

CSc 437  
Homework 3 (100 pts.)  
Due: 10/24/11

**Instructions.** All assignments are to be completed on separate paper. Use only one side of the paper. Assignments will be due at the beginning of class, or via email. To receive full credit, you must show all of your work.

Unless otherwise specified, all questions are taken from the textbook (second edition)

1. 4.10
2. 4.13
3. 4.15
4. 4.16
5. Given a set  $S$  of  $n$  points in the plane, find in expected time  $O(n)$ , the smallest area ellipse  $e$ , which is axis-parallel, its ratio of its height to its width is 1 : 3, and contains  $S$ . (hint — there is a one-line solution).
6. Given two sets  $R$  and  $B$ , each containing  $n$  points in  $\mathbb{R}^d$ , suggest an algorithm that finds in expected time  $O(n)$  if there is a halfspace that contains all the points of  $R$  and none of the points of  $B$ . A halfspace is the region of all points lying on one of a side of a hyperplane. You can assume that the dimension  $d$  is fixed, so any functions that depends only on  $d$  can be considered a constant.
7. Give an example of a set  $S$  of points in the plane, and the  $kD$  – tree constructing on them, and an example on a reporting query on  $S$ , such that the query takes  $\Omega(\sqrt{n})$  time, even though the output size  $k$  is only zero (no point is reported)
8. 5.4
9. 5.11
10. 5.13 (only (a) and (b))
11. You are given a set of  $n$  unit disks in the plane, all containing a point  $p$ . Preprocess them into a data structure so that given a query point  $q$ , you can answer in  $O(\log n)$  time whether *any* disks of  $S$  containing  $q$ . The preprocessing and storage are  $O(n \log^2 n)$  and  $O(n \log n)$  respectively.  
Hint — All you need to do is to store the vertices of the boundary of the union of the disks. Understand how the union “typically” looks like. Prove that its complexity is  $O(n)$  (recall that the disks all have the same radius)

and use a question from a previous homework to compute the boundary of this union. Use polar coordinate system (a points  $x$  is represented by the angle that the segment  $xO$  is created with the positive  $x$ -axis, and the distance  $\|x - O\|$ , where  $O$  is the origin.