Tries and suffixes trees

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Trie: A data-structure for a set of words

All words over the alphabet Σ={a,b,..z}.
In the slides, the alphabet is only {a,b,c,d}.
S – set of words = {a,aba, a, aca, addd}.
Need to support the operations

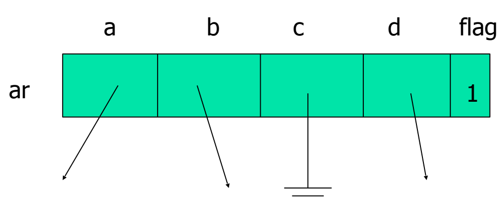
- insert(*w*) – add a new word *w* into *S*.
- delete(*w*) – delete the word *w* from *S*.
- find(*w*) is *w* in S ?
  - Future operation:
    - Given text (many words) where is *w* in the text.

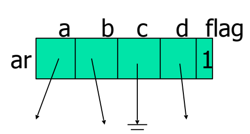
- The time for each operation should be O(*k*), where *k* is the number of letters in *w*

- Usually each word is associated with addition info – not discussed here.

Trie (Tree+Retrive) for S

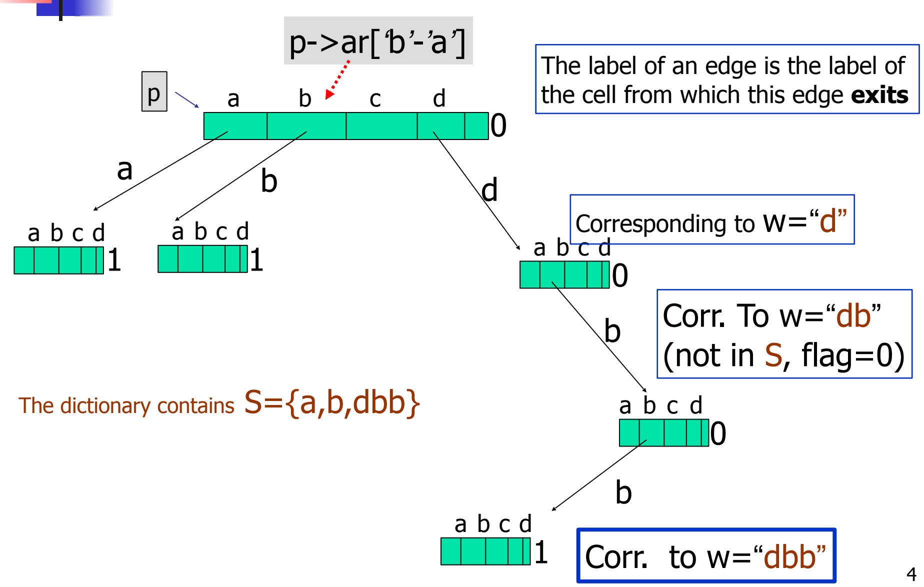
- A tree where each node is a struct consist
- Struct node {
  - char[4] *ar;
  - char flag ; /* 1 if a word ends at this node. Otherwise 0 */





Rule:
Each node corresponds to a word w.
w∈ S **iff** flag=1

A trie - example



Finding if word *w* is in the tree

p=root; i =0 // remember - each string ends with `\0'

While(1){

- If w[i] == '\0' //we have scanned all letters of w
  - then return the flag of p ; **else**
- If $(p.a[w[i]-'a']) == NULL$ //the entry of p correspond to w[i] is NULL

  return **false**;
- $p = (p.a[w[i]-'a'])$ //Set p to be the node pointed by this entry
- i++;

}

Inserting a word *w*

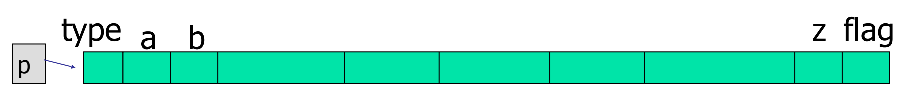
- Try to perform find(*w*).
  - If runs into a NULL pointers, create new nodes along the path.
  - The flag fields of all new nodes is 0.
- Set the flag of the last node to 1

Deleting a word *w*

- Find the node p corresponding to w (using `find' operation).
- Set the flag field of p to 0.
- If p is dead (I.e. flag==0 and all pointers are NULL) then free(p), set p=parent(p) and repeat this check.

