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07.363

Functional and Logic Programming

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Structures in Prolog

Prolog Structures

- Aka, *structured* or *compound* objects
- An object with several components.
- Similar to Pascal's *Record*-type.
- Used to group things together.
- $\overbrace{\texttt{course}}^{\text{functor}}(\overbrace{\texttt{prolog,chris,mon,11}}^{\text{arguments}})$
- The *arity* of a functor is the number of arguments.

Structures – Courses I

```
lectures(Lecturer, Day) :-
   course(Course, time(Day,_,_), Lecturer, _).

duration(Course, Length) :-
   course(Course, time(Day,Start,Finish), Lec, Loc),
   Length is Finish - Start.

occupied(Room, Day, Time) :-
   course(Course, time(Day,Start,Finish), Lec, Room),
   Start =< Time, Time =< Finish.
```


Structures – Courses II

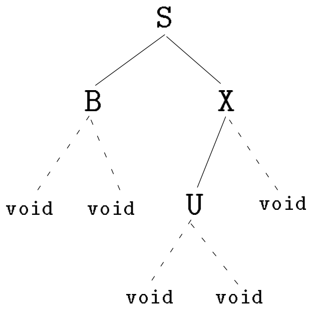
```
course(c231, time(mon,4,5), cc, plt1).
course(c231, time(wed,10,11), cc, plt1).
course(c231, time(thu,4,5), cc, plt1).
course(c363, time(mon,11,12), cc, slt1).
course(c363, time(thu,11,12), cc, slt1).

?- occupied(slt1, mon, 11).
yes
?- lectures(cc, mon).
yes
```

Binary Trees I

```
tree(Element, Left, Right)

tree(s,
   tree(b, void, void),
   tree(x,
      tree(u, void, void),
      void).
```



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Binary Trees II – Counting Nodes

```
size_of_tree(Tree, Size) :-
   size_of_tree(Tree, 0, Size).

size_of_tree(void, Size, Size).
size_of_tree(tree(_, L, R), SizeIn, SizeOut) :-
   Size1 is SizeIn + 1,
   size_of_tree(L, Size1, Size2),
   size_of_tree(R, Size2, SizeOut).
```

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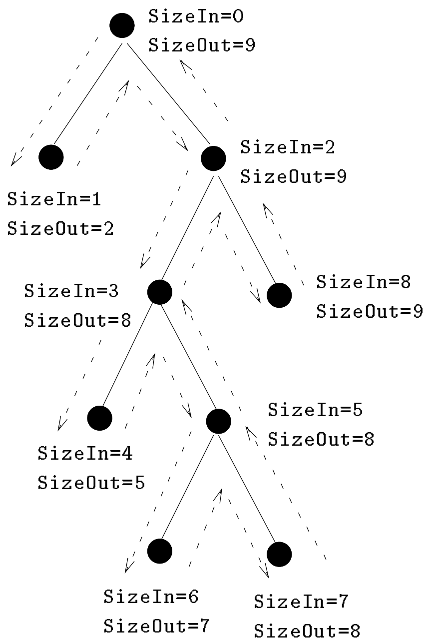
Binary Trees III – Size



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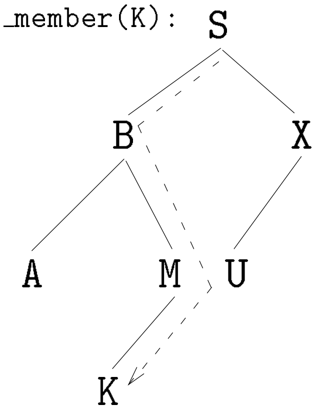
Binary Trees IV – BST

tree_member(K):




```
tree_member(X, tree(X,_, _)).
tree_member(X, tree(Y,Left,_)) :-
   X < Y, tree_member(Y, Left).
tree_member(X, tree(Y,_,Right)) :-
   X > Y, tree_member(Y, Right).
```

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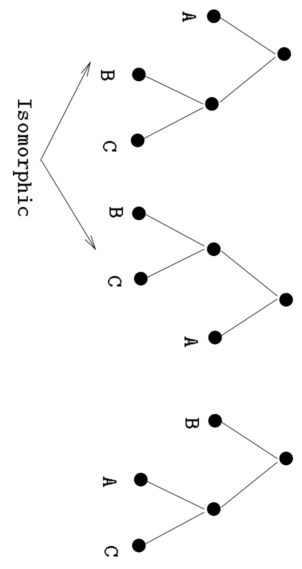
Binary Trees V – Isomorphism

Tree isomorphism:



Two binary tree T_1 and T_2 are *isomorphic* if T_2 can be obtained by reordering the branches of the subtrees of T_1.

