Geographically Correlated Failures in Power Networks – Survivability Analysis

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The Power Grid

- A failure will have a significant effect on many interdependent systems - oil/gas, water, transportation, telecommunications
- Extremely complex network
- Relies on physical infrastructure
  - Vulnerable to physical attacks
- Failures can cascade
Large Scale Physical Attacks/Disasters

- EMP (Electromagnetic Pulse) attack
- Solar Flares - in 1989 the Hydro-Quebec system collapsed within 92 seconds leaving 6 Million customers without power

- Other natural disasters

- Physical attacks or disasters affect a specific geographical area


FERC, DOE, and DHS, Detailed Technical Report on EMP and Severe Solar Flare Threats to the U.S. Power Grid, 2010
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Hurricane Sandy Update:
The effects of Hurricane Sandy are profound throughout the eastern seaboard of the United States, including the New York City metro area and vast portions of New Jersey. Our thoughts and prayers are with our members and employees affected by this calamity.

IEEE is experiencing significant power disruptions to our U.S. facilities in New Jersey and New York. As a result, you may experience disruptions in service from IEEE. We apologize for any inconvenience and thank you for your patience.

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Related Work


- Cascading failures in the power grid
  - Dobson et al. (2001-2010), Hines et al. (2007-2011), Chassin and Posse (2005), Xiao and Yeh (2011), ...
  - The $N$-$k$ problem where the objective is to find the $k$ links whose failures will cause the maximum damage: Bienstock et al. (2005, 2009)
  - Interdiction problems: Bier et al. (2007), Salmeron et al. (2009), ...
  - Do not consider geographical correlation of initial failing links
Power Grid Vulnerability and Cascading Failures

- Power flow follows the laws of physics
- Control is difficult
  - It is difficult to “store packets” or “drop packets”
- Modeling is difficult
  - Final report of the 2003 blackout - cause #1 was “inadequate system understanding” (stated at least 20 times)
- Power grids are subject to **cascading failures**:  
  - Initial failure event  
  - Transmission lines fail due to overloads  
  - Resulting in subsequent failures
- Large scale geographically correlated failures have a different effect than a single line outage
- Objectives:  
  - Assess the vulnerability of different locations in the grid to **geographically correlated failures**  
  - Identify properties of the cascade model
Outline

- Background
- Power flows and cascading failures
- Numerical results - single event
- Cascade properties
- Vulnerability analysis and numerical results
Power Flow Equations - DC Approximation

- Exact solution to the AC model is infeasible

\[
P_{ij} = U_i^2 g_{ij} - U_i U_j g_{ij} \cos \theta_{ij} - U_i U_j b_{ij} \sin \theta_{ij}
\]
\[
Q_{ij} = -U_i^2 b_{ij} + U_i U_j b_{ij} \cos \theta_{ij} - U_i U_j g_{ij} \sin \theta_{ij}
\]
and \( \theta_{ij} = \theta_i - \theta_j \).

- Non-linear, non-convex, intractable,
- May have multiple solutions

- We use DC approximation which is based on:

\[
U_i \equiv 1, \forall i
\]
\[
f_i, d_i
\]
\[
P_i = f_i - d_i
\]
\[
\sin \theta_{ij} \approx \theta_{ij}
\]

- \( U_i = \) 1 p.u. for all \( i \)
- Pure reactive transmission lines - each line is characterized only by its reactance \( x_{ij} = -1/b_{ij} \)
- Phase angle differences are "small", implying that \( \sin \theta_{ij} \approx \theta_{ij} \)
The active power flow $P_{ij}$ can be found by solving:

$$f_i + \sum_{j : P_{ji} > 0} P_{ji} = \sum_{j : P_{ij} > 0} P_{ij} + d_i$$
for each node $i$

$$P_{ij} = \frac{\theta_i - \theta_j}{x_{ij}}$$
for each line $(i, j)$

Lemma (Bienstock and Verma, 2010):
Given the supply and demand vectors $\{f_i\}$ and $\{d_i\}$ with $\sum_i f_i = \sum_i d_i$ for each connected component of the network, the above equations have unique solution in $\{P_{ij}, \theta_i\}$