Heuristics for saving space

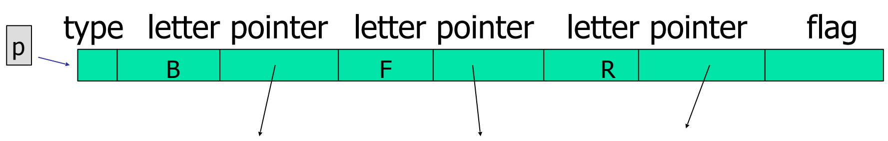
- The space required is Θ(|Σ| |S|).
- To save some space, if Σ is larger, there are a few heuristics we can use. Assume Σ={a,b..z} .
- We use two types of nodes
  - Type "A", which is used when the number of children of a node is more than 3



Note – the letters are not stores explicitly

Heuristics for space saving

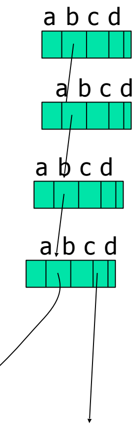
- Type “B” is used if there are 3 or less children:
- The “letter” of the child is also stored:

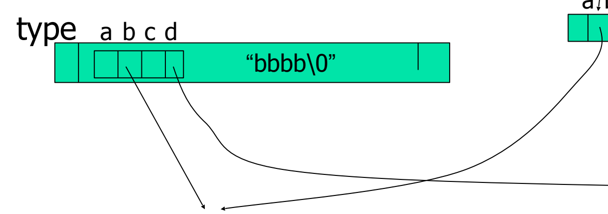


•The rule of the flag is the same as in type “A” nodes.
•We only store the 3 pointers, but we need to know to which letters they corresponds to.

Another Heuristics – path compression

- Replace a long sequence of nodes, all having only one a single child, with a single node (of type “pointer to string”) that maintains
  - a point to the next node,
  - a point to the string.





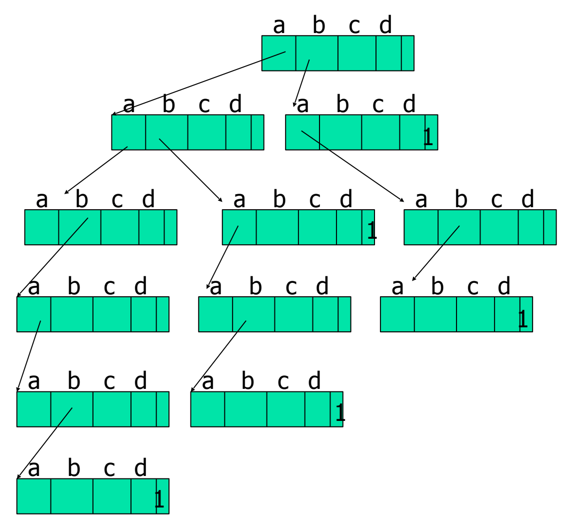
Suffix tree.

- Assume *B* (for book) is a very long text.
- Want to preprocess *B*, so when a word *w* is given, we can quickly find if it is in *B*.
- We can find it in O(|*w*|).
- Idea:
  - Consider *B* as a long string.
  - Create a trie *T* of all suffixes of *B*.
  - In addition to the flag (specifying if a word ends at node), we also stored the index in *B* where this word begins.
  - Example *B=“aabab”*
    S={“aabab”, “abab”, “bab”, “ab”, “b”}

Observation: w appears in B ⇔
w is the prefix of a suffix of B.
Example: B=“hello**nice**world”, w=“nice”.

