CSc 453

Compilers and Systems Software

6 : Top-Down Parsing I

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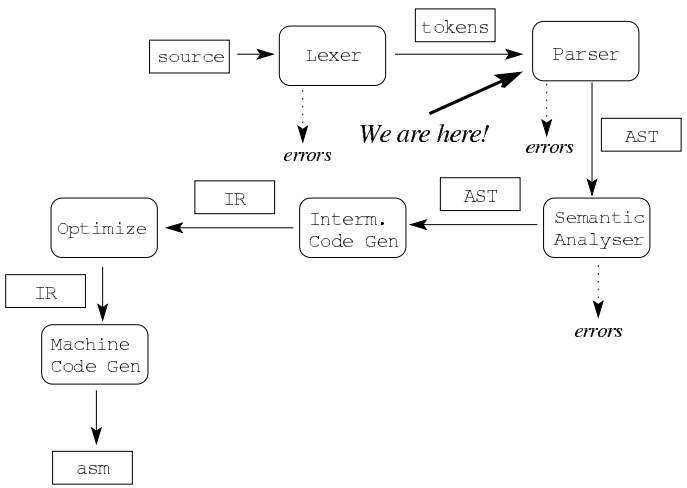
Overview

Compiler Phases



Grammars

Context Free Grammars

- CFGs are used to describe the syntax of programming languages. A production

$$S \rightarrow \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S_1\ \underline{\texttt{else}}\ S_2$$

in a CFG says

"If S_1 and S_2 are statements and E an expression then '$\underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S_1\ \underline{\texttt{else}}\ S_2$' is a statement".

Notice that this production is recursive; it allows if-statements to occur within if-statements.

Context Free Grammars...

$$S \rightarrow \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S_1\ \underline{\texttt{else}}\ S_2$$

- $\underline{\texttt{if}}$, $\underline{\texttt{then}}$, and $\underline{\texttt{else}}$ are terminal symbols or tokens.
- S, S_1, S_2, and E are non-terminals. They are like "variables", that represent the kinds of strings that the grammar defines as statements or expressions, respectively.

CFG Notation

terminals:

$\underline{\texttt{a}}, \underline{\texttt{b}}, \underline{\texttt{c}}, \cdots, \underline{+}, \underline{-}, \cdots, \underline{0}, \underline{1}, \cdots, \underline{\texttt{if}}, \underline{\texttt{do}}$.

nonterminals:

$A, B, C, \cdots, S, \cdots$, expr, stmt.

grammar symbols:

$X, Y, Z, \cdots$ (either terminals or nonterminals).

strings of terminals:

$u, v, w \cdots$.

strings of grammar symbols:

$\alpha, \beta, \gamma, \cdots$ (strings of terminals or nonterminals).

productions:

$A \rightarrow \alpha_1, A \rightarrow \alpha_2, \cdots, A \rightarrow \alpha_k$, or
$A \rightarrow \alpha_1 \mid \alpha_2 \cdots \mid \alpha_k$.

Derivations — Productions as *Rewrite Rules*

1. Start with the start symbol, S.
2. Pick any production $S \rightarrow \alpha$, eg. $S \rightarrow \underline{\texttt{id}}\ \underline{:=}\ E$.
3. We say that S derives $\underline{\texttt{id}}\ \underline{:=}\ E$, or $S \Rightarrow \underline{\texttt{id}}\ \underline{:=}\ E$. '$\underline{\texttt{id}}\ \underline{:=}\ E$' is a sentential form derived from S.
4. Repeat: pick a nonterminal A from the sentential form, replace with the RHS of a production $A \rightarrow \alpha$:
$S \Rightarrow \underline{\texttt{id}}\ \underline{:=}\ E \Rightarrow \underline{\texttt{id}}\ \underline{:=}\ E\underline{+}E \Rightarrow$
$\underline{\texttt{id}}\ \underline{:=}\ \underline{\texttt{id}} + E \Rightarrow \underline{\texttt{id}}\ \underline{:=}\ \underline{\texttt{id+num}}$. $S \stackrel{*}{\Rightarrow} \underline{\texttt{id}}\ \underline{:=}\ \underline{\texttt{id+num}}$.

$$\begin{aligned} S &\rightarrow \underline{\texttt{id}}\ \underline{:=}\ E \mid \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \\ E &\rightarrow E\underline{+}E \mid \underline{\texttt{id}} \mid \underline{\texttt{num}} \end{aligned}$$

Terminology

- A grammar is a 4-tuple

  (non-terminals, terminals, productions, start-symbol)

  or

  $$(N, \Sigma, P, S)$$

- A production is of the form $\alpha \to \beta$ where α, β are taken from $N \bigcup \Sigma$.
- Read $\alpha \to \beta$ as "rewrite α with β".
- Read $\Rightarrow$ as "directly derives".
- Read $\overset{r}{\Rightarrow}$ as "directly derives using rule r".
- Read $\overset{*}{\Rightarrow}$ as "derives in zero or more steps".

Derivations...

