## CSc 453

Syntax Analysis
(Parsing)

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## Overview



Main Task: Take a token sequence from the scanner and verify that it is a syntactically correct program.
Secondary Tasks:

- Process declarations and set up symbol table information accordingly, in preparation for semantic analysis.
- Construct a syntax tree in preparation for intermediate code generation.


## Context-free Grammars

- A context-free grammar for a language specifies the syntactic structure of programs in that language.
- Components of a grammar:
- a finite set of tokens (obtained from the scanner);
- a set of variables representing "related" sets of strings, e.g., declarations, statements, expressions.
- a set of rules that show the structure of these strings.
- an indication of the "top-level" set of strings we care about.


## Context-free Grammars: Definition

Formally, a context-free grammar $G$ is a 4-tuple $G=(\mathrm{V}, \mathrm{T}, \mathrm{P}, \mathrm{S})$, where:

- V is a finite set of variables (or nonterminals). These describe sets of "related" strings.
- T is a finite set of terminals (i.e., tokens).
- P is a finite set of productions, each of the form

$$
A \rightarrow \alpha
$$

where $A \in \mathrm{~V}$ is a variable, and $\alpha \in(\mathrm{V} \cup \mathrm{T})^{*}$ is a sequence of terminals and nonterminals.

- $S \in \mathrm{~V}$ is the start symbol.


## Context-free Grammars: An Example

A grammar for palindromic bit-strings:
$\mathrm{G}=(\mathrm{V}, \mathrm{T}, \mathrm{P}, \mathrm{S})$, where:

- $V=\{S, B\}$
- $T=\{0,1\}$
- $P=\{S \rightarrow B$,

$$
S \rightarrow \varepsilon,
$$

$$
S \rightarrow 0 \text { S } 0,
$$

$$
S \rightarrow 1 S 1,
$$

$$
\mathrm{B} \rightarrow 0,
$$

$$
B \rightarrow 1
$$

        \}
    
## Context-free Grammars: Terminology

- Derivation: Suppose that
- $\alpha$ and $\beta$ are strings of grammar symbols, and
- $A \rightarrow \gamma$ is a production.

Then, $\alpha A \beta \Rightarrow \alpha \gamma \beta$ (" $\alpha A \beta$ derives $\alpha \gamma \beta$ ").

- $\Rightarrow$ : "derives in one step"
$\Rightarrow^{*}$ : "derives in 0 or more steps"

$$
\alpha \Rightarrow{ }^{*} \alpha
$$

(0 steps)
$\alpha \Rightarrow^{*} \beta$ if $\alpha \Rightarrow \gamma$ and $\gamma \Rightarrow^{*} \beta \quad(\geq 1$ steps $)$

## Derivations: Example

- Grammar for palindromes: $\mathrm{G}=(\mathrm{V}, \mathrm{T}, \mathrm{P}, \mathrm{S})$,
- $V=\{S\}$,
- $T=\{0,1\}$,
- $P=\{S \rightarrow 0 S 0|1 S 1| 0|1| \varepsilon\}$.
- A derivation of the string 10101:

S
$\Rightarrow 1 \mathrm{~S} 1 \quad$ (using $S \rightarrow$ 1S1)
$\Rightarrow 10 \mathrm{SO} 1 \quad$ (using $S \rightarrow 0 \mathrm{SO}$ )
$\Rightarrow 10101$ (using $S \rightarrow 1$ )

## Leftmost and Rightmost Derivations

- A leftmost derivation is one where, at each step, the leftmost nonterminal is replaced.
(analogous for rightmost derivation)
- Example: a grammar for arithmetic expressions: $E \rightarrow E+E|E * E| i d$
- Leftmost derivation:
$\mathrm{E} \Rightarrow \mathrm{E} * \mathrm{E} \Rightarrow \mathrm{E}+\mathrm{E} * \mathrm{E} \Rightarrow \mathrm{id}+\mathrm{E}^{*} \mathrm{E} \Rightarrow \mathbf{i d}+\mathbf{i d}{ }^{*} \mathrm{E} \Rightarrow \mathbf{i d}+\mathbf{i d}^{*}$ id
- Rightmost derivation:
$\mathrm{E} \Rightarrow \mathrm{E}+\mathrm{E} \Rightarrow \mathrm{E}+\mathrm{E}^{*} \mathrm{E} \Rightarrow \mathrm{E}+\mathrm{E}^{*}$ id $\Rightarrow \mathrm{E}+\mathrm{id}{ }^{*}$ id $\Rightarrow \mathrm{id}+\mathrm{id}^{*}$ id


