Operations on Patterns

Many interesting patterns can be created by operations on other patterns, changing and combining them in various ways.

Patterns can be represented as grids of cells. When a pattern is interpreted as a drawdown, black cells indicate where the warp is on top and white cells where the weft is on top. Figure 1 shows an example. [Covered elsewhere.]

Figure 1. A Drawdown Pattern

Cells in lines across the pattern from top to bottom are called columns, while cells in lines from left to right are called rows. The word lines is used for both in situations in which orientation is not important. See Figure 2.

Figure 2. Columns and Rows

A variety of operations can be performed on such patterns. They can be changed by geometrical transformations, such as rotation. Two patterns can be concatenated (adjointed) to form a larger pattern. A portion of a pattern can be replaced by another pattern. The rows and columns can be rearranged. And a pattern can be turned over to show its back side, as in the back of a woven fabric.

Notation

Uppercase italic letters, like \( P \), \( Q \), and \( R \), are used to name patterns for the purpose of identification.

Some operations on patterns require integer values. Lowercase italic letters, such as \( i \), \( j \), and \( k \), are used for these.

Various symbols are used to stand for operations on patterns.

Pattern Properties

Two properties of patterns are important in many operations:
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- width, the number of columns, in a pattern $P$ is denoted by $\omega(P)$
- height, the number of rows; in a pattern $P$ is denoted by $\eta(P)$

Sometimes it is useful to know the total number of cells in a pattern. The number of cells in a pattern $P$ is denoted by $\sigma(P)$. $\sigma(P) = \omega(P) \times \eta(P)$. Figure Ω.3 illustrates these properties.

![Figure Ω.3. Pattern Dimensions](image)

Another important property of a pattern is the number of different colors it has. For a pattern $P$, the number of colors is denoted by $\kappa(P)$. For drawdowns, $\kappa(P) = 2$.

For “color drawdowns” (patterns in which the colors of the warp and weft threads are shown), $\kappa(P)$ may be greater than 2. Figure Ω.4 shows a 6-color pattern.

![Figure Ω.4. $\kappa(P) = 6$](image)

Geometrical Transformations

There are two kinds of geometrical transformations that can be performed on patterns: rotations and flips.

Rotations

A pattern can be rotated in increments of 90°: 90°, 180°, and 270°. Rotation by 360° leaves the pattern unchanged. Rotation is measured in the clockwise direction by convention.
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The rotations of a pattern $P$ by $90^\circ$, $180^\circ$, and $-90^\circ$ ($270^\circ$) are denoted by $\Box P$, $\mathcal{O} P$, and $\mathcal{F} P$, respectively.

Figure Ω.5 shows the rotations of a square pattern and Figure Ω.6 shows the rotations of an oblong pattern.

![Rotations of a Square Pattern](image1)

![Rotations of an Oblong Pattern](image2)

Note that:

\[
\eta(P) = \omega(\Box P) = \omega(\mathcal{F} P) \\
\omega(P) = \eta(\mathcal{O} P) = \eta(\mathcal{F} P) \\
\eta(P) = \eta(\mathcal{O} P) \\
\omega(P) = \omega(\Box P).
\]
Flips

There are four flips:

- horizontal, around a vertical axis
- vertical, around a horizontal axis
- right, around the left diagonal (from the upper-left corner to the lower-right corner)
- left, around the right diagonal (from the upper-right corner to the lower-left corner)

These flips of a pattern $P$ are denoted by $\Box P$, $\downarrow P$, $\square P$, and $\triangle P$, respectively.

Figure Ω.7 shows these flips for a square pattern and Figure Ω.8 shows them for an oblong pattern.
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Note that:

\[ \eta(P) = \eta(\square P) = \eta(\bigcirc P) \]
\[ \omega(P) = \omega(\square P) = \omega(\bigcirc P) \]
\[ \eta(P) = \omega(\bigcirc P) = \omega(\square P) \]
\[ \omega(P) = \eta(\square P) = \eta(\bigcirc P) \]

Compound Operations

For rotations in increments of 90°, only one operation, \( \square P \), is needed. Applying it twice results in rotation by 180°, and applying three times results in rotation by 270°:

\[ \square \square P = \square P \]
\[ \square \square \square P = \bigcirc P \]

There also are relationships between rotations and flips. For example,

\[ \square \square P = \bigcirc P \]

In fact, all the geometrical operations can be obtained by using just \( \bigcirc \) and \( \square \) or by using just \( \square \), \( \bigcirc \), and \( \square \). For a discussion of these relationships, see Reference 2.

