CSc 372

Comparative Programming Languages

25 : Prolog — Techniques

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Generate & Test – Integer Division

Generate & Test

A generate-and-test procedure has two parts:

1. A generator which can generate a number of possible solutions.
2. A tester which succeeds iff the generated result is an acceptable solution.

When the tester fails, the generator will backtrack and generate a new possible solution.

Generate & Test – Division

- We can define integer arithmetic (inefficiently) in Prolog:

```
% Integer generator.
is_int(0).
is_int(X) :- is_int(Y), X is Y+1.

% Result = N1 / N2.
divide(N1, N2, Result) :-
   is_int(Result),
   P1 is Result*N2,
   P2 is (Result+1)*N2,
   P1 =< N1, P2 > N1, !.

| ?- divide(6,2,R).
      R = 3
```

Generate & Test – Division. . .

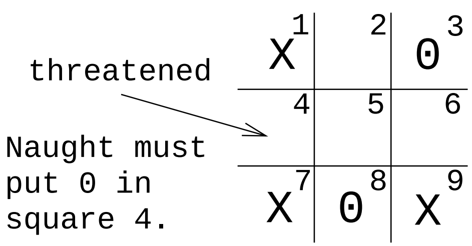
```
is_int(0).
is_int(X) :- is_int(Y), X is Y+1.
divide(N1, N2, Result) :-
   is_int(Result),
   P1 is Result*N2, P2 is (Result+1)*N2,
   P1 =< N1, P2 > N1, !.
```

divide(6,2,R) --- N1=6, N2=2				
Res	P1	P2	P1 =< N1	P2 > N1
0	0	2	True	False
1	2	4	True	False
2	4	6	True	False
3	6	12	True	True

Generate & Test – Tic-Tac-Toe

Generate & Test – Tic-Tac-Toe

- This is a part of a program to play Tic-Tac-Toe (Naughts and Crosses).
- Two players take turns to put down X and O on a 3x3 board. Whoever gets a line of 3 (horizontal, vertical, or diagonal) markers has won.



Generate & Test – Tic-Tac-Toe. . .

- We'll look at the predicate `forced_move` which answers the question:
  - Am I (the naught-person) forced to put a marker at a particular position?
- The program tries to find a line with two crosses.
- It only makes sense to find one forced move, hence the cut.

Generate & Test – Tic-Tac-Toe...

- `aline(L)` is a generator – it generates all possible lines(L).
- `threatening(L,B,Sq)` is a tester – it succeeds if `Sq` is a threatened square in line `L` of board `B`.

```
forced_move(Board, Sq) :-
   aline(Line),
   threatening(Line, Board, Sq), !.

?- forced_move(b(x,_,o,_,_,_,x,o,x),4).
   yes

aline([1,2,3]).  aline([4,5,6]).  aline([7,8,9]).
aline([1,4,7]).  aline([2,5,8]).  aline([3,6,9]).
aline([1,5,9]).  aline([3,5,7]).
```

Gen. & Test – Tic-Tac-Toe...

- `threatening` succeeds if it finds a line with two crosses and one empty square.

```
threatening([X,Y,Z],B,X) :-
   empty(X,B), cross(Y,B), cross(Z,B).
threatening([X,Y,Z],B,Y) :-
   cross(X,B), empty(Y,B), cross(Z,B).
threatening([X,Y,Z],B,Z) :-
   cross(X,B), cross(Y,B), empty(Z,B).
```

Gen. & Test – Tic-Tac-Toe...

- A square is empty if it is an uninstantiated variable.
- `arg(N,S,V)` returns the `N`:th element of a structure `S`.

```
empty(Sq, Board) :-
   arg(Sq,Board,Val), var(Val).
cross(Sq, Board) :-
   arg(Sq,Board,Val), nonvar(Val), Val=x.
naught(Sq, Board) :-
   arg(Sq,Board,Val), nonvar(Val), Val=o.
```

