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07.363

Functional and Logic Programming

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Logic I

What is Logic I?

Logic is the study of methods and principles used to distinguish *correct* from *incorrect* reasoning.

An *argument* consists of a number of *propositions*:

- some *premisses*, and
- a *conclusion*.

If the conclusion *follows* from the premisses then the argument is *sound*, otherwise it is *unsound*.

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What is Logic II?

- An argument might be sound, even though the conclusion or some of the premisses are false.

$$\frac{\text{Napoleon was German} \quad \text{All Germans are European}}{\text{Napoleon was European}}$$

$$\frac{NapoleonwasGerman \quad AllGermansareAsians}{NapoleonwasAsian}$$

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Propositional Logic I

- Propositional logic consists of
  - propositions $(P, Q, R, ...)$
  - logical connectives $(\wedge, \vee, \neg, \Rightarrow, \iff)$.
  - inference rules
- Propositions are either true or false

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Propositional Logic II

Conjunction (**and**, $\wedge$)

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

- *conjunct* **and** *conjunct*

Examples:

John likes apples **and** Lisa has braces.

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Propositional Logic III

Disjunction (**or**, $\vee$)

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

- *disjunct* **or** *disjunct*
- $\vee$ = *inclusive* disjunction
- English: *or*, *unless*

Examples:

The picnick will be held **unless** it rains.

Either the picnick will be held **or** it rains.

You can have ice-cream **or** cake.

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Propositional Logic IV

Implication (**implies**, $\Rightarrow$)

P	Q	$P \Rightarrow Q$	$\neg(P \wedge \neg Q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

- *antecedent* $\Rightarrow$ *consequent*
- $P \Rightarrow Q \equiv \neg(P \wedge \neg Q) \equiv \neg P \vee Q$

Examples:

If he has a good lawyer then he will be acquitted.

If he has a good lawyer he will be acquitted.

He will be acquitted if he has a good lawyer.

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Propositional Logic V

Bi-implication (**iff**, $\iff$)

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

- *antecedent* $\Rightarrow$ *consequent*
- $P \iff Q \equiv P \Rightarrow Q \wedge Q \Rightarrow P$
- "P if and only if Q"

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Propositional Logic VI

Negation (**not**, $\neg$)

P	$\neg P$
T	F
F	T

- "It is false that ..."
- "It is not the case that ..."
- "X is not ..."

Examples:

Not all humans are mortals.

Some humans are not mortals.

Daisy is not a duck.

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Propositional Logic Examples I

proposition	symbol
Egypt's food shortage worsens	E
Iran raises the price of oil	I
Jordan requests more A. aid	J
Libya raises the price of oil	L
Saudi buys 500 warplanes	S

- "Iran raises the price of oil but Libya does not raise the price of oil." $I \wedge \neg L$
- "Iran and Libya both do not raise the price of oil." $\neg I \wedge \neg L$

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Propositional Logic Examples II

- "It is not the case that either Egypt's food shortage worsens or Jordan requests more american aid" $\neg(E \vee J) \equiv \neg E \wedge \neg J$
- "Iran won't raise the price of oil unless Libya does so." $\neg I \vee L$
- Either Iran raises the price of oil and Egypt's food shortage worsens or it is not the case both that Jordan requests more American aid and that Saudi buys 500 more warplanes." $(I \wedge E) \vee \neg(J \wedge S)$

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Propositional Logic Examples III

proposition	symbol
Argentina mobilizes	A
Brazil protests to the UN	B
Chile will call for a meeting	C

- If Argentina mobilizes then if Brazil protests to the UN then Chile will call for a meeting of all the Latin American States. $A \Rightarrow (B \Rightarrow C)$
- If Argentina mobilizes and Brazil protests to the UN then Chile will call for a meeting of all the Latin American States. $(A \wedge B) \Rightarrow C$

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Propositional Logic Examples IV