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Binary Trees VI – Isomorphism

```
tree_iso(void, void).

tree_iso(tree(X, L1, R1), tree(X, L2, R2)) :-
   tree_iso(L1, L2), tree_iso(R1, R2).

tree_iso(tree(X, L1, R1), tree(X, L2, R2)) :-
   tree_iso(L1, R2), tree_iso(R1, L2).
```

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Binary Trees VII – Tree Substitution

Replace all occurrences of X by Y.

```
subs(X, Y, void, void).
subs(X, Y, tree(X, L1, R1), tree(Y, L2, R2)) :-
   subs(X, Y, L1, L2),
   subs(X, Y, R1, R2).
subs(X, Y, tree(Z, L1, R1), tree(Z, L2, R2)) :-
   X =\= Y, subs(X, Y, L1, L2),
   subs(X, Y, R1, R2).
```

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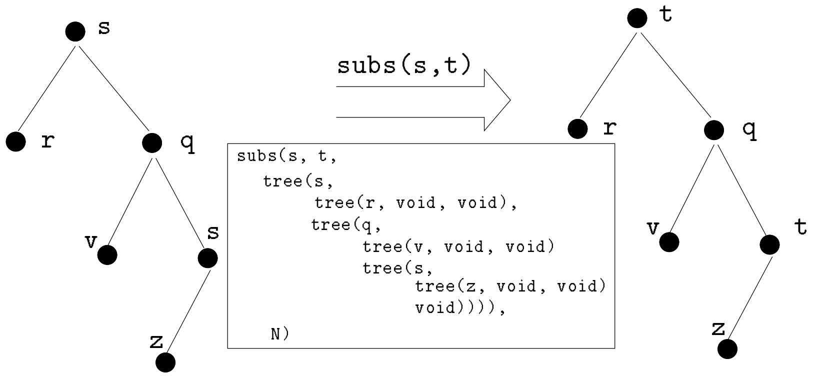
Binary Trees VIII – Tree Substitution



Symbolic Differentiation I

$$\frac{\mathrm{d}c}{\mathrm{d}x} = 0 \quad (1)$$
$$\frac{\mathrm{d}x}{\mathrm{d}x} = 1 \quad (2)$$
$$\frac{\mathrm{d}(U^c)}{\mathrm{d}x} = cU^{c-1}\frac{\mathrm{d}U}{\mathrm{d}x} \quad (3)$$
$$\frac{\mathrm{d}(-U)}{\mathrm{d}x} = -\frac{\mathrm{d}U}{\mathrm{d}x} \quad (4)$$
$$\frac{\mathrm{d}(U+V)}{\mathrm{d}x} = \frac{\mathrm{d}U}{\mathrm{d}x} + \frac{\mathrm{d}V}{\mathrm{d}x} \quad (5)$$
$$\frac{\mathrm{d}(U-V)}{\mathrm{d}x} = \frac{\mathrm{d}U}{\mathrm{d}x} - \frac{\mathrm{d}V}{\mathrm{d}x} \quad (6)$$
$$\frac{\mathrm{d}(cU)}{\mathrm{d}x} = c\frac{\mathrm{d}U}{\mathrm{d}x} \quad (7)$$

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Symbolic Differentiation II

$$\frac{\mathrm{d}(UV)}{\mathrm{d}x} = U\frac{\mathrm{d}V}{\mathrm{d}x} + V\frac{\mathrm{d}U}{\mathrm{d}x} \quad (8)$$
$$\frac{\mathrm{d}(\frac{U}{V})}{\mathrm{d}x} = \frac{V\frac{\mathrm{d}U}{\mathrm{d}x} - U\frac{\mathrm{d}V}{\mathrm{d}x}}{V^2} \quad (9)$$
$$\frac{\mathrm{d}(\ln U)}{\mathrm{d}x} = U^{-1}\frac{\mathrm{d}U}{\mathrm{d}x} \quad (10)$$
$$\frac{\mathrm{d}(\sin(U))}{\mathrm{d}x} = \frac{\mathrm{d}U}{\mathrm{d}x}\cos(U) \quad (11)$$
$$\frac{\mathrm{d}(\cos(U))}{\mathrm{d}x} = -\frac{\mathrm{d}U}{\mathrm{d}x}\sin(U) \quad (12)$$

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Symbolic Differentiation III

$$\frac{\mathrm{d}c}{\mathrm{d}x} = 0 \quad (1)$$
$$\frac{\mathrm{d}x}{\mathrm{d}x} = 1 \quad (2)$$
$$\frac{\mathrm{d}(U^c)}{\mathrm{d}x} = cU^{c-1}\frac{\mathrm{d}U}{\mathrm{d}x} \quad (3)$$

```
deriv(C, X, 0) :- number(C).

deriv(X, X, 1).

deriv(U ^C, X, C * U ^L * DU) :-
      number(C), L is C - 1,
      deriv(U, X, DU).
```

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Symbolic Differentiation IV

$$\frac{\mathrm{d}(-U)}{\mathrm{d}x} = -\frac{\mathrm{d}U}{\mathrm{d}x} \quad (4)$$
$$\frac{\mathrm{d}(U+V)}{\mathrm{d}x} = \frac{\mathrm{d}U}{\mathrm{d}x} + \frac{\mathrm{d}V}{\mathrm{d}x} \quad (5)$$
$$\frac{\mathrm{d}(U-V)}{\mathrm{d}x} = \frac{\mathrm{d}U}{\mathrm{d}x} - \frac{\mathrm{d}V}{\mathrm{d}x} \quad (6)$$

```
deriv(-U, X, -DU) :-
   deriv(U, X, DU).

deriv(U+V, X, DU + DV) :-
   deriv(U, X, DU), deriv(V, X, DV).

deriv(U-V, X, DU - DV) :-
   deriv(U, X, DU), deriv(V, X, DV).
```