Known as a good approximation
Frequently used for contingency analysis
- Do the assumptions hold during a cascade?
Line Outage Rule

- Different factors can be considered in modeling outage rules
  - The main is thermal capacity $u_{ij}$
- Simplistic approach: fail lines with $|P_{ij}| > u_{ij}$
  Not part of the power flow problem constraints
- More realistic policy:
  Compute the moving average
  $\tilde{P}_{ij} := \alpha |P_{ij}| + (1 - \alpha) \tilde{P}_{ij}$
  ($0 \leq \alpha \leq 1$ is a parameter)
  Fail lines (possibly randomly)
  if $\xi_{ij} = \tilde{P}_{ij}/u_{ij}$ is close to or above 1

- In the following examples - deterministic outage rule:
  Fail lines with $\frac{\tilde{P}_{ij}}{u_{ij}} > 1$

- More generally:
  - Each line $(i,j)$ is characterized by its state $\xi_{ij} = \tilde{P}_{ij}/u_{ij}$
  - An outage rule $O(\xi_{ij}) \in [0,1]$ specifies the probability that $(i,j)$ will fail given that its current state is $\xi_{ij}$
Cascading Failure Model

- **Input:** Fully connected network graph $G$, supply/demand vectors with $\sum_i f_i = \sum_i d_i$, lines states $\xi_{ij}$
- **Failure Event:** At time step $t = 0$, a failure of a subset of lines occurs
- **Until no more lines fail do:**
  - Adjust the total demand to the total supply within each component of $G$
  - Use the power flow model to compute the flows in $G$
  - Update the state of lines $\xi_{ij}$ according to the new flows
  - Remove the lines from $G$ according to a given outage rule $O$
Example of a Cascading Failure

- Until no more lines fail do:
  - Adjust the total demand to the total supply within each component of
  - Use the power flow model to compute the flows in
  - Update the state of lines $\xi_{ij}$ according to the new flows
  - Remove the lines from $\xi_{ij}$ according to a given outage rule

Initial failure causes disconnection of load 3 from the generators in the rest of the network

As a result, line (2,3) becomes overloaded
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Numerical Results - Power Grid Map

- Obtained from the GIS (Platts Geographic Information System)
- Substantial processing of the raw data
- Used a modified Western Interconnect system, to avoid exposing the vulnerability of the real grid

- 13,992 nodes (substations), 18,681 lines, and 1,920 power stations.
- 1,117 generators (red), 5,591 loads (green)
- Assumed that demand is proportional to the population size
Determining The System Parameters

- The GIS does not provide the power capacities and reactance values.
- We use the length of a line to determine its reactance.
  - There is a linear relation.
- We estimate the power capacity by solving the power flow problem of the original power grid graph.
  - Without failures - $N$-Resilient grid.
  - With all possible single failures - $(N-1)$-Resilient grid.
- We set the power capacity $u_{ij} = K P_{ij}$.
  - $P_{ij}$ is the flow of line $(i,j)$ and the constant $K$ is the grid’s Factor of Safety (FoS).

$$u_{13} = 1680 \text{ MW}$$
$$P_{13} = 1400 \text{ MW}$$

$P_1 = f_1 = 2000 \text{ MW}$
$K = 1.2$

$P_{12} = 600 \text{ MW}$
$u_{12} = 720 \text{ MW}$
$x_{12} = 10 \Omega$

$P_2 = f_2 = 1000 \text{ MW}$

$P_{23} = 1600 \text{ MW}$
$u_{23} = 1920 \text{ MW}$
$x_{23} = 5 \Omega$

$P_3 = -d_3 = -3000 \text{ MW}$

We use $K = 1.2$ in most of the following examples.
Cascade Development - San Diego area

