1. Create a trie for the set of words \( S = \{ab, ba, ca, caa, caaa, baaa\} \) over the alphabet \( \Sigma = \{a, b, c\} \).

2. In the version of tries presented in class, every leaf of the tree is an array of size \( 1 + |\Sigma| \) (where \( \Sigma \) is the alphabet). Note that this is quite wasteful in space, since by definition, every pointer cell of a leaf contains a NULL.

   (a) Suggest a modifications of the trie at which leafs are not stores at all, and the parent of a leaf store a special boolean field per every cell.

   (b) Assume for simplicity that \( |\Sigma| = 32 \). Show how to implement the improvement while adding one extra computer word per each node.

   (c) Show how to implement your improvement without any extra fields. Assume that there are areas in the computer memory at which cells of the tries cannot be stored.

   (d) Show that for every value of an integer \( h \), there is a trie of height \( h \), for which your improvement saves at least \( 1/2 \) of the memory needed, comparing to trie of the same set of words, that does not use this improvement. Prove your answer.

3. Let \( T \) be a tree with \( m \) leaves, and each internal (non-leaf) node has two or more children. Prove that \( T \) has \( \leq m \) internal nodes.

4. Consider a text \( B \), and the suffix trie \( T \) you are constructed for \( B \). Show that a word \( w \) appears as a substring in \( B \) if and only if there is a path in \( T \) from the root to some nodes, and this path corresponds to \( w \).

5. How would you change the structure of the trie, so that you can perform the following operations on this trie:

   (a) Given a set \( S = \{w_1 \ldots w_n\} \) of words, construct the trie for \( S \) in time \( O(\Sigma_{i=1}^n |w_i|) \).
(b) Given a word $w$ (not necessarily of $S$), find how many words in $S$ have $w$ as a prefix. You should be able to answer this query in time $O(|w|)$.

6. **Bonus:** Given positive integers $n$ and $k$, construct a set $S = \{w_1 \ldots w_n\}$ of words over an alphabet $\Sigma$, such that $|w_i| \leq k$ for every $w_i \in S$, and the number of nodes in the trie $T$ constructed for $S$ is as large as possible. What is this number? Assume for simplicity that $n = |\Sigma|^\ell$ for some integer $\ell$. 