## Context-free Grammars: Terminology

- The language of a grammar $G=(\mathrm{V}, \mathrm{T}, \mathrm{P}, \mathrm{S})$ is $L(G)=\left\{w \mid w \in \mathrm{~T}^{*}\right.$ and $\left.\mathrm{S} \Rightarrow^{*} w\right\}$.
The language of a grammar contains only strings of terminal symbols.
- Two grammars $G_{1}$ and $G_{2}$ are equivalent if $L\left(G_{1}\right)=L\left(G_{2}\right)$.


## Parse Trees

- A parse tree is a tree representation of a derivation.
- Constructing a parse tree:
- The root is the start symbol $S$ of the grammar.
- Given a parse tree for $\alpha X \beta$, if the next derivation step is $\alpha X \beta \Rightarrow \alpha \gamma_{1} \ldots \gamma_{n} \beta$ then the parse tree is obtained as:



## Approaches to Parsing

- Top-down parsing:
- attempts to figure out the derivation for the input string, starting from the start symbol.
- Bottom-up parsing:
- starting with the input string, attempts to "derive in reverse" and end up with the start symbol;
- forms the basis for parsers obtained from parser-generator tools such as yacc, bison.


## Top-down Parsing

- "top-down:" starting with the start symbol of the grammar, try to derive the input string.
- Parsing process: use the current state of the parser, and the next input token, to guide the derivation process.
- Implementation: use a finite state automaton augmented with a runtime stack ("pushdown automaton").


## Bottom-up Parsing

- "bottom-up:" work backwards from the input string to obtain a derivation for it.
- Parsing process: use the parser state to keep track of:
- what has been seen so far, and
- given this, what the rest of the input might look like.
- Implementation: use a finite state automaton augmented with a runtime stack ("pushdown automaton").


## Parsing: Top-down vs. Bottom-up



## Parsing Problems: Ambiguity

- A grammar $G$ is ambiguous if some string in $L(G)$ has more than one parse tree.
- Equivalently: if some string in $\mathrm{L}(\mathrm{G})$ has more than one leftmost (rightmost) derivation.
- Example: The grammar

$$
\mathrm{E} \rightarrow \mathrm{E}+\mathrm{E}|\mathrm{E} * \mathrm{E}| \text { id }
$$

is ambiguous, since "id+id*id" has multiple parses:


## Dealing with Ambiguity

1. Transform the grammar to an equivalent unambiguous grammar.
2. Use disambiguating rules along with the ambiguous grammar to specify which parse to use.
Comment: It is not possible to determine algorithmically whether:

- Two given CFGs are equivalent;
- A given CFG is ambiguous.


## Removing Ambiguity: Operators

- Basic idea: use additional nonterminals to enforce associativity and precedence:
- Use one nonterminal for each precedence level:
- $E \rightarrow E * E|E+E|$ id needs 2 nonterminals (2 levels of precedence).
- Modify productions so that "lower precedence" nonterminal is in direction of precedence:
$\mathrm{E} \rightarrow \mathrm{E}+\mathrm{E} \Rightarrow \mathrm{E} \rightarrow \mathrm{E}+\mathrm{T}$ (+ is left-associative)


## Example

- Original grammar.
$E \rightarrow E * E|E / E| E+E|E-E|(E) \mid$ id precedence levels: $\left\{{ }^{*}, /\right\}>\{+,-\}$ associativity: *, l, +, - are all left-associative.
- Transformed grammar:

$$
\begin{aligned}
& \mathrm{E} \rightarrow \mathrm{E}+\mathrm{T}|\mathrm{E}-\mathrm{T}| \mathrm{T} \quad \text { (precedence level for: }{ }^{+,,- \text {) }} \\
& \mathrm{T} \rightarrow \mathrm{~T}^{*} \mathrm{~F}|\mathrm{~T} / \mathrm{F}| \mathrm{F} \\
& \mathrm{~F} \rightarrow(\mathrm{E}) \mid \text { id }
\end{aligned}
$$

## Bottom-up parsing: Approach

1. Preprocess the grammar to compute some info about it. (FIRST and FOLLOW sets)
2. Use this info to construct a pushdown automaton for the grammar:

- the automaton uses a table ("parsing table") to guide its actions;
- constructing a parser amounts to constructing this table.