$N$-Resilient, Factor of Safety $K = 1.2$
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\[ N \text{-Resilient, Factor of Safety } K = 1.2 \rightarrow \text{Yield} = 0.33 \]
\[ \text{For } (N-1) \text{-Resilient } \rightarrow \text{Yield} = 0.35 \]
\[ \text{For } K = 2 \rightarrow \text{Yield} = 0.7 \]

(Yield - the fraction of the demand which is satisfied at the end of the cascade)
Cascade Development - 5 Rounds,
Idaho-Montana-Wyoming border

\[ N\text{-Resilient, Factor of Safety } K = 1.2 \rightarrow \text{Yield} = 0.39 \]

For \((N-1)\)-Resilient \rightarrow \text{Yield} = 0.999 \quad \text{For } K = 2 \rightarrow \text{Yield} = 0.999

(Yield - the fraction of the demand which is satisfied at the end of the cascade)
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Latest Major Blackout Event: San Diego, Sept. 2011

Blackout description (source: California Public Utility Commission)
Pacific Southwest Balancing Authority

SDG&E

Imperial Co.

San Diego Co.

Arizona

WAPA

APS

Mexico

CFE

Otay Mesa

Larkspur

Tijuana

La Rosita

Path 44

SONGS

Tallega

Central

SWPL
## Blackout Statistics

<table>
<thead>
<tr>
<th>Utility Company</th>
<th>Generation Lost (MW)</th>
<th>Demand Interrupted (MW)</th>
<th>Number of Customers Affected</th>
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<tbody>
<tr>
<td>SDG&amp;E</td>
<td>2229</td>
<td>4293</td>
<td>1,387,336</td>
</tr>
<tr>
<td>SCE</td>
<td>2428</td>
<td>0</td>
<td>117*</td>
</tr>
<tr>
<td>CFE Comision Federal de Electricidad</td>
<td>1915</td>
<td>2205</td>
<td>1,157,000</td>
</tr>
<tr>
<td>IID Imperial Irrigation District</td>
<td>333</td>
<td>929</td>
<td>144,000</td>
</tr>
<tr>
<td>APS Arizona Public Service</td>
<td>76</td>
<td>389</td>
<td>69,694</td>
</tr>
<tr>
<td>WAPA Western Area Power Association</td>
<td>0</td>
<td>74</td>
<td>18,000</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>6982 MW</strong></td>
<td><strong>7890 MW</strong></td>
<td><strong>2,776,147</strong></td>
</tr>
</tbody>
</table>
Event Timeline


15:27:58 to 15:30:00 – CCM tripped in CFE area (needed emergency assistance of 158 MW). IID experienced problems with Imperial Valley-El Centro line resulting in 100MW swing.

15:32:00 to 15:33:44 – IID transformer bank and two units trip. Also two 161 kV lines trip at Niland-WAPA and Niland-Coachella Valley.

15:35:40 to 15:36:45 – Two APS 161 kV lines to Yuma tripped and electrically separated from IID and WAPA. SDG&E now fed power into Yuma area.

15:37:56 – IID’s 161 kV tie to WAPA tripped. Import power into Yuma, Imperial Valley, Baja Norte, and San Diego wholly dependant on Path 44.

15:37:58 to 15:38:07 – El Centro Substation (IID) trip due to under frequency. Two units at La Rosita plant (CFE) trip resulting in a loss of 420 MW.

15:38:21 – Path 44 exceeded safety setting of 8000 Amps. Overload relay protection initiated to separate Path 44 between SCE and SDG&E at SONGS switchyard.

15:38:22 to 15:38:38 – SONGS and local power plants trip. 230kV lines open.