Suffix tree.

Example *B=“aabab” S={“aabab”, “abab”, “bab”, “ab”, “b”}*



To know **where** a word appear in B, we store with each node the index of the beginning of the suffix in B.

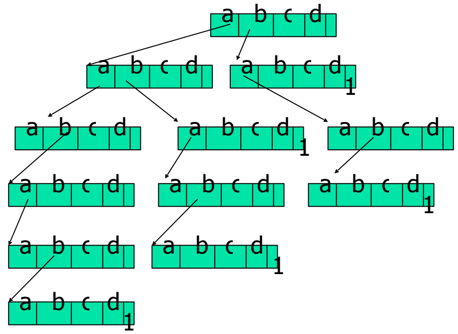
(we can store only the first appearance of the word in the text)

Size of suffix tree

Example *B="aabab"* *S={"aabab", "abab", "bab", "ab", "b"}*

Assume n=|B|.
Total length of all string $\Theta(n^2)$
Size of a node is $|\Sigma|$
So size of the tree is $\Theta(n^2 |\Sigma|)$.

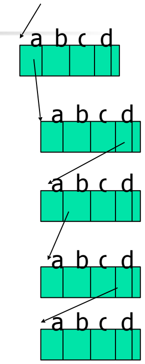
Time to construct the tree $\Theta(n^2)$



We can save some space.

Example *B="aabab"*
S={"aabab", "abab", "bab", "ab", "b"}

Suffix tries on a diet

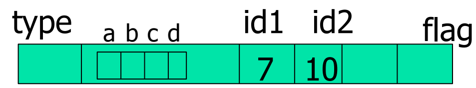


Def: a *thread* is a path from node *u* to node *v* in the *trie*, consisting of nodes of outdegree *1* (except maybe the last one) and *flag=0*.

Obs: There is a contagious part of *B*, identical to the string the shred represents. We call this part the shred-string

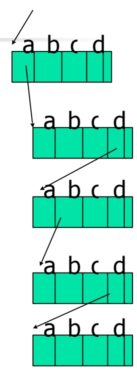
We stores the book *B* itself as an array.

We use a new type of nodes, called thread-nodes, maintain the first (*id1*) and last (*id2*) indexes of the shred-string in *B*.



1 7 10
B="cadbdaadbd

Suffix tries on a diet - cont

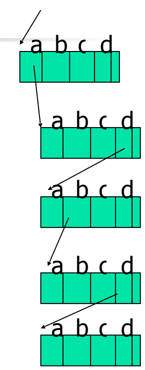


Algorithm for constructing a "thin" trie:

Given B – create an empty trie T, and insert all *n* suffixes of *B* into *T* --- generating a trie of size $\Theta(n^2)$.

Traverse the tries, and each time that a shred is seen, replace all nodes of the shred with a single shred-node.

Suffix tries on a diet - cont



- Clearly the use of thread-nodes saves some-but can we prove something ?
- Observations: Every leaf of T must be the end of some prefix of B. So the number of number of leaves of T is $\leq n$.
- n=|B|
- To bound the size of T, we will need to bound the number of internal nodes.
- Observations:
  - T might contain special nodes whose flag=1 (a suffix terminates at these nodes).
  - The number of special nodes is $\leq n$ (since this is the number of suffixes).
- What about other internal nodes of T ?

Suffix tries on a diet - cont

Lemma: Let T′ be a rooted tree with m leaves, where each internal node has ≥ 2 children. Then T′ has $\leq m$ internal nodes. (proof - easy induction. Homework)

Back to thin suffix tries *T*:

- *T has* $\leq n$ special nodes (with flag=1) and
- *T has* $\leq n$ leaves.
- Every other nodes has ≥ 2 children. (with flag=1). Applying the Lemma in this case, implies that the total number of internal nodes $\leq 2n$.

- Conclusion: The number of nodes in T is $\leq 3n$ (much better than the uncompressed version that could have $\Theta(n^2)$ nodes.
- So the size of the trie is only a constant more than the size of the book.