Arbitrage

Generate & Test – Arbitrage

From the Online Webster's:

arbitrage *simultaneous purchase and sale of the same or equivalent security in order to profit from price discrepancies*

```
?- arbitrage.
   dollar dmark yen 1.03751
   yen dollar dmark 1.03751
   dmark yen dollar 1.03751
```

```
exchange(pound, dollar, london, 1.550).
exchange(pound, dollar, new_york, 1.555).
exchange(pound, dollar, tokyo, 1.559).
exchange(pound, yen, london, 153.97).
exchange(pound, yen, new_york, 154.05).
exchange(pound, yen, tokyo, 154.3).
exchange(pound, dmark, london, 2.4075).
exchange(pound, dmark, new_york, 2.44).
exchange(pound, dmark, tokyo, 2.408).
exchange(dollar, yen, london, 98.3).
exchange(dollar, yen, new_york, 98.35).
exchange(dollar, yen, tokyo, 98.25).
exchange(dollar, dmark, london, 1.537).
exchange(dollar, dmark, new_york, 1.58).
exchange(dollar, dmark, tokyo, 1.57).
exchange(yen, dmark, london, 0.015635).
exchange(yen, dmark, new_york, 0.0155).
exchange(yen, dmark, tokyo, 0.0158).
```

Generate & Test – Arbitrage...

```
arbitrage :-
   profit3(From, Via, To, Profit), % Gen
   Profit > 1.03, % Test
   write(From), write(' '),
   write(Via), write(' '),
   write(To), write(' '),
   write(Profit), nl, fail.
arbitrage.

% Find three currencies, and the profit:
profit3(From, Via, To, Profit) :-
   best_rate(From, Via, P1, R1),
   best_rate(Via, To, P2, R2),
   best_rate(To, From, P3, R3),
   Profit is R1 * R2 * R3.
```

Generate & Test – Arbitrage...

```
% We can convert back and forth
% between currencies:
rate(From, To, P, R) :-
   exchange(From, To, P, R).
rate(From, To, P, R) :-
   exchange(To, From, P, S), R is 1/S.

% Find the best place to convert
% between currencies From & To:
best_rate(From, To, Place,Rate):-
   rate(From, To, Place, Rate),
   not((rate(From, To, P1, R1), R1>Rate)).
```

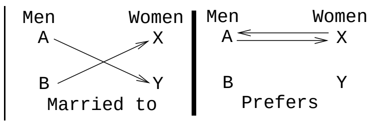
Stable Marriages

Stable Marriages

- Suppose there are N men and N women who want to get married to each other.
- Each man (woman) has a list of all the women (men) in his (her) preferred order. The problem is to find a set of marriages that is stable.

A set of marriages is `unstable` if two people who are not married both prefer each other to their spouses. If A and B are men and X and Y women, the pair of marriages $A - Y$ and $B - X$ is unstable if

- A prefers X to Y, and
- X prefers A to B.



Stable Marriages – Example

Person	Sex	1st choice	2nd choice	3rd choice
Avraham	M	Chana	Ruth	Zvia
Binyamin	M	Zvia	Chana	Ruth
Chaim	M	Chana	Ruth	Zvia
Zvia	F	Binyamin	Avraham	Chaim
Chana	F	Avraham	Chaim	Binyamin
Ruth	F	Avraham	Binyamin	Chaim

- Chaim-Ruth, Binyamin-Zvia, Avraham-Chana is stable.
- Chaim-Chana, Binyamin-Ruth, Avraham-Zvia is unstable, since Binyamin prefers Zvia over Ruth and Zvia prefers Binyamin over Avraham.

Stable Marriages...

- Write a program which takes a set of people and their preferences as input, and produces a set of stable marriages as output.

Input Format:

```
prefer(avraham, man,
        [chana,tamar,zvia,ruth,sarah]).

men([avraham,binyamin,chaim,david,elazar]).
women([zvia, chana, ruth, sarah, tamar]).
```

- The first rule, says that `avraham` is a man and that he prefers `chana` to `tamar`, `tamar` to `zvia`, `zvia` to `ruth`, and `ruth` to `sarah`.

```
prefer(avraham, man, [chana, tamar, zvia, ruth, sarah]).
prefer(binyamin, man, [zvia, chana, ruth, sarah, tamar]).
prefer(chaim, man, [chana, ruth, tamar, sarah, zvia]).
prefer(david, man, [zvia, ruth, chana, sarah, tamar]).
prefer(elazar, man, [tamar, ruth, chana, zvia, sarah]).
prefer(zvia, woman, [elazar, avraham, david, binyamin, chaim]).
prefer(chana, woman, [david, elazar, binyamin, avraham, chaim]).
prefer(ruth, woman, [avraham, david, binyamin, chaim, elazar]).
prefer(sarah, woman, [chaim, binyamin, david, avraham, elazar]).
prefer(tamar, woman, [david, binyamin, chaim, elazar, avraham]).
```

Stable Marriages...