15:38:38 – Blackout
Failures indeed “skip” over a few hops
The following properties hold:

- **Consecutive failures may happen within arbitrarily long distances of each other**
  - Very different from the epidemic-percolation-based cascade models

- **Cascading failures can last arbitrarily long time**

- **Proofs for simple graphs**
  - Based on the observation that for all parallel paths \( \sum_{\text{path } 1} P_{ij} x_{ij} = \sum_{\text{path } 2} P_{ij} x_{ij} \)
Power Flow Cascading Failures Model*

The following properties hold:

- Consider failure events $F$ and $F'$ ($F$ is a subset of $F'$) - The damage after $F$ can be greater than after $F'$

- Consider graphs $G$ and $G'$ ($G$ is a subgraph of $G'$) - $G$ may be more resilient to failures than $G'$

- Observation (without proof): In large scale geographically correlated failures we do not experience the slow start phenomena that follows single line failures

* Proofs for simple graphs
Outline

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Identification of Vulnerable Locations

- **Circular and deterministic failure model**: All lines and nodes within a radius $r$ of the failure's epicenter are removed from the graph (this includes lines that pass through the affected area).

- Theoretically, there are infinite attack locations.
- We would like to consider a finite subset.
Identification of Vulnerable Locations

- Utilizing observations regarding the attack locations - $O(n^2)$
  - e.g., all attacks that affect only a single link are equivalent
- Candidates for the most vulnerable locations are the intersection points of the hippodromes:

- Identifying the intersections, using computational geometric tools - $O(m^2)$ ($m$ - the number of faces in the arrangement)*
  - Can be extended to probabilistic attack models
- For $r=50$ km, ~70,000 candidate locations were produced for the part of the Western Interconnect that we used

* based on Agarwal, Efrat, Ganjugunte, Hay, Sankararaman, and Zussman (2011)
Computational Workload

- Eight core server was used to perform computations and simulations.
- The identification of failure locations was performed in parallel, on different sections of the map.
  - For a given radius - was completed in less than 24 hours.
- The simulation of each cascading failure required solving large scale systems of equations (using the Gurobi Optimizer).
  - Completed in less than 8 seconds for each location.
- When parallelized, the whole simulation was completed in less than 24 hours.
Performance Metrics

- **The yield**: the fraction of the original total demand which remained satisfied at the end of the cascading failure
- **The number of rounds until stability**
- **The number of failed lines**
- **The number of connected components in the resulting graph**
Yield Values, $N$-Resilient

The color of each point represents the yield value of a cascade whose epicenter is at that point.
Number of Rounds until Stability, $N$-Resilient
The color of each point represents the yield value of a cascade whose epicenter is at that point.
Number of Failed Lines, $N-1$ Resilient

The color of each point represents the yield value of a cascade whose epicenter is at that point.
Scatter Graphs - after 5 Rounds

- Number of faulted lines vs. number of initially faulted lines
- Number of connected components vs. number of initially faulted lines
Scatter Graphs - Unlimited Number of Rounds

# Rounds Until Stability

# Initially Faulted Lines

0 10 20 30 40 50 60 70

0 1200

0.2 0.3 0.35 0.5

The Yield

0 2000 4000 6000 8000

# Faulted Lines
Compute moving average $\tilde{P}_{ij} = \alpha |P_{ij}| + (1-\alpha)\tilde{P}_{ij}$. Fail line if $\tilde{P}_{ij} > u_{ij}$.
Sensitivity Analysis - Stochastic Rule

- Specific attack - 100 repetitions for each $\varepsilon$, $q=1/2$

- 25 different attacks - comparison between deterministic and stochastic ($\varepsilon = 0.04$), $q=1/2$

P{Line \((i,j)\) faults at round \(t\)} = \begin{cases} 
1, & \tilde{p}_{ij}^t > (1 + \varepsilon)u_{ij} \\
0, & \tilde{p}_{ij}^t \leq (1 - \varepsilon)u_{ij} \\
q, & \text{otherwise}
\end{cases}
Conclusions

- Using network survivability tools developed efficient algorithms to identify vulnerable locations in the power grid
  - Based on the DC approximation and computational geometry
- Showed that cascade propagation models differ from the classical epidemic/percolation-based models
- Performed an extensive numerical study along with a sensitivity analysis
  - Can serve as input for smart-grid monitoring and strengthening efforts