## FIRST Sets

Defn: For any string of grammar symbols $\alpha$,

- $\operatorname{FIRST}(\alpha)=\left\{\mathbf{a} \mid \mathrm{a}\right.$ is a terminal and $\left.\alpha \Rightarrow^{*} \mathrm{a} \beta\right\}$.
- if $\alpha \Rightarrow^{*} \varepsilon$ then $\varepsilon$ is also in $\operatorname{FIRST}(\alpha)$.
- Example: $\mathrm{E} \rightarrow \mathrm{TE}$
$\mathrm{E}^{\prime} \rightarrow+\mathrm{T} \mathrm{E}^{\prime} \mid \varepsilon$
$T \rightarrow \mathrm{~T}^{\prime}$
$\mathrm{T}^{\prime} \rightarrow{ }^{*} \mathrm{FT}^{\prime} \mid \varepsilon$
$F \rightarrow(E) \mid$ id
$\operatorname{FIRST}(\mathrm{E})=\operatorname{FIRST}(\mathrm{T})=\operatorname{FIRST}(\mathrm{F})=\{($, id $\}$
$\operatorname{FIRST}\left(\mathrm{E}^{\prime}\right)=\{+, \varepsilon\}$
$\operatorname{FIRST}\left(\mathrm{T}^{\prime}\right)=\left\{{ }^{*}, \varepsilon\right\}$


## Computing FIRST Sets

Given a sequence of grammar symbols $A$ :

- if $A$ is a terminal or $A=\varepsilon$ then $\operatorname{FIRST}(A)=\{A\}$.
- if $A$ is a nonterminal with productions $A \rightarrow \alpha_{1}|\ldots| \alpha_{n}$ then:
- $\operatorname{FIRST}(A)=\operatorname{FIRST}\left(\alpha_{1}\right) \cup \ldots \cup \operatorname{FIRST}\left(\alpha_{n}\right)$.
- if $A$ is a sequence of symbols $Y_{1} \ldots Y_{k}$ then:
- $\underline{\text { for } i=1 \text { to } k \text { do: }}$
- add each $a \in \operatorname{FIRST}\left(Y_{i}\right)$, such that $a \neq \varepsilon$, to $\operatorname{FIRST}(A)$.
- if $\varepsilon \notin \operatorname{FIRST}\left(Y_{i}\right)$ then break;
- if $\varepsilon$ is in each of $\operatorname{FIRST}\left(\mathrm{Y}_{1}\right), \ldots, \operatorname{FIRST}\left(\mathrm{Y}_{k}\right)$ then add $\varepsilon$ to $\overline{\operatorname{FIRST}}(A)$.


## Computing FIRST sets: cont'd

- For each nonterminal $A$ in the grammar, initialize $\operatorname{FIRST}(A)=\varnothing$.
- repeat \{
for each nonterminal $A$ in the grammar \{ compute $\operatorname{FIRST}(A) ; \quad{ }^{*}$ as described previously */ \}
\} until there is no change to any FIRST set.


## Example (FIRST Sets)

$$
\begin{aligned}
& X \rightarrow Y Z \mid a \\
& Y \rightarrow b \mid \varepsilon \\
& Z \rightarrow c \mid \varepsilon
\end{aligned}
$$

- $X \rightarrow \mathbf{a}$, so add $\mathbf{a}$ to $\operatorname{FIRST}(X)$.
- $X \rightarrow Y Z, b \in \operatorname{FIRST}(Y)$, so add $b$ to FIRST(X).
- $Y \rightarrow \varepsilon$, i.e. $\varepsilon \in \operatorname{FIRST}(\mathrm{Y})$, so add non- $\varepsilon$ symbols from FIRST(Z) to $\operatorname{FIRST}(\mathrm{X})$.
- add c to $\mathrm{FIRST}(\mathrm{X})$.
- $\varepsilon \in \operatorname{FIRST}(\mathrm{Y})$ and $\varepsilon \in \operatorname{FIRST}(Z)$, so add $\varepsilon$ to $\operatorname{FIRST}(\mathrm{X})$.