Quadtrees

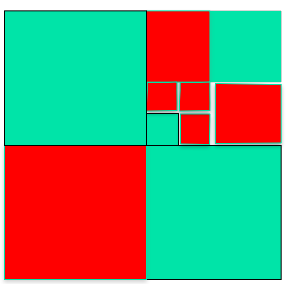
:

A simple data structure for geometric objects (e.g. points, houses, an image, 3D scene)

Support efficiently a very wide variety of queries.

Shares similarities with tries, hence taught together.

QuadTrees

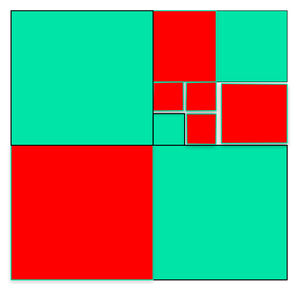


Assume we are given a red/green picture defined a $2^h \times 2^h$ grid. E.g. pixels. Each pixel is either **green** or **red**.

(more general and interesting examples – soon)

Need to represent the shape “compactly”

QuadTrees



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Need a data structure that could answers multiple types of queries. For example:

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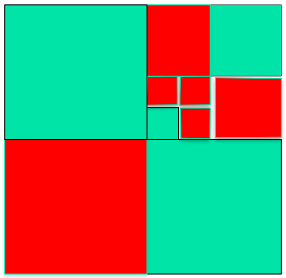

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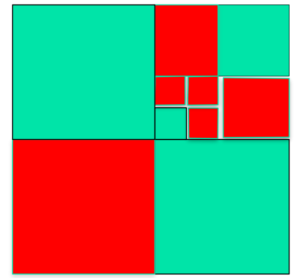

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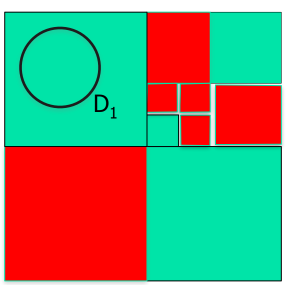
2.For a given query disk D, are there any green points in D ?

QuadTrees



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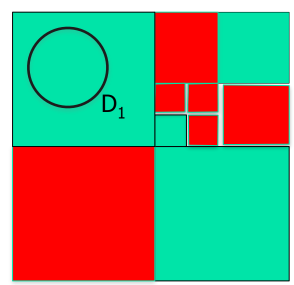

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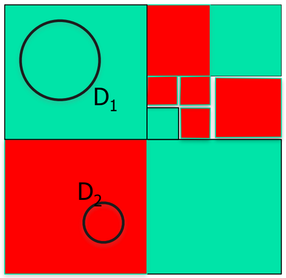
3.How many green points are there in D ?
4.Etc etc

QuadTrees



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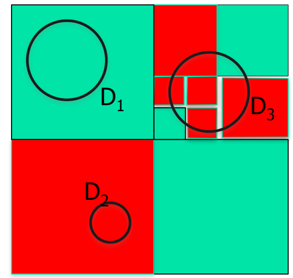

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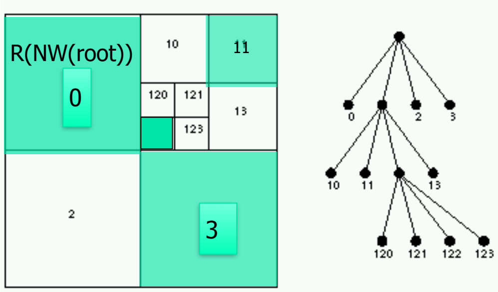
Regions of nodes



R(root))

A tree where each internal node has 4 children.

In general, every node v is associated with a region of the plane. Lets denote this region by R(v).