- `gen` generates all possible sets of marriages, `unstable` tests if they are stable.

```
go :-
   men(ML), women(WL),
   gen(ML, WL, [], L), \+unstable(L)),
   show(L), fail.
go.

?- men(ML), women(WL), gen(ML,WL,[],L).
   L = [m(elazar,tamar),m(david,sarah),
      m(chaim,ruth),m(binyamin,chana),
      m(avraham,zvia)] ?  ;
         .........
```

Stable Marriages — Generate

```
gen([A|M1], W, In, Out) :-
   delete(B, W, W1),
   gen(M1, W1, [m(A,B)|In], Out).
gen([],[],L,L).

delete(A, [A|L], L).
delete(A, [X|L], [X|L1]) :-
   delete(A, L, L1).
```

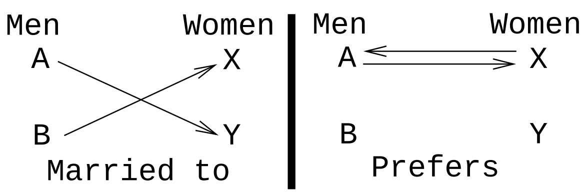
Stable Marriages — Test

```
% A prefers B to C.
pref(A, B, C) :-
   prefer(A, _, L),
   append(_, [B|S], L), !,
   member(C, S), !.

unstable(L) :-
   append(_, [A|R], L),
   member(B, R),
   (is_unstable(A,B);
      is_unstable(B,A)).

is_unstable(m(A,Y), m(B,X)) :-
   pref(A, X, Y),
   pref(X, A, B).
```

Stable Marriages. . .



Bedtime Story

Puzzles – Bedtime Story

"Helder, a poor scientist, was in love with the daughter of an admiral. One day, a general captured the girl. Helder rode to the general's barrack and killed the general. The girl was grateful and fell in love with Helder. The admiral was so happy to have his daughter back he gave Helder half of all his boats."

- "Who is the father of the girl?"
- "Who is rich?"
- "Who loves who?"
- "Who is poor?"
- "Who captured who?"
- "Who killed who?"

Puzzles – Bedtime Story. . .

```
:- op(500, xfy, 'is_').
:- op(500, yfx, 'loves').
:- op(500, yfx, 'kills').
:- op(500, yfx, 'to').
:- op(500, yfx, 'captures').
:- op(500, yfx, 'rides_to').
:- op(500, yfx, 'gives').
:- op(500, yfx, 'is_father_of').
:- op(800, yfx, 'and').

X and Y :- X, Y.
```

Puzzles – Bedtime Story. . .

```
helder is_ poor.
helder is_ scientist.
admiral is_ happy.
admiral is_father_of girl.
helder loves girl.
girl loves helder.
general captures girl.
helder kills general.
admiral gives half_boats to helder.
```

Puzzles – Bedtime Story. . .

```
% Who loves who?
?- Z loves Y, write(Z), write(' loves '),
      write(Y), nl, fail.
   helder loves girl
   girl loves helder

% Who captures who?
?- Z captures Y.
   Z = general
   Y = girl
```

Puzzles – Bedtime Story. . .

```
% Who kills who?
?- Z kills Y.
   Z = helder
   Y = general

% Who loves who's daughter?
?- Z loves G and F is_father_of G.
   Z = helder
   G = girl
   F = admiral
```