- It is not the case that if Argentina mobilizes then both Brazil will protest to the UN and Chile will call for a meeting of all the Latin American States.
  $\neg(A \Rightarrow (B \wedge C))$
- Brazil will protest to the UN only if either Argentina mobilizes or Chile calls for a meeting of all the Latin American States.
  $B \Rightarrow (A \vee C)$
- Unless Chile calls for a meeting of all the Latin American States, Brazil will protest to the UN.
  $B \vee C$

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The Semantic Sequent I

The *semantic sequent*

$$P_1, P_2, \cdots, P_n \models Q$$

is valid if there is no way that Q can be false when all the P_i's are true.

- P_i is a proposition.
- $P_1, P_2, \cdots, P_n$ are the *premisses.*
- Q is the conclusion.
- $\models$ (the semantic turnstile) is read as "therefore".

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The Semantic Sequent II

P	Q	P	$P \Rightarrow Q$	$\models$	Q
T	T	T	T		T
T	F	T	F		F
F	T	F	T		T
F	F	F	T		F

P	Q	$P \Rightarrow Q$	$\models$	$\neg P \vee Q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

P	Q	$P \Rightarrow Q$	$\models$	$\neg P \wedge Q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		F

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The Semantic Sequent III

If Britain leaves the EC or if the trade deficit is reduced, then the price of butter will fall. If Britain stays in the EC, then exports will not increase. The trade deficit will increase unless exports are increased. Therefore, the price of butter will not fall.

proposition	symbol
Britain leaves the EC	P_1
The trade deficit is reduced	P_2
The price of butter falls	P_3
Exports increase	P_4

$P_1 \vee P_2 \Rightarrow P_3, \neg P_1 \Rightarrow \neg P_4, \neg P_2 \vee P_4 \models \neg P_3$

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The Semantic Sequent III

$P_1 \vee P_2 \Rightarrow P_3, \neg P_1 \Rightarrow \neg P_4, \neg P_2 \vee P_4 \models \neg P_3$

P_1	P_2	P_3	P_4	$P_1 \vee P_2 \Rightarrow P_3$	$\neg P_1 \Rightarrow \neg P_4$	$\neg P_2 \vee P_4$	$\models$	$\neg P_3$
T	T	T	T	T	T	T		F
T	T	T	F	T	T	F		F
T	T	F	T	F	T	T		T
T	T	F	F	F	T	F		T
T	F	T	T	T	T	T		F
T	F	T	F	T	T	T		F
					⋮			
F	F	F	F	T	T	T		T

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Predicate Logic I

Daisy is a duck. Ducks like water. Therefore Daisy likes water.

- `duck(Daisy)`
- `likes(Daisy, water)`
- $\forall X \bullet \texttt{duck}(X) \Rightarrow \texttt{likes}(X, \texttt{water})$

$P \wedge Q, P \vee Q, \neg P$ Conjunction, Disjunction, Negation

$P \Rightarrow Q, P \iff Q$ Implication, Bi-implication

$\forall x \bullet P(x)$ Universal quantifier. "$P(x)$ is true for all x". x is a variable.

$\exists x \bullet P(x)$ Existential quantifier. "There exists an x, for which $P(x)$ is true".

$\exists_1 x \bullet P(x)$ Unique existential quantifier. "There exists *exactly one* x, $\cdots$".

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Predicate Logic II

Example 1

- $\forall x \bullet \texttt{man}(x) \Rightarrow \texttt{human}(x)$
- "For all x, if x is a man then x is human."
- "All men are human"

Example 2

- $\exists x \bullet \texttt{father}(\texttt{John}, x) \wedge \texttt{female}(x)$
- "There exists an x such that John is the father of x and x is female."
- "John has a daughter"

Example 3

- $\forall x \bullet \texttt{animal}(x) \Rightarrow \exists y \bullet \texttt{motherof}(y, x)$
- "For all x such that x is an animal, there exists a y such that y is the mother of x "
- "Every animal has a mother"

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