Although the relationships between the geometrical operations are interesting, for pattern construction, it’s more convenient to have the whole set available.

Concatenation

Concatenation is the adjoining (juxtaposition) of two patterns to form a larger one. There are two forms of pattern concatenation: horizontal, in which patterns are adjoined at their vertical edges, and vertical, in which patterns are adjoined at their horizontal edges.

Horizontal and vertical concatenation are denoted by the symbols \( \rightarrow \) and \( \downarrow \) respectively.

In order for concatenation to be possible, the adjoining edges must be of the same length:

For the horizontal concatenation of patterns \( P \) and \( Q \), \( \eta(P) = \eta(Q) \).

For the vertical concatenation of patterns \( P \) and \( Q \), \( \omega(P) = \omega(Q) \).

Figures \( \Omega.9 \) and \( \Omega.100 \) show examples of horizontal and vertical concatenation.
Duplicate Edges

A potential design problem arises when the adjoining edges in concatenation are the same, cell by cell. This produces a duplication at the boundary, which may be undesirable for aesthetic and structural reasons. Therefore, if the adjoining edges are the same, one edge is discarded. Figure Ω.11 shows an example.
Duplicate removal is automatic in concatenation of patterns. If duplicate removal is not desired, the operations $+\text{H}$ and $\text{T}$ can be used.

Figure Ω.12 shows an example of horizontal concatenation without duplicate removal.

Note: Duplicate edges are removed only at adjoining boundaries. Any other duplicate rows or columns are not affected.

**Repetition and Extension**

Repetition is a common way to extend a small pattern to make a larger one. Repetition is a special case of concatenation.

Like concatenation, repetition can be horizontal or vertical. The notation $P \downarrow_i$ to denotes the repetition of $P$ horizontally $i$ times. Similarly, the notation $P \uparrow_i$ denotes the repetition of $P$ vertically $i$ times.

Figures Ω.13 and Ω.14 illustrate examples of these operations.
Duplicate Edges

As in concatenation, duplicate edges at boundaries are discarded by the repetition operations. Figure Ω.15 shows an example.
Duplicate removal is automatic in the repetition of patterns. If duplicate removal is not desired, the operations $\rightarrow_+$ and $\uparrow_+$ can be used. Figure $\Omega.16$ shows an example of horizontal repetition without duplicate removal.

Note: Duplicate edges are removed only at adjoining boundaries. Any other duplicate rows or columns are not affected.

Extension

Sometimes it is useful to extend a pattern by repetition to a width or height that is not an even multiple of that dimension of the pattern. The operations $\Rightarrow i$ and $P \downarrow i$ extend $P$ by repetition to a total of $i$ columns and $i$ rows, respectively. Figures $\Omega.17$ and $\Omega.18$ show examples of extension.
Figure Ω.17. Horizontal Extension

Figure Ω.18. Vertical Extension

Note: If \( i \) is less than the dimension of the pattern in the given direction, the pattern is truncated at the right or bottom, accordingly.

As with repetition, duplicate edges at the boundaries of patterns are removed. The operations \( P \Rightarrow_{i} \) and \( P \downarrow_{i} \) do not remove duplicate edges.
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Summary of Operations

- $\omega(P)$ width
- $\eta(P)$ height
- $\sigma(P)$ number of cells
- $\kappa(P)$ number of colors
- $\square P$ rotation by $90^\circ$
- $\bigcirc P$ rotation by $180^\circ$
- $\bigodot P$ rotation by $-90^\circ$
- $\leftrightarrow P$ horizontal flip
- $\uparrow P$ vertical flip
- $\rightarrow P$ right flip
- $\leftarrow P$ left flip
- $\sqcup$ horizontal concatenation
- $\sqcap$ vertical concatenation
- $\sqcup_\ast$ horizontal concatenation without duplicate removal
- $\sqcap_\ast$ vertical concatenation without duplicate removal
- $\blackarrow i$ horizontal repetition
- $\rightarrow i$ vertical repetition
- $\Rightarrow i$ horizontal extension
- $\Downarrow i$ vertical extension
- $\blackarrow_\ast i$ horizontal repetition without duplicate removal
- $\rightarrow_\ast i$ vertical repetition without duplicate removal
- $\Rightarrow_\ast i$ horizontal extension without duplicate removal
- $\Downarrow_\ast i$ vertical extension without duplicate removal

[More to come, but not yet written.]