R(root) is the whole region of interest (e.g. input image or USA)

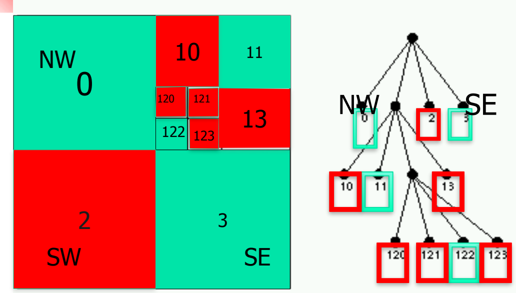
The smallest possible area of R(v) is a single **pixel**.

For every non-root node v, we have $R(v) \subset R(parent(v))$

Let NW(v) denote the North West child of v.
(similarly NE, SW, SE)

R(v) = is the union of
R(NW(v)), R(NE(v)) R(SW(v)), R(SE(v))

QuadTrees



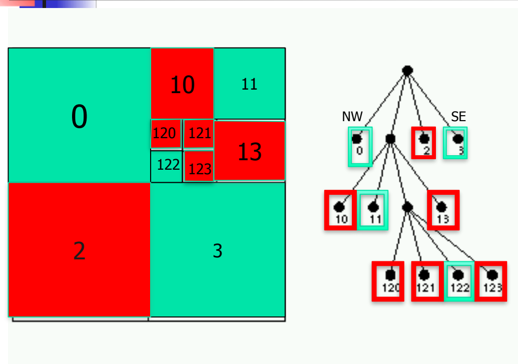
- Assume we are given a red/green picture defined on a $2^h \times 2^h$ grid of **pixels**.
- Each pixel has as a unique color (Green or Red)
- **Every node $v \in T$ is associated with a geometric region $R(v)$**

Alg constructQT for a shape S.

- **input** – a node $v \in T$, and a shape S.
- **Output** – a Quadtree T_v representing the shape of S within $R(v)$).

- If S is fully green in $R(v)$, or S is fully red in $R(v)$ – then
- v is a leaf, labeled Green or Red. Return ;
- Otherwise, divide $R(v)$ into 4 equal-sized quadrants, corresponding to nodes v.*NW,* v.*NE,* v.*SW,* v.*SE*.
- Call constructQT recursively for each quadrant.

QuadTrees



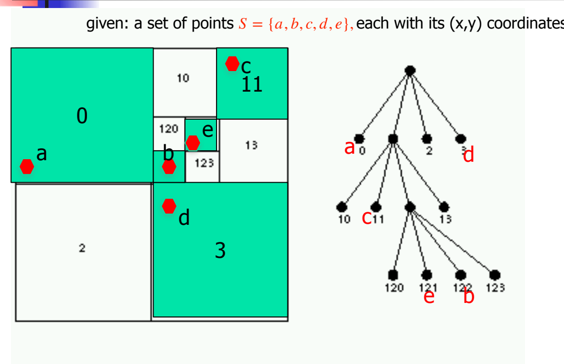
Consider a picture stored on an $2^h \times 2^h$ grid. Each pixel is either red or green.

We can represent the shape "compactly" using a QT.

Height – at most h.
Point location operation – given a point q, is it black or white
– takes time O(h)
- could it be much smaller ?

Many other operations are very simple to implement.

QuadTree for a set of points



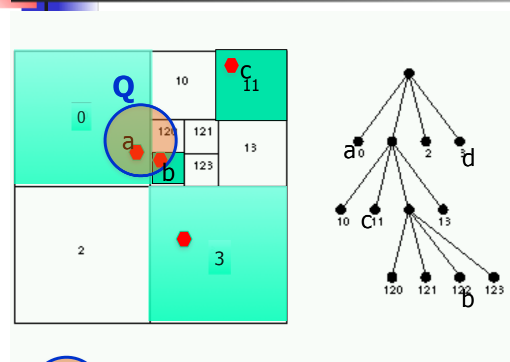
Now consider a set of points (red) but on a $2^h \times 2^h$ grid.

