$$\alpha = \pi - \sin^{-1}(\psi)$$

$$\sin^{-1}(\psi) = \frac{\sin((1-\alpha)x + \alpha y)}{||((1-\alpha)x + \alpha y)||}$$

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https://www.geogebra.org/m/mwuczhjw
Spherical linear interpolation ("slerp")

\[ \alpha + \beta = \psi \]
\[ \mathbf{v}(t) = w_0 \mathbf{v}_0 + w_1 \mathbf{v}_1 \]
\[ \sin \alpha \]
\[ \frac{w_1}{w_0} = \frac{\sin \beta}{\sin \psi} \]
\[ w_0 = \sin \beta / \sin \psi \]
\[ w_1 = \sin \alpha / \sin \psi \]
\[ \psi = \cos^{-1}(\mathbf{v}_0 \cdot \mathbf{v}_1) \]

Quaternion Interpolation

- Spherical linear interpolation naturally works in any dimension
- Traverses a great arc on the sphere of unit quaternions
  - Uniform angular rotation velocity about a fixed axis

\[ \psi = \cos^{-1}(q_0 \cdot q_1) \]
\[ q(t) = \frac{q_0 \sin(1 - t) \psi + q_1 \sin t \psi}{\sin \psi} \]

https://www.geogebra.org/m/mwuczjhjw

Character Animation

Animating w/ Skeletal Hierarchies
Forward vs. Inverse Kinematics

After hip rotation

Original

After knee rotation

IK solver connection

Hip and knee joint angles computed automatically

Effector motion

Inverse Kinematics Solves for all Intermediate Constraints

https://youtu.be/0a9ql7kwiA?t=50