Puzzles – Trees

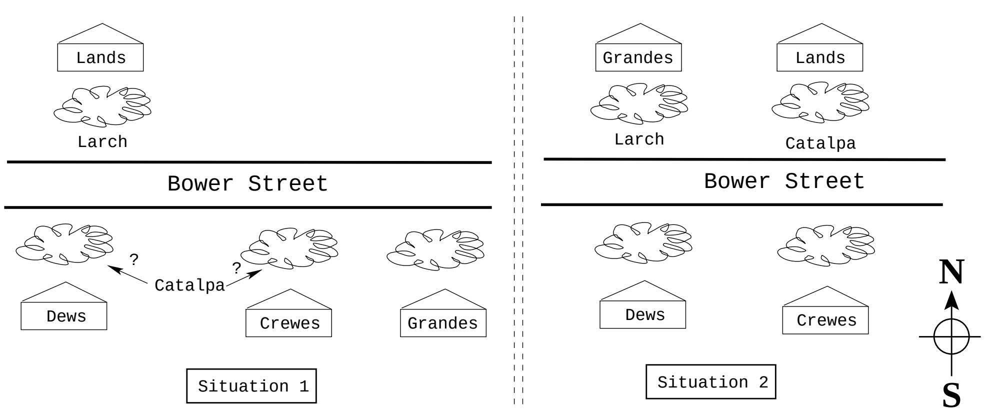
Puzzles – Trees

- The Crewes, Dews, Grandes, and Lands of Bower Street each have a front-yard tree: Catalpa, Dogwood, Gingko, Larch.
- The Grandes' tree and the Catalpa are on the same side of the street.
- The Crewes live across the street from the Larch.
- The Larch is across the street from the Dews' house.
- No tree starts with the same letter as its owner's name.
- Who owns which tree?

Puzzles – Trees

```
| ?- solve.
   Grandes owns the Larch
   Crewes owns the Dogwood
   Dews owns the Ginko
   Lands owns the Catalpa
```

Puzzles – Trees. . .



Puzzles – Trees. . .

```
% Let's assume that the Larch is on the
% north side of the street.
northside('Larch').

% The Crewes live across the street from
% the Larch.  The Larch is across the
% street from the Dews' house.
southside('Crewes').
southside('Dews').

% The Grandes' tree and the 'Catalpa'
% are on the same side of the street.
northside('Catalpa') :-
   northside('Grandes').
```

Puzzles – Trees. . .

```
% If Grandes have a 'Larch', then they
% must live on the north side.
northside('Grandes') :-
   have('Grandes', 'Larch').

% Grandes have a 'Larch', if noone
% else does.
have('Grandes','Larch') :-
   not_own('Crewes','Larch'),
   not_own('Dews','Larch'),
   not_own('Lands','Larch')
```

Puzzles – Trees. . .

```
% then the Dews' and Crews' will be
% on the south side.  Also, if the
% Catalpa is on the north the Dogwood
% and Ginko must both be on the south
% side (since each house has one tree).
southside('Dogwood') :-
   northside('Larch'),
   northside('Catalpa').
southside('Ginko') :-
   northside('Larch'),
   northside('Catalpa').
```

Puzzles – Trees. . .

```
% Are you a tree or a plant?
person(X) :- member(X,
   ['Grandes','Crewes','Dews','Lands']).
tree(X) :- member(X,
   ['Catalpa','Ginko','Dogwood','Larch']).

% No tree starts with the same letter as
% its owner's name.
not_own(X,Y) :-
   name(X, [A|_]), name(Y,[A|_]).

% The Grandes' tree and the 'Catalpa'
% are on the same side of the street.
not_own('Grandes','Catalpa').
```

Puzzles – Trees. . .

```
% Only a person can own a tree.
not_own(X,Y) :- person(X), person(Y).
not_own(X,Y) :- tree(X), tree(Y).

% A person can only own a tree that's on
% the same side of the street as
% themselves.
not_own(X,Y) :- northside(X),southside(Y).
not_own(X,Y) :- southside(X),northside(Y).
```

Puzzles – Trees...

```
% You can't own what someone else owns.
not_own('Crewes', X) :- owns('Dews', X).
not_own('Lands', X) :- owns('Crewes', X).
not_own('Lands', X) :- owns('Dews',X).

owns(X,Y) :-
   person(X), tree(Y),
   not(not_own(X,Y)).

solve :-
   owns(Person,Tree),
   write(Person), write(' owns the '),
   write(Tree),nl,fail.
solve.
```

Logic Arithmetic

Arithmetic In Logic

- Arithmetic in Prolog is just like arithmetic in imperative languages. We can't do `25 is X + Y` and hope to get `X` and `Y` instantiated to every pair of numbers that sum to 25.
- There are cases when we need the power of logic arithmetic, rather than the efficient built-in operators. That is no problem, we can always define the logic arithmetic predicates ourselves.
- For example, how do we split a number into the two parts Note that this is similar to splitting a list using append.