- $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if
  - $\boxed{A \to \gamma}$ is a production, and
  - α and β are strings of grammar symbols.

$\Rightarrow$: Derives in one step.

$\overset{*}{\Rightarrow}$: Derives in 0 or more steps.

$\overset{+}{\Rightarrow}$: Derives in 1 or more steps.

$\underset{lm}{\Rightarrow}$: Leftmost derivation.

$\underset{rm}{\Rightarrow}$: Rightmost derivation.

$L(G)$: The language generated by grammar G. This is the set of strings w, such that there is a derivation $S \overset{+}{\Rightarrow} w$, where S is G's start-symbol.

Derivations...

The string of terminal symbols $\boxed{\texttt{id:=id+num}}$ is generated by a leftmost derivation:

$$\begin{aligned} S &\underset{lm}{\Rightarrow} \texttt{id} := E \underset{lm}{\Rightarrow} \texttt{id} := E + E \\ &\underset{lm}{\Rightarrow} \texttt{id} := \texttt{id} + E \underset{lm}{\Rightarrow} \texttt{id} := \texttt{id} + \texttt{num} \\ S &\overset{+}{\Rightarrow} \texttt{id:=id+num} \end{aligned}$$

Example Grammar:

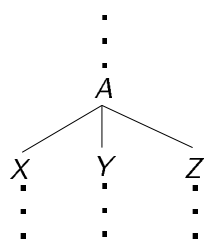
$$\begin{aligned} S &\to \texttt{id} := E \mid \texttt{if}\ E\ \texttt{then}\ S \\ E &\to E + E \mid \texttt{id} \mid \texttt{num} \end{aligned}$$

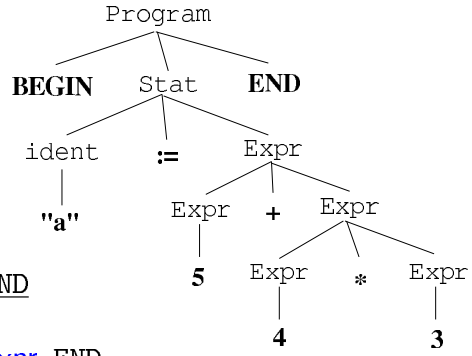
Parse Trees...

- If one step of our derivation is

  $$\cdots A \cdots \Rightarrow \cdots X\ Y\ Z \cdots$$

  (i.e, we used the rule $A \to XYZ$) then we'll get a parse (sub-)tree





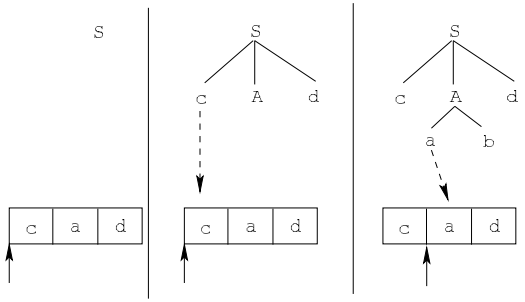
Program ⇒ BEGIN Stat END
⇒ BEGIN ident := Expr END
⇒ BEGIN "a" := Expr END
⇒ BEGIN "a" := Expr + Expr END
⇒ BEGIN "a" := 5 + Expr END
⇒ BEGIN "a" := 5 + Expr * Expr END
⇒ BEGIN "a" := 5 + 4 * Expr END
⇒ BEGIN "a" := 5 + 4 * 3 END

Top-Down Parsing

Top-Down Backtracking Parser

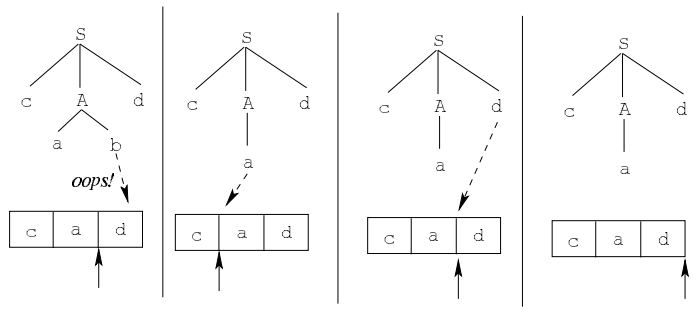
- Top-down parsing involves building a parse tree for the input string by starting at the root and adding nodes in preorder.

$S \rightarrow cAd$ $A \rightarrow ab|a$



Top-Down Backtracking Parser...

- If a backtracking top-down parser chooses the wrong production rule to expand a node it backs up over the input, and undoes some of the parse tree construction:



Grammar Rewriting

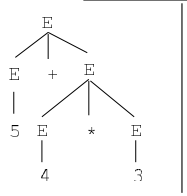
Ambiguous Grammars

- A grammar is ambiguous if some string of tokens can produce two (or more) different parse trees.