Final: $\operatorname{FIRST}(X)=\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \varepsilon\}$.

## FOLLOW Sets

Definition: Given a grammar $\mathrm{G}=(\mathrm{V}, \mathrm{T}, \mathrm{P}, \mathrm{S})$, for any nonterminal $A \in \mathrm{~V}$ :

- $\operatorname{FOLLOW}(A)=\left\{\mathbf{a} \in \mathrm{T} \mid S \Rightarrow^{*} \alpha A a \beta\right.$ for some $\left.\alpha, \beta\right\}$.
i.e., FOLLOW(A) contains those terminals that can appear after $A$ in something derivable from the start symbol S .
- if $S \Rightarrow^{*} \alpha A$ then $\$$ is also in FOLLOW $(A)$. ( $\$ \equiv E O F$, "end of input.")


## Example:

$$
\begin{aligned}
& E \rightarrow E+E \mid \text { id } \\
& \operatorname{FOLLOW}(E)=\{+, \$\} .
\end{aligned}
$$

## Computing FOLLOW Sets

Given a grammar $\mathrm{G}=(\mathrm{V}, \mathrm{T}, \mathrm{P}, \mathrm{S})$ :

1. add \$ to FOLLOW(S);
2. repeat $\{$

- for each production $A \rightarrow \alpha B \beta$ in P , add every non- $\varepsilon$ symbol in $\operatorname{FIRST}(\beta)$ to $\operatorname{FOLLOW}(B)$.
- for each production $A \rightarrow \alpha B \beta$ in $P$, where $\varepsilon \in \operatorname{FIRST}(\beta)$, add everything in $\operatorname{FOLLOW}(A)$ to $\operatorname{FOLLOW}(B)$.
- for each production $A \rightarrow \alpha B$ in $P$, add everything in $\operatorname{FOLLOW}(A)$ to $\operatorname{FOLLOW}(B)$.
\} until no change to any FOLLOW set.


## Example (FOLLOW Sets)

$X \rightarrow Y Z \mid a$
$\mathrm{Y} \rightarrow \mathrm{b} \mid \varepsilon$
$Z \rightarrow \mathbf{c} \mid \varepsilon$

- $X$ is start symbol: add $\$$ to FOLLOW $(X)$;
- $X \rightarrow Y Z$, so add everything in FOLLOW $(X)$ to FOLLOW(Z).
$>$ add $\$$ to FOLLOW(Z).
- $X \rightarrow Y Z$, so add every non- $\varepsilon$ symbol in FIRST(Z) to FOLLOW(Y).
- add c to FOLLOW(Y).
- $X \rightarrow Y Z$ and $\varepsilon \in \operatorname{FIRST}(Z)$, so add everything in FOLLOW $(X)$
to FOLLOW $(\mathrm{Y})$.
- add \$ to FOLLOW(Y).


## Shift-reduce Parsing

- An instance of bottom-up parsing
- Basic idea: repeat

1. in the string being processed, find a substring $\alpha$ such that $A \rightarrow \alpha$ is a production;
2. replace the substring $\alpha$ by $A$ (i.e., reverse a derivation step).
until we get the start symbol.

- Technical issues: Figuring out

1. which substring to replace; and
2. which production to reduce with.

