A Network Calculus for Multi-Hop Fading Channels

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Intermediate nodes are store and forward relays
A fading channel is characterised by its channel capacity
Fading Channel Capacity

- Channel capacity [Shannon 1948]
  \[ C(\gamma) = W \log(1 + \gamma) \]
- \( \gamma = \bar{\gamma}|h|^2 \) for fading channels
- Channel gain \( h \) is a complex r.v.

Q: How do fading channel properties affect multihop network performance?
Network Model

- Fluid-flow traffic, discrete time
- Arrival and service are independent
- I.i.d. cross traffic at each node
- Time-varying random service that is equal to the instantaneous channel capacity

\[ C(\gamma_t) = W \log (g(\gamma_t)), \quad \gamma_t = \bar{\gamma}|h_t|^2 \]

- Computing this service distribution is hard!
Related Work: Multihop network performance analysis

- **Simplified channel models**
  - FSMC model [Wang and Moayeri 1995][Sadeghi et al 2008]
    - more than two states models may not be tractable
    - not easily extended to multihop networks
  - ON-OFF model
    - tractable but very simplified model
  - used in queuing theory [Ishizaki 2007], network calculus [Ciucu 2011], effective bandwidth [Hasan, Krunz, Matta 2004]

- **Effective capacity** [Wu and Negi 2003]
  - log-MGF of the channel capacity
  - tractable only for low SNR where $\log(1 + \gamma) \approx \gamma$

- **Physical layer models** [Hasna and Alouini 2003]
  - outage probability for AF wireless relay network
  - expression for MGF of end-to-end SNR
  - not suitable for network analysis
Network Calculus

- \((\min, +)\) dioid algebra
- Backlog: \(B(s) = A(0, s) - D(0, s)\)
- Delay: \(W(s) = \inf\{u \geq 0 : A(0, s) \leq D(0, s + u)\}\)

- Dynamic server [Chang 2000]
  \[D(0, t) \geq \inf_{u \leq t} \{A(0, u) + S(u, t)\}\]
  \[= A \ast S(0, t)\]

- Network service:
  \[S_{\text{net}}(\tau, t) = S_1 \ast S_2 \ast \cdots \ast S_N(\tau, t)\]
Bit domain

- Arrivals and departures are measured in bits
- For fading channels, service is given in terms of $\log(g(\gamma_t))$
- Distribution of $S$ is not easy to work with
- Service in terms of $g(\gamma_t)$ rather than $\log(g(\gamma_t))$ – more tractable
- SNR service $S(\tau, t) = \prod_{i=\tau}^{t-1} g(\gamma_i)$ resides in the SNR domain
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SNR service $S(\tau, t) = \prod_{i=\tau}^{t-1} g(\gamma_i)$ resides in the SNR domain
Our Approach

- SNR domain is governed by (min, ×) dioid algebra
- Network SNR server

\[ S_{\text{net}}(\tau, t) = S_1 \otimes S_2 \otimes \cdots \otimes S_N(\tau, t) \]
Network Calculus

- Service: $S(\tau, t) = \prod_{i=\tau}^{t-1} g(\gamma_i)$
- Arrival: $A(\tau, t) = \prod_{i=\tau}^{t-1} e^{ai}$
- Departure: $D(0, t) \geq A \otimes S(\tau, t) = \inf_{\tau \leq u \leq t} \{ A(\tau, u) \cdot S(u, t) \}$
- Backlog: $B(t) = \log \left( \frac{A(0,t)}{D(0,t)} \right)$
- Delay: $W(t) = \inf \{ u \geq 0 : A(0, t) \leq D(0, t + u) \}$
Computation of $S_1 \otimes S_2$

- Mellin transform: $\mathcal{M}_X(s) = E[X^{s-1}]$
- For two independent servers
  \[
  \mathcal{M}_{S_1 \otimes S_2}(s, \tau, t) \leq \sum_{u=\tau}^{t} \mathcal{M}_{S_1}(s, \tau, u) \cdot \mathcal{M}_{S_2}(s, u, t)
  \]
- For $N$ i.i.d. fading channels
  \[
  \mathcal{M}_{S_{\text{net}}}(s, \tau, t) \leq \binom{N - 1 + t - \tau}{t - \tau} \cdot (\mathcal{M}_{g(\gamma)}(s))^{t-\tau}, \quad \forall s < 1
  \]
- Moment bound: $\Pr(X \geq a) \leq a^{-s} \mathcal{M}_X(1 + s), \quad \forall a, s > 0$
Main Result: Statistical Performance Bounds

Define

\[ M(s, \tau, t) = \min(\tau, t) \sum_{u=0}^{\min(\tau, t)} M_A(1 + s, u, t) \cdot M_S(1 - s, u, \tau) \]

- **Backlog:** \( Pr(B(t) > b^\varepsilon) \leq \varepsilon \), where

  \[ b^\varepsilon = \inf_{s > 0} \left\{ \frac{1}{s} (\log M(s, t, t) - \log \varepsilon) \right\} \]

- **Delay:** \( Pr(W(t) > w^\varepsilon) \leq \varepsilon \), where

  \[ \inf_{s > 0} \left\{ M(s, t + w^\varepsilon, t) \right\} \leq \varepsilon \]
Cascade of $N$ i.i.d. Rayleigh Channels

- Service for Rayleigh channels
  - $g(\gamma) = 1 + \gamma = 1 + \bar{\gamma}|h|^2$
  - $|h| \sim \text{Rayleigh r.v.}$
  - For i.i.d. Rayleigh fading channel
    
    $M_S(s, \tau, t) = \left( e^{1/\bar{\gamma}\bar{\gamma}^{-1}} \Gamma(s, \bar{\gamma}^{-1}) \right)^{t-\tau}$