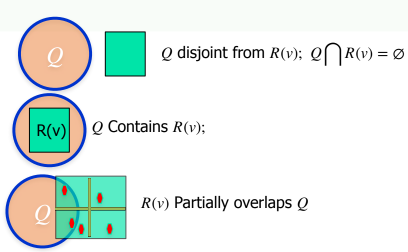
Splitting policy: Split until each quadrant contains ≤1 point.

Build a similar QT, but we stop splitting a quadrant when it contain ≤1 point (or some other small constant)
Point location operation – given a point q, is it black or white
– takes time O(h) (in practice, usually much less)

Many other **splitting polices** are very simple to implement.
(eg. A leaf could contain contains ≤17 points)

QuadTrees for a set of points

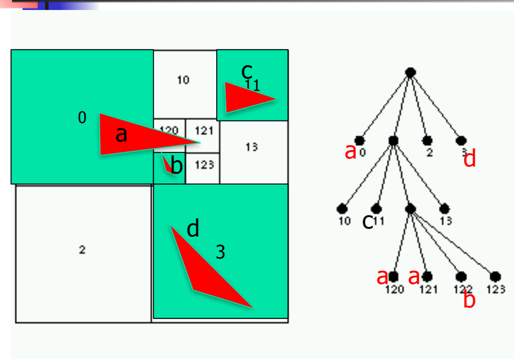




Report(Q,v)
// Q – a query disk
/* report all the points in stored at the subtree rooted at v, which are contained inside Q. */

1. If v is NULL – **return**.
2. If R(v) is disjoint from Q –**return** NULL.
3. If R(v) is fully contained in Q – report all points in the subtree rooted at v.
4. If v is a leaf – check each point in R(v) if inside Q
5. Else //$R(v)$ Partially overlaps Q
   Report(Q, NW(v)) and
   Report(Q, NE(v)) and
   Report(Q, SW(v)) and
   Report(Q, SE(v))

QuadTrees for shape



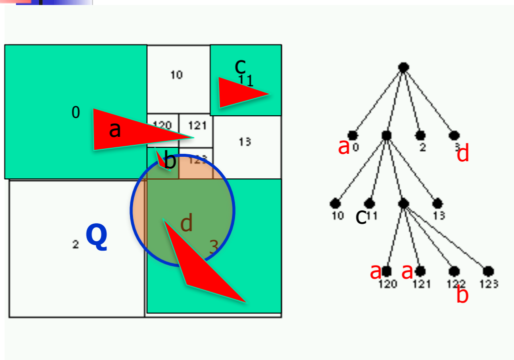
Input: Set S of triangles $S=\{t_{1 \ldots} t_n\}$

Splitting policy: Split quadrant if it intersects more than 1 triangle of S.

Note – a triangle might be stored in multiple leaves.
Some leaves might store no triangles.

Finding all triangles inside a query region Q – essentially same Report Report(Q,v) as before
(minor modifications)

QuadTrees for shape



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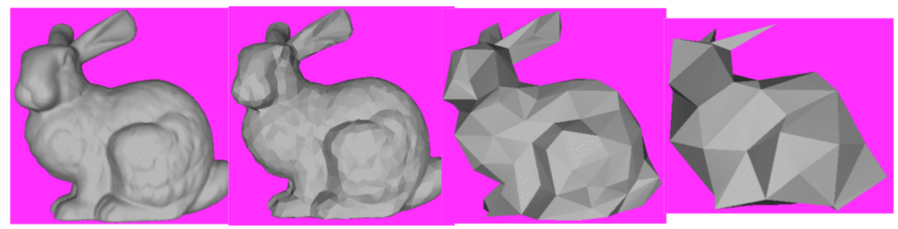
Level Of Details

- Idea – the same object is stored several times, but with a different level of details
- Coarser representations for distant objects
- Decision which level to use is accepted `on the fly'
  (eg in graphics applications, if we are far away from a terrain, we could tolerate usually large error)



69,451 polys | 2,502 polys | 251 polys | 76 polys