## Shift-reduce Parsing: Example

Grammar: $\quad S \rightarrow \mathbf{a A B e}$
$A \rightarrow A b c \mid \mathbf{b}$
$B \rightarrow \mathbf{d}$

| Input: | abbcde | (using $A \rightarrow \mathbf{b}$ ) |
| :---: | :--- | :--- |
| $\Rightarrow$ | $\mathbf{a} A \mathbf{b c d e}$ | (using $A \rightarrow A \mathbf{b c}$ ) |
| $\Rightarrow$ | $\mathbf{a} A d \mathbf{e}$ | (using $B \rightarrow \mathbf{d}$ ) |
| $\Rightarrow$ | $\mathbf{a} A B \mathbf{e}$ | (using $S \rightarrow \mathbf{a} A B e$ ) |
| $\Rightarrow$ | $S$ |  |

## Shift-Reduce Parsing: cont'd

- Need to choose reductions carefully:

$$
\text { abbcde } \Rightarrow \text { aAbcde } \Rightarrow \mathrm{a} A \mathrm{bcBe} \Rightarrow \ldots
$$ doesn't work.

- A handle of a string $s$ is a substring $\alpha$ s.t.:
- $\alpha$ matches the RHS of a rule $A \rightarrow \alpha$; and
- replacing $\alpha$ by $A$ (the LHS of the rule) represents a step in the reverse of a rightmost derivation of $s$.
- For shift-reduce parsing, reduce only handles.


## Shitt-reduce Parsing: Implementation

- Data Structures:
- a stack, its bottom marked by '\$'. Initially empty.
- the input string, its right end marked by '\$'. Initially $w$.
- Actions:
repeat
Shift some $(\geq 0)$ symbols from the input string onto the stack, until a handle $\beta$ appears on top of the stack.

2. Reduce $\beta$ to the LHS of the appropriate production.
until ready to accept.

- Acceptance: when input is empty and stack contains only the start symbol.


## Example

| Stack ( $\rightarrow$ ) | Input | Action |
| :---: | :---: | :---: |
| \$ | abbcde\$ | shift |
| \$a | bbcde\$ | shift |
| \$ab | bcde\$ | reduce: $A \rightarrow \mathbf{b}$ |
| \$a $A$ | bcde\$ | shift |
| \$aAb | cde\$ | shift |
| \$aAbc | de\$ | reduce: $A \rightarrow$ Abc |
| \$a $A$ | de\$ | shift |
| \$aAd | e\$ | reduce: $B \rightarrow \mathbf{d}$ |
| \$a $A B$ | e\$ | shift |
| \$aABe | \$ | reduce: $S \rightarrow \mathbf{a} A B \mathbf{e}$ |
| \$S | \$ | accept |

## Conflicts

- Can't decide whether to shift or to reduce both seem OK ("shift-reduce conflict").

Example: $S \rightarrow$ if $E$ then $S \|$ if $E$ then $S$ else $S \mid \ldots$

- Can't decide which production to reduce with - several may fit ("reduce-reduce conflict").

Example: Stmt $\rightarrow$ id ( args ) | Expr

$$
\text { Expr } \rightarrow \text { id ( args ) }
$$

## LR Parsing

- A kind of shift-reduce parsing. An $\mathrm{LR}(k)$ parser:
- scans the input L-to-R;
- produces a Rightmost derivation (in reverse); and
- uses $k$ tokens of lookahead.
- Advantages:
- very general and flexible, and handles a wide class of grammars;
- efficiently implementable.
- Disadvantages:
- difficult to implement by hand (use tools such as yacc or bison).


## LR Parsing: Schematic

stack symbols:
$\mathrm{s}_{i}$ : parser states
$\mathrm{X}_{i}$ : grammar symbols


- The driver program is the same for all LR parsers (SLR(1), $\operatorname{LALR}(1), \operatorname{LR}(1), \ldots)$. Only the parse table changes.
- Different LR parsing algorithms involve different tradeoffs between parsing power, parse table size.


## LR Parsing: the parser stack

- The parser stack holds strings of the form
$s_{0} X_{1} s_{1} X_{2} s_{2} \ldots X_{m} s_{m} \quad$ ( $s_{m}$ is on top)
where $s_{i}$ are parser states and $X_{i}$ are grammar symbols.
(Note: the $X_{i}$ and $s_{i}$ always come in pairs, with the state component $s_{i}$ on top.)
- A parser configuration is a pair
<stack contents, unexpended input〉


## LR Parsing: Roadmap

- LR parsing algorithm:
- parse table structure
- parsing actions
- Parse table construction:
- viable prefix automaton
- parse table construction from this automaton
- improving parsing power: different LR parsing algorithms


## LR Parse Tables

- The parse table has two parts: the action function and the goto function.
- At each point, the parser's next move is given by action $\left[s_{m}, \mathbf{a}_{i}\right]$, where:
- $s_{m}$ is the state on top of the parser stack, and
- $\mathrm{a}_{i}$ the next input token.
- The goto function is used only during reduce moves.