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Symbolic Differentiation V

$$\frac{\mathrm{d}(cU)}{\mathrm{d}x} = c\frac{\mathrm{d}V}{\mathrm{d}x} \qquad (7)$$

$$\frac{\mathrm{d}(UV)}{\mathrm{d}x} = U\frac{\mathrm{d}V}{\mathrm{d}x} + V\frac{\mathrm{d}U}{\mathrm{d}x} \qquad (8)$$

$$\frac{\mathrm{d}(\frac{U}{V})}{\mathrm{d}x} = \frac{V\frac{\mathrm{d}U}{\mathrm{d}x} - U\frac{\mathrm{d}V}{\mathrm{d}x}}{V^2} \qquad (9)$$

```
deriv(C*U, X, C*DU) :-
   number(C), deriv(U, X, DU).

deriv(U*V, X, U*DV + V*DU) :-
   deriv(U, X, DU), deriv(V, X, DV).

deriv(U/V, X, (V*DU - U*DV)/(V*V)) :-
   deriv(U, X, DU), deriv(V, X, DV).
```

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Symbolic Differentiation VI

$$\frac{\mathrm{d}(\ln U)}{\mathrm{d}x} = U^{-1}\frac{\mathrm{d}U}{\mathrm{d}x} \qquad (10)$$

$$\frac{\mathrm{d}(\sin(U))}{\mathrm{d}x} = \frac{\mathrm{d}U}{\mathrm{d}x}\cos(U) \qquad (11)$$

$$\frac{\mathrm{d}(\cos(U))}{\mathrm{d}x} = -\frac{\mathrm{d}U}{\mathrm{d}x}\sin(U) \qquad (12)$$

```
deriv(log(U), X, DU/U) :-
      deriv(U, X, DU).

deriv(sin(U), X, DU*cos(U)) :-
      deriv(U, X, DU).

deriv(cos(U), X, -DU*sin(U)) :-
      deriv(U, X, DU).
```

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Symbolic Differentiation VII

```
?- deriv(x, x, D).
   D = 1

?- deriv(sin(x), x, D).
   D = 1*cos(x)

?- deriv(sin(x) + cos(x), x, D).
   D = 1*cos(x)+ (-1*sin(x))

?- deriv(sin(x) * cos(x), x, D).
   D = sin(x)* (-1*sin(x))
      +cos(x)* (1*cos(x))

?- deriv(1 / x, x, D).
   D = (x*0-1*1)/ (x*x)
```

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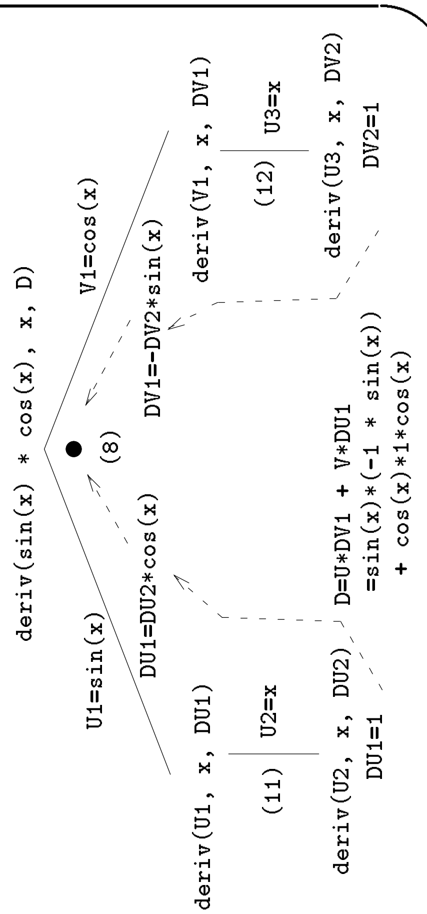
Symbolic Differentiation VIII



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Symbolic Differentiation IX

```
?- deriv(1/sin(x), x, D).
   D = (sin(x)*0-1* (1*cos(x)))
      (sin(x)*sin(x))

?- deriv(x ^3, x, D).
   D = 1*3*x^2

?- deriv(x^3 + x^2 + 1, x, D).
   D = 1*3*x^2+1*2*x^1+0
```

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Symbolic Differentiation X

```
?- deriv(3 * x ^3, x, D).
   D = 3* (1*3*x^2)+x^3*0

?- deriv(4* x ^3 +
   4 * x^2 + x - 1, x, D).
   D = 4* (1*3*x^2)+x^3*0+
      (4* (1*2*x^1)+x^2*0)+1-0
```

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Prolog So Far II

- Prolog *terms*:
  - atoms (`a, 1, 3.14`)
  - structures
    `guitar(ovation, 1111, 1975)`
- Infix expressions are abbreviations of "normal" Prolog terms:

infix	prefix
`a + b`	`+(a, b)`
`a + b* c`	`+(a, *(b, c))`

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