- Arrivals: $(\sigma(s), \rho(s))$ bounded arrivals [Chang 2000]
  
  $M_A(s, \tau, t) \leq e^{(s-1)\cdot(\rho(s-1)\cdot(t-\tau)+\sigma(s-1))}, \quad s > 1$

- This traffic class includes Markov-modulated processes, effective bandwidth, etc.
Performance Bounds of $N$ Rayleigh Channels

Define:

$$V(s) \triangleq e^{s \rho(s)} e^{1/\bar{\gamma}} \bar{\gamma}^{-s} \Gamma(1 - s, \frac{1}{\bar{\gamma}})$$

- **BACKLOG**: $\Pr(B(t) > b_{\text{net}}^\varepsilon) \leq \varepsilon$, where

$$b_{\text{net}}^\varepsilon = \inf_{s > 0} \left\{ \sigma(s) - \frac{1}{s} (N \log(1 - V(s)) + \log \varepsilon) \right\}$$

- **DELAY**: $\Pr(W(t) > w^\varepsilon) \leq \varepsilon$, where

$$\inf_{s > 0} \left\{ \frac{e^{s(-\rho(s)w^\varepsilon + \sigma(s))}}{(1 - V(s))^N} \cdot \min \left\{ 1, (V(s))^{w^\varepsilon} (w^\varepsilon)^{N-1} \right\} \right\} \leq \varepsilon$$
Numerical Results for $N$ Rayleigh Channels

Model parameters

- $\Delta t = 1$ ms
- $W = 20$ kHz
- $(\sigma, \rho)$ bounded traffic
- $\sigma = 50$ kb
- $\rho = 0$ to 60 kbps
- $\bar{\gamma} = 0$ to 40 dB
- $N = 1$ to 100

We used deterministically bounded traffic, hence, the only source of randomness is the fading channel!
Backlog Bounds for $N$ Rayleigh Channels

- $b^\varepsilon_{\text{net}}$ vs. $\bar{\gamma}$
  - $\rho = 30$ kbps
  - $\varepsilon = 10^{-4}$

- $b^\varepsilon_{\text{net}}$ vs. $\rho$
  - $\bar{\gamma} = 10$ dB
  - $\varepsilon = 10^{-4}$
Backlog and Delays

(i) $\varepsilon(b)$ vs. $\bar{\gamma}$
- buffer size = 400kb

Waterfall curves for loss probability

(ii) $\varepsilon(w)$ vs. $\bar{\gamma}$
- $N = 10$
- $\rho = 20$ kbps

Tighter delay bounds at higher SNR
Conclusions

- New approach to analyze cascade of fading channels
- Analysis in SNR domain using \((\min, \times)\) dioid algebra
- Use Mellin transform and moment bound to compute end-to-end bounds
- Application to cascade of i.i.d. Rayleigh channels
  - Explicit bounds in terms of the physical channel parameters
  - Bounds scale linearly in \(N\)
- \((\min, \times)\) dioid algebra has potential applications in models with time varying channel models
Thank you
Q & A
Delay bounds

(iii) $\varepsilon(w)$ vs. EtoE delay
- $\rho = 20$ kbps
- Effect of $N$ on the violation prob. at low SNR is huge!

(iv) $w_{\text{net}}^\varepsilon$ vs. $\bar{\gamma}$
- $\rho = 30$ kbps
- $\varepsilon = 10^{-4}$
Fading Channels With Cross Traffic

- Leftover SNR service:
  \[ S_o(\tau, t) = \frac{S(\tau, t)}{A_c(\tau, t)} \]

- Dynamic SNR server:
  \[ M_{S_o}(s, \tau, t) = M_{S/A_c}(s, \tau, t) = M_S(s, \tau, t) \cdot M_{A_c}(2 - s, \tau, t) \]

- N-node:
  \[ M_{S_{o,net}}(s, \tau, t) \leq e^{(1-s) \cdot N \sigma_c(1-s) \left( N - 1 + t - \tau \right)} \cdot \left( M_{g(\gamma)}(s) e^{(1-s) \cdot \rho_c(1-s)} \right)^{t-\tau}, \quad s < 1 \]
Bounds of Rayleigh Channels With Cross Traffic

1. End-to-end Backlog of the through flow

\[
b_{o,\text{net}}^\varepsilon(t) \leq \inf_{s>0} \left\{ \sigma_o(s) + N\sigma_c(s) - \frac{1}{s} \left[ N \log (1 - V_o(s)) + \log \varepsilon \right] \right\}
\]

2. Delay bound, we estimate for \( w^\varepsilon \geq 0 \)

\[
\inf_{s>0} \left\{ \frac{e^{s(-\rho_o(s)w+\sigma_o(s)+N\sigma_c(s))}}{(1 - V_o(s))^N} \cdot \min \left\{ 1, (V_o(s))^{w^\varepsilon} (w^\varepsilon)^{N-1} \right\} \right\} \leq \varepsilon
\]

where,

\[
V_o(s) = e^{s(\rho_o(s)+\rho_c(s))} e^{s \gamma^{-1}} \Gamma(1 - s, \gamma^{-1})
\]
Numerical results

- $\varepsilon = 10^{-4}$
- $W = 20$ kHz
- $\Delta t = 1$ msec.
- $(\sigma, \rho)$ bounded through and cross traffic
- $\sigma_o = \sigma_c = 50$ kb

(i) $b_{o,\text{net}}^\varepsilon$ vs. $\bar{\gamma}$
- $\rho_o = 30$ kbps

(ii) $b_{o,\text{net}}^\varepsilon$ vs. $\rho_o$
- $\bar{\gamma} = 10$ dB
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