## LR Parser Actions: shift

- Suppose:
- the parser configuration is $\left\langle s_{0} X_{1} s_{1} \ldots X_{m} s_{m}, \mathbf{a}_{i} \ldots \mathbf{a}_{n}\right\rangle$, and
- $\quad \underline{\text { action }}\left[s_{m}, \mathbf{a}_{i}\right]=$ 'shift $s_{n}$ '.
- Effects of shift move:

1. push the next input symbol $\mathbf{a}_{i}$; and
2. push the state $s_{n}$

- New configuration: $\left\langle s_{0} X_{1} s_{1} \ldots X_{m} s_{m} \mathbf{a}_{i} s_{n}, \mathbf{a}_{i+1} \ldots \mathbf{a}_{n}\right\rangle$


## LR Parser Actions: reduce

- Suppose:
- the parser configuration is $\left\langle s_{0} X_{1} s_{1} \ldots X_{m} s_{m}, \mathbf{a}_{i} \ldots \mathbf{a}_{n}\right\rangle$, and
- action $\left[s_{m}, \mathbf{a}_{i}\right]=$ 'reduce $A \rightarrow \beta$ '.
- Effects of reduce move:

1. pop $n$ states and $n$ grammar symbols off the stack ( $2 n$ symbols total), where $n=|\beta|$.
2. suppose the (newly uncovered) state on top of the stack is $t$, and goto $[t, A]=u$.
3. push $A$, then $u$.

- New configuration: $\left\langle s_{0} X_{1} s_{1} \ldots X_{m-n} s_{m-n} \boldsymbol{A} u, \mathbf{a}_{i} \ldots \mathbf{a}_{n}\right\rangle$


## LR Parsing Algorithm

1. set ip to the start of the input string $w \$$.
2. while TRUE do:
3. let $s=$ state on top of parser stack, $\mathbf{a}=$ input symbol pointed at by ip.
4. if action $[s, \mathbf{a}]==$ 'shift $t$ ' then: (i) push the input symbol a on the stack, then the state $t ;(i i)$ advance ip.
5. if action $[s, \mathbf{a}]==$ 'reduce $A \rightarrow \beta^{\prime}$ then: (i) pop $2^{*}|\beta|$ symbols off the stack; (ii) suppose $t$ is the state that now gets uncovered on the stack; (iii) push the LHS grammar symbol $A$ and the state $u$ $=$ goto $[A, t]$.
6. if action $[s, \mathbf{a}]==$ 'accept' then accept;
7. else signal a syntax error.

## LR parsing: Viable Prefixes

- Goal: to be able to identify handles, and so produce a rightmost derivation in reverse.
- Given a configuration $\left\langle s_{0} X_{1} s_{1} \ldots X_{m} s_{m}, \mathbf{a}_{i} \ldots \mathbf{a}_{n}\right\rangle$ :
- $X_{1} X_{2} \ldots X_{m} \mathbf{a}_{i} \ldots \mathbf{a}_{n}$ is obtainable on a rightmost derivation.
- $X_{1} X_{2} \ldots X_{m}$ is called a viable prefix.
- The set of viable prefixes of a grammar are recognizable using a finite automaton.
This automaton is used to recognize handles.


## Viable Prefix Automata

- An $L R(0)$ item of a grammar $G$ is a production of G with a dot "." somewhere in the RHS.
- Example: The rule $A \rightarrow \mathbf{a} A \mathbf{b}$ gives these $\operatorname{LR}(0)$ items:
- $A \rightarrow \bullet \mathbf{a} A \mathbf{b}$
- $A \rightarrow \mathbf{a} \cdot A \mathbf{b}$
- $A \rightarrow \mathbf{a} A \cdot \mathbf{b}$
- $A \rightarrow \mathbf{a} A \mathbf{b}$ -
- Intuition: ' $A \rightarrow \alpha$ • $\beta$ ' denotes that:
- we've seen something derivable from $\alpha$; and
- it would be legal to see something derivable from $\beta$ at this point.