Arithmetic In Logic...

- We can always write our own **logic** arithmetic predicates.

```
% Represent S as the sum of 2 numbers.
% minus(S, D1, D2) -- S − D1 = D2
minus(S, S, 0).
minus(S, D1, D2) :-     % Note that
   S > 0, S1 is S-1,    % S must be
   minus(S1, D1, D3),   % instantiated.
   D2 is D3 + 1.

?- minus(3, X, Y).
   X = 3, Y = 0 ;
   X = 2, Y = 1 ;
   X = 1, Y = 2 ;
   X = 0, Y = 3
```

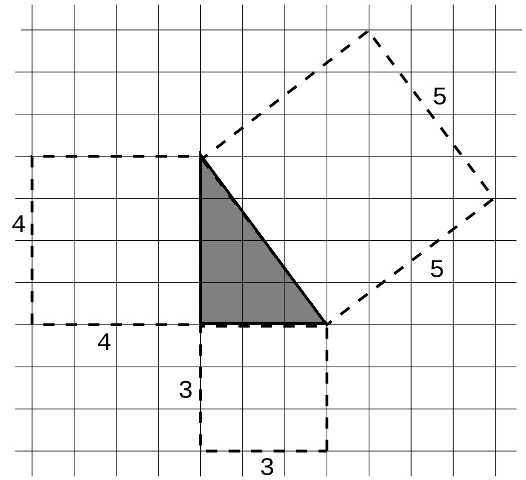
Arithmetic In Logic...

- The `minus` predicate splits `S` into `D1 + D2`. Why does it work? Well, look at this:

$$
\begin{aligned}
S1 &= S-1 \;\; \texttt{first line} \\
D3 &= S1-D1 \;\; \texttt{second line} \\
D2 &= D3+1 \;\; \texttt{third line} \\
S &= S1+1 \\
&= (D3+D1)+1 \\
&= ((D2-1)+D1)+1 \\
&= D2+D1
\end{aligned}
$$

- Note that the `minus` predicate require the first argument to be instantiated, but not the second and third. `minus`, below, is a lot like `append`.

Pythagorean Triples



Pythagorean Triples...

```
?- pythag(X, Y, Z).
   X = 4, Y = 3, Z = 5 ;
   X = 3, Y = 4, Z = 5 ;
   X = 8, Y = 6, Z = 10 ;
   X = 6, Y = 8, Z = 10 ;
   X = 12, Y = 5, Z = 13 ;
   X = 5, Y = 12, Z = 13 ;
   X = 12, Y = 9, Z = 15
```

Pythagorean Triples...

- `is_int` is used to generate a sequence of numbers.
- `int_triple` splits the generated integer `S` into the sum of three integer `X, Y, Z`.
- In other words, first we check all triples that sum to 1 to see if any of them are pythagorean triples, then all triples that sum to 2, etc. This obviously will eventually check "all" triples. It also will make sure that we get them "in order", with the smallest triples first.

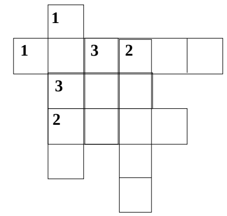
Pythagorean Triples. . .

```
% Generate a sequence of numbers.
is_int(0).
is_int(X) :- is_int(Y), X is Y+1.

pythag(X, Y, Z) :-
      int_triple(X, Y, Z),
      Z*Z =:= X*X + Y*Y.

% Generate integer triples:  S=X+Y+Z.
int_triple(X, Y, Z) :-
      is_int(S),
      minus(S, X, S1), X > 0,
      minus(S1, Y, Z), Y > 0, Y > 0.
```

Exercise: Crossword Puzzle



Across

1. The Fifth Element.
2. Mumintroll mum.
3. Beer.

Down

1. Kills at chess.
2. Best drummer. Ever.
3. Electric Light Orchestra.

Write a program that solves the crossword puzzle above, assuming this database of words:

```
word(leeloo). word(death). word(ale).
word(tove). word(levon). word(elo).
```

Exercise: Crossword Puzzle

1. Now, assume that you have a much bigger database of words.
2. How would you organize the database for much faster searching?
3. How would you rewrite your code to make use of the new database structure?