CSc 372

Comparative Programming Languages

10 : Haskell — Curried Functions

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Infix Functions

Declaring Infix Functions

- Sometimes it is more natural to use an infix notation for a function application, rather than the normal prefix one:
  - `5 + 6` (infix)
  - `(+) 5 6` (prefix)
- Haskell predeclares some infix operators in the standard prelude, such as those for arithmetic.
- For each operator we need to specify its precedence and associativity. The higher precedence of an operator, the stronger it binds (attracts) its arguments: hence:

  `3 + 5*4` $\equiv$ `3 + (5*4)`

  `3 + 5*4` $\not\equiv$ `(3 + 5) * 4`

Declaring Infix Functions...

- The associativity of an operator describes how it binds when combined with operators of equal precedence. So, is

  ```
  5-3+9     ≡ (5-3)+9 = 11
              OR
  5-3+9     ≡ 5-(3+9) = -7
  ```

  The answer is that + and - associate to the left, i.e. parentheses are inserted from the left.
- Some operators are right associative: `5^3^2` $\equiv$ `5^(3^2)`
- Some operators have free (or no) associativity. Combining operators with free associativity is an error:

  ```
  5 == 4 < 3           ⇒ ERROR
  ```

Declaring Infix Functions...

- The syntax for declaring operators:

```
infixr prec oper   -- right assoc.
infixl prec oper   -- left assoc.
infix prec oper    -- free assoc.
```

From the standard prelude:

```
infixl 7 *
infix 7  /, `div`, `rem`, `mod`
infix 4  ==, /=, <, <=, >=, >
```

- An infix function can be used in a prefix function application, by including it in parenthesis. Example:

```
?  (+) 5 ((*) 6 4)
      29
```

Multi-Argument Functions

Multi-Argument Functions

- Haskell only supports one-argument functions.
- An n-argument function $f(a_1, \cdots, a_n)$ is constructed in either of two ways:
  1. By making the one input argument to f a tuple holding the n arguments.
  2. By letting f "consume" one argument at a time. This is called currying.

Tuple	Currying
`add :: (Int,Int)->Int`	`add :: Int->Int->Int`
`add (a, b) = a + b`	`add a b = a + b`

Currying

- Currying is the preferred way of constructing multi-argument functions.
- The main advantage of currying is that it allows us to define specialized versions of an existing function.
- A function is specialized by supplying values for one or more (but not all) of its arguments.
- Let's look at Haskell's plus operator `(+)`. It has the type

  `(+) :: Int -> (Int -> Int)`.

- If we give two arguments to `(+)` it will return an `Int`:

  `(+) 5 3` $\Rightarrow$ `8`

Currying...

- If we just give one argument (5) to `(+)` it will instead return a function which "adds 5 to things". The type of this specialized version of `(+)` is `Int -> Int`.
- Internally, Haskell constructs an intermediate – specialized – function:

```
add5 ::  Int -> Int
add5 a = 5 + a
```

- Hence, `(+) 5 3` is evaluated in two steps. First `(+) 5` is evaluated. It returns a function which adds 5 to its argument. We apply the second argument 3 to this new function, and the result 8 is returned.

Currying...

- To summarize, Haskell only supports one-argument functions. Multi-argument functions are constructed by successive application of arguments, one at a time.
- Currying is named after logician Haskell B. Curry (1900-1982) who popularized it. It was invented by Schönfinkel in 1924. Schönfinkeling doesn't sound too good...
- Note: Function application (`f x`) has higher precedence (10) than any other operator. Example:

  `f 5 + 1` $\Leftrightarrow$ `(f 5) + 1`
  `f 5 6` $\Leftrightarrow$ `(f 5) 6`

Currying Example

- Let's see what happens when we evaluate `f 3 4 5`, where `f` is a 3-argument function that returns the sum of its arguments.

```
f ::  Int -> (Int -> (Int -> Int))
f x y z = x + y + z
```

`f 3 4 5` $\equiv$ `((f 3) 4) 5`

Currying Example. . .

- `(f 3)` returns a function `f' y z` (`f'` is a specialization of `f`) that adds 3 to its next two arguments.

`f 3 4 5` $\equiv$ `((f 3) 4) 5` $\Rightarrow$ `(f' 4) 5`

```
f' ::  Int -> (Int -> Int)
f' y z = 3 + y + z
```

Currying Example...

- `(f' 4)` ($\equiv$ `(f 3) 4`) returns a function `f''z` (`f''` is a specialization of `f'`) that adds (3+4) to its argument.

