

CSc 437  
Homework 4 (100 pts.)  
Due: 11/7/11

**Instructions.** All assignments are to be completed on separate paper. Use only one side of the paper. Assignments will be due at the beginning of class, or via email. To receive full credit, you must show all of your work.

All questions are taken from the textbook (second edition)

1. 6.8
2. **10 points bonus** 6.9. In particular 1) prove what is the complexity of the planar map (as a function of  $n$  and  $A$ ). 2) Explain which modifications are needed (both algorithmic and data structures) to the algorithm studied in class so it can handle intersecting segments, 3) Argue that the expected path length in the graph is  $O(\log n)$ , and 4) What is the total time needed to insert all segments.
3. 6.10
4. 6.16
5. Let  $p$  be a point in the plane, and let  $s_i = NN_S(p)$ . Prove directly, without using bisections, that any point  $x$  on the segment  $s_i p$  satisfies  $s_i = NN_S(x)$ . Prove this fact to bound the number of cells in  $VD(S)$ . Use this fact to bound the complexity of  $VD(S)$ .
6. Let  $S = \{s_1 \dots s_n\}$ , be a set of sites, and let  $\{w_1 \dots w_n\}$  be a set of real numbers. We define the distance of a point  $x$  from  $d_i(x, s_i)$ , the *weighted distance* from  $x$  to  $s_i$  to be  $d_i(x) = w_i + \|x - s_i\|$  where  $\|x - s_i\|$  is the Euclidean distance between  $x$  and  $s_i$ . Repeat the previous question in this case. Which family of curves represent the bisectors between sites? What is the answer to this question if all  $w_i = 1$  for every  $i$ .
7. Prove that a point  $s_i \in S$  is a vertex of  $CH(S)$  iff the cell of  $s_i$  in  $VD(S)$  is unbounded.
8. Prove that if not all the sites of  $S$  lie on a line, then the edges of  $VD(S)$  form a connected graph
9. Assume  $VD(S)$  is given to you, and let  $s'$  be a site not in  $S$ . Suggest an  $O(n)$  time algorithm for computing  $VD(S \cup s')$ .
10. Given  $S$  as above, suggest an  $O(n \log n)$  algorithm to determine if you can continuously translate a unit disk from a starting position when it centers is on site  $s_i$  to a position where its center is a different site  $s_j$ , so that at any intermediate stage the disk is disjoint to all sites of  $S$ .

11. 7.3

12. 7.11

13. 9.2

14. 9.13