## Overall Approach

Given a grammar $G$ with start symbol $S$ :

- Construct the augmented grammar by adding a new start symbol $S^{\prime}$ and a new production $S^{\prime} \rightarrow S$.
- Construct a finite state automaton whose start state is labeled by the $\operatorname{LR}(0)$ item $S^{\prime \prime} \rightarrow \bullet S$.
- Use this automaton to construct the parsing table.


## Viable Prefix NFA for LR(0) items

- Each state is labeled by an LR(0) item. The initial state is labeled $S^{\prime} \rightarrow \bullet S$.
- Transitions:

1. 


where $X$ is a terminal or nonterminal.
2.

where $X$ is a nonterminal, and $X \rightarrow \gamma$ is a production.

## Viable Prefix NFA: Example

## Grammar :

$S \rightarrow 0 S 1$
$S \rightarrow \varepsilon$


## Viable Prefix NFA $\Rightarrow$ DFA

- Given a set of $\mathrm{LR}(0)$ items $I$, the set closure( $($ ) is constructed as follows: repeat

1. add every item in I to closure( $I$ );
2. if $A \rightarrow \alpha \bullet B \beta \in \underline{\text { closure }(I) \text { and } B \text { is a nonterminal, then for each }}$ production $B \rightarrow \gamma$, add the item $B \rightarrow \bullet \gamma$ to closure $(l)$.
until no new items can be added to closure( $I$ ).

- Intuition:
$A \rightarrow \alpha \bullet B \beta \in \underline{\text { closure(l) }}$ ) means something derivable from $\mathrm{B} \beta$ is legal at this point. This means that something derivable from $B$ (and thus $\gamma$ ) is also legal.


## Viable Prefix NFA $\Rightarrow$ DFA (cont'd)

- Given a set of $\operatorname{LR}(0)$ items $I$, the set goto $(I, X)$ is defined as

$$
\text { goto }(I, X)=\underline{\operatorname{closure}}(\{A \rightarrow \alpha X \bullet \beta \mid A \rightarrow \alpha \bullet X \beta \in I\})
$$

## - Intuition:

- if $A \rightarrow \alpha \bullet X \beta \in I$ then ( $a$ ) we've seen something derivable from $\alpha$; and ( $b$ ) something derivable from $X \beta$ would be legal at this point.
- Suppose we now see something derivable from $X$.

The parser should "go to" a state where ( $a$ ) we've seen something derivable from $\alpha X$; and ( $b$ ) something derivable from $\beta$ would be legal.

## Example



- Let $I_{0}=\left\{\mathbf{S}^{\prime} \rightarrow\right.$ •S $\}$.
- $I_{1}=\operatorname{closure}\left(I_{0}\right)=\left\{\mathrm{S}^{\prime} \rightarrow \cdot \mathrm{S}\right.$,
/* from $I_{0}^{* /}$
$S \rightarrow \bullet 0 S 1, S \rightarrow \bullet\}$
- goto $\left(I_{1}, \mathbf{0}\right)=$ closure $(\{S \rightarrow \mathbf{0} \cdot \mathrm{~S} \mathbf{1}\})$

$$
=\{S \rightarrow 0 \cdot S \text { 1, } S \rightarrow \bullet 0 S 1, S \rightarrow \bullet\}
$$

## Viable Prefix DFA for LR(0) Items

1. Given a grammar $G$ with start symbol $S$, construct the augmented grammar with new start symbol $S^{\prime}$ and new production $S^{\prime} \rightarrow S$.
2. $C=\left\{\underline{\text { closure }}\left(\left\{S^{\prime} \rightarrow \bullet \mathbf{\bullet}\right\}\right)\right\} ; / / C=$ a set of sets of items $=$ set of parser states
3. repeat \{
for each set of items $I \in C\{$
for each grammar symbol $X\{$
if $($ goto $(I, X) \neq \varnothing$ \&\& goto $(1, X) \notin C)\{\quad / /$ new state add goto( $1, X$ ) to $C$;
\}
\}
\}
\} until no change to $C$;
4. return $C$.