`f 3 4 5` $\equiv$ `((f 3) 4) 5` $\Rightarrow$ `(f' 4) 5`
$\Rightarrow$ `f'' 5`

```
f'' ::  Int -> Int
f'' z = 3 + 4 + z
```

- Finally, we can apply `f''` to the last argument (5) and get the result:

`f 3 4 5` $\equiv$ `((f 3) 4) 5` $\Rightarrow$ `(f' 4) 5`
$\Rightarrow$ `f'' 5` $\Rightarrow$ 3+4+5 $\Rightarrow$ 12

Currying Example

The Combinatorial Function:

- The combinatorial function $\binom{n}{r}$ "n choose r", computes the number of ways to pick r objects from n.

$$\binom{n}{r} = \frac{n!}{r! * (n-r)!}$$

In Haskell:

```
comb ::  Int -> Int -> Int
comb n r = fact n/(fact r*fact(n-r))

?  comb 5 3
   10
```

Currying Example...

```
comb ::  Int -> Int -> Int
comb n r = fact n/(fact r*fact(n-r))
```

comb 5 3 $\Rightarrow$ (comb 5) 3 $\Rightarrow$
comb^5 3 $\Rightarrow$
120 / (fact 3 $*$ (fact 5-3)) $\Rightarrow$
120 / (6 $*$ (fact 5-3)) $\Rightarrow$
120 / (6 $*$ fact 2) $\Rightarrow$
120 / (6 $*$ 2) $\Rightarrow$
120 / 12 $\Rightarrow$
10

comb^5 r = 120 / (fact r $*$ fact(5-r))

- comb^5 is the result of partially applying comb to its first argument.

Associativity

- Function application is left-associative:
  `f a b = (f a) b` | `f a b` $\neq$ `f (a b)`
- The function space symbol '`->`' is right-associative:

  `a -> b -> c = a -> (b -> c)`

  `a -> b -> c` $\neq$ `(a -> b) -> c`
- `f` takes an `Int` as argument and returns a function of type `Int -> Int`. `g` takes a function of type `Int -> Int` as argument and returns an `Int`:

  `f' ::  Int -> (Int -> Int)`

  $\Updownarrow$

  `f ::  Int -> Int -> Int`

  $\not\Updownarrow$

  `g ::  (Int -> Int) -> Int`

What's the Type, Mr. Wolf?

- If the type of a function `f` is

$$\mathtt{t}_1 \texttt{ -> } \mathtt{t}_2 \texttt{ -> } \cdots \texttt{ -> } \mathtt{t}_n \texttt{ -> } \mathtt{t}$$

- and `f` is applied to arguments

$$\mathtt{e}_1\texttt{::}\mathtt{t}_1\texttt{, } \mathtt{e}_2\texttt{::}\mathtt{t}_2\texttt{, } \cdots\texttt{, } \mathtt{e}_k\texttt{::}\mathtt{t}_k\texttt{,}$$

- and $\mathtt{k} \leq \mathtt{n}$
- then the result type is given by cancelling the types $\mathtt{t}_1 \cdots \mathtt{t}_k$:

$$\cancel{\mathtt{t}}_1 \texttt{ -> } \cancel{\mathtt{t}}_2 \texttt{ -> } \cdots \texttt{ -> } \cancel{\mathtt{t}}_k \texttt{ -> } \mathtt{t}_{k+1} \texttt{ -> } \cdots \texttt{ -> } \mathtt{t}_n \texttt{ -> } \mathtt{t}$$

- Hence, $\mathtt{f}\ \mathtt{e}_1\ \mathtt{e}_2\ \cdots\ \mathtt{e}_k$ returns an object of type

$$\mathtt{t}_{k+1} \texttt{ -> } \cdots \texttt{ -> } \mathtt{t}_n \texttt{ -> } \mathtt{t}.$$

- This is called the Rule of Cancellation.

`flip`

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  = f y x
```

- The `flip` function takes a function `f x y` (`f` is the function and `x` and `y` its two arguments, and reorders the arguments!
- Or, more correctly, `flip` returns a new function `f y x`.
- You can use this when you want to specialize a function by supplying an argument, but the function takes its arguments in the "wrong order."

`flip`...

- Consider the `(!!)` function, for example:

```
> :type (!!)
(!!) :: [a] -> Int -> a
> :type flip(!!)
flip (!!) :: Int -> [a] -> a
> (!!) [1..10] 2
3
> (flip (!!)) 2 [1..10]
3
```

- Now you can write a function `fifth` using `(!!)` which returns the fifth element of a list:

```
fifth :: [a] -> a
fifth = (flip (!!)) 5
```

Homework

- Define an operator `$$` so that `x $$ xs` returns `True` if `x` is an element in `xs`, and `False` otherwise.

Example:

```
? 4 $$ [1,2,5,6,4,7]
  True

? 4 $$ [1,2,3,5]
  False

? 4 $$ []
  False
```

Homework

- Define an function `drop3` which takes a list as argument and returns a new list with the first three elements removed.
- Use currying!

Homework

```
> :type elem
elem :: Eq a => a -> [a] -> Bool
> elem 3 [1..10]
```

- The `elem` function returns true if the first argument is a member of the second (a list).
- Write a function `has3 xs` which returns true if `xs` (a list) contains the number 3.
- Write a function `isSmallPrime x` which returns true if `x` is one of the numbers 2,3,5,7.
- Use currying!

```
> isSmallPrime 2
True
> has3 [1]
False
```