## SLR(1) Parse Table Construction I

Given a grammar $G$ with start symbol $S$ :

- Construct the augmented grammar $G^{\prime}$ with start symbol $S^{\prime}$.
- Construct the set of states $\left\{I_{0}, I_{1}, \ldots, I_{n}\right\}$ for the Viable Prefix DFA for the augmented grammar $\mathrm{G}^{\prime}$.
- Each DFA state $I_{i}$ corresponds to a parser state $s_{i}$.
- The initial parser state $s_{0}$ coresponds to the DFA state $I_{0}$ obtained from the item $S^{\prime \prime} \rightarrow \bullet S$.
- The parser actions in state $s_{i}$ are defined by the items in the DFA state $I_{i}$.


## SLR(1) Parse Table Construction II

Parsing action for parser state $s_{i}$ :

- action table entries:
- if DFA state $I_{i}$ contains an item $A \rightarrow \alpha \bullet \mathbf{a} \beta$ where $\mathbf{a}$ is a terminal, and goto $\left(I_{i}, \mathbf{a}\right)=I_{j}$ : set action $[i, \mathbf{a}]=$ shift $j$.
- if DFA state $I_{i}$ contains an item $A \rightarrow \alpha \bullet$, where $A \neq S^{\prime}$ : for each $\mathbf{b} \in \operatorname{FOLLOW}(A)$, set action $[i, \mathbf{b}]=$ reduce $A \rightarrow \alpha$.
- if state $I_{i}$ contains the item $S^{\prime} \rightarrow S \bullet$ set action $[i, \$]=$ accept.
- goto table entries:
- for each nonterminal $A$, if $\operatorname{goto}\left(I_{i}, A\right)=I_{j}$, then goto $[i, A]=j$.
- any entry not defined by these steps is an error state.
- if any state has multiple entries, the grammar is not SLR(1).


## SLR(1) Shortcomings

- $\operatorname{SLR}(1)$ parsing uses reduce actions too liberally. Because of this it fails on many reasonable grammars.
- Example (simple pointer assignments):
$S \rightarrow R \mid L=R$
$L \rightarrow{ }^{*} R \mid$ id
$R \rightarrow L$
The SLR parse table has a state $\{S \rightarrow L \bullet=R, R \rightarrow L \bullet\}$, and FOLLOW(L) = $\{=, \$\}$.
$\Rightarrow$ shift-reduce conflict.


## Improving LR Parsing

- SLR(1) parsing weaknesses can be addressed by incorporating lookahead into the LR items in parser states.
The lookahead makes it possible to remove some "spurious" reduce actions in the parse table.
The $L A L R(1)$ parsers produced by bison and yacc incorporate such lookahead items.
- This improves parsing power, but at the cost of larger parse tables.


## Error Handling

Possible reactions to lexical and syntax errors:

- ignore the error. Unacceptable!
- crash, or quit, on first error. Unacceptable!
- continue to process the input. No code generation.
- attempt to repair the error: transform an erroneous program into a similar but legal input.
- attempt to correct the error: try to guess what the programmer meant. Not worthwhile.


## Error Reporting

- Error messages should refer to the source program.
prefer "line 11: X redefined" to "conflict in hash bucket 53"
- Error messages should, as far as possible, indicate the location and nature of the error. avoid "syntax error" or "illegal character"
- Error messages should be specific. prefer "x not declared in function foo" to "missing declaration"
- They should not be redundant.


## Error Recovery

- Lexical errors: pass the illegal character to the parser and let it deal with the error.
- Syntax errors: "panic mode error recovery"
- Essential idea: skip part of the input and pretend as though we saw something legal, then hope to be able to continue.
- Pop the stack until we find a state $s$ such that goto $[s, A]$ is defined for some nonterminal $A$.
- discard input tokens until we find some token a that can legitimately follow $A$ (i.e., $\mathbf{a} \in \operatorname{FOLLOW}(A)$ ).
- push the state goto $[s, A]$ and continue parsing.