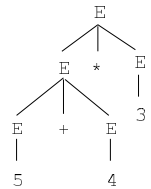
$$\mathtt{E} ::= \mathtt{E} \underline{+} \mathtt{E} \mid \mathtt{E} \underline{*} \mathtt{E} \mid \underline{\mathtt{number}}$$

$5 + 4 * 3$

$$\begin{aligned}
\mathtt{E} &\Rightarrow \mathtt{E} \underline{+} \mathtt{E} \\
&\Rightarrow 5 \underline{+} \mathtt{E} \\
&\Rightarrow 5 \underline{+} \mathtt{E} \underline{*} \mathtt{E} \\
&\Rightarrow 5 \underline{+} 4 \underline{*} \mathtt{E} \\
&\Rightarrow 5 \underline{+} 4 \underline{*} 3
\end{aligned}$$



$$\begin{aligned}
\mathtt{E} &\Rightarrow \mathtt{E} \underline{*} \mathtt{E} \\
&\Rightarrow \mathtt{E} \underline{*} 3 \\
&\Rightarrow \mathtt{E} \underline{+} \mathtt{E} \underline{*} 3 \\
&\Rightarrow \mathtt{E} \underline{+} 4 \underline{*} 3 \\
&\Rightarrow 5 \underline{+} 4 \underline{*} 3
\end{aligned}$$



Operator Precedence

- The precedence of an operator is a measure of its binding power, i.e. how strongly it attracts its operands.
- Usually $*$ has higher precedence than $+$:

$$4 + 5 * 3$$

means

$$4 + (5 * 3),$$

not

$$(4 + 5) * 3.$$

- We say that $\underline{*}$ binds harder than $\underline{+}$.

Operator Associativity

- The associativity of an operator describes how operators of equal precedence are grouped.
- $+$ and $-$ are usually left associative:

$$4 - 2 + 3$$

means

$$(4 - 2) + 3 = 5,$$

not

$$4 - (2 + 3) = -1.$$

We say that $+$ associates to the left.

- ^ associates to the right:

$$2\text{^}3\text{^}4 = 2\text{^}(3\text{^}4).$$

Expression Grammars

- We must write unambiguous expression grammars that reflect the associativity and precedence of all operators.
- The next slide gives the algorithm for writing such grammars.

Resulting Expression Grammar:

expr ::= expr + term | term

term ::= term * factor | factor

factor ::= (expr) | number

1. Create one non-terminal for each precedence level, for example $p_1, p_2, \cdots, p_n$, where p_n has the highest precedence level.
2. For operator *op* at precedence level i construct the following production if the operator is
   - left associative:

   $p_i ::= p_i \ op \ p_{i+1} \mid p_{i+1}$
   - right associative:

   $p_i ::= p_{i+1} \ op \ p_i \mid p_{i+1}$
3. Construct a production for nonterminal p_{n+1} which represents primary expressions such as identifiers, numbers, parenthesized expressions, etc:

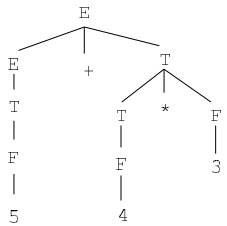
   $p_{n+1} ::= (p_1) \mid num \mid id$

5 + 4 * 3

E ::= E + T | T

T ::= T * F | F

F ::= number



E ⇒ E + T
⇒ T + T
⇒ F + T
⇒ 5 + T
⇒ 5 + T * F
⇒ 5 + F * F
⇒ 5 + 4 * F
⇒ 5 + 4 * 3

E ⇒ E + T
⇒ E + T * F
⇒ E + T * 3
⇒ E + F * 3
⇒ E + 4 * 3
⇒ T + 4 * 3
⇒ F + 4 * 3
⇒ 5 + 4 * 3

Top-Down Parsing

Recursive Descent Parsing

```
PROCEDURE S ();
   IF curr_tok = if THEN
      match(if); E();
      match(then); S();
   ELSIF curr_tok = id THEN
      match(id); match(:=); E();
   ELSE syntax error ENDIF;
PROCEDURE E ();
   IF curr_tok = id THEN match(id);
   ELSE IF curr_tok = num THEN match(num);
   ELSE E(); match(+); E();
   ENDIF;
```

$$
\begin{array}{rcl}
S & \rightarrow & \underline{\texttt{id}} := E \\
 & | & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \\
E & \rightarrow & E + E \\
 & | & \underline{\texttt{id}} \mid \underline{\texttt{num}}
\end{array}
$$

Recursive Descent—*Small Problem 1*

We may loop forever:

```
PROCEDURE E ();
   IF ···
   ELSE E(); match(+); E();
   ···
```

Recursive Descent—*Small Problem 2*

What about productions that start out similarly:

$$
\begin{array}{rcl}
S & \rightarrow & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \mid \\
 & & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ \underline{\texttt{else}}\ S
\end{array}
$$

```
PROCEDURE S ();
  IF curr_tok = if THEN
    match(if); E(); match(then); S();
  ELSIF curr_tok = if THEN
    match(if); E(); match(then);
    S(); match(else); S();
  ELSIF ··· ENDIF
```

Recursive Descent—*Small Problem 3*

What if there are several possible "next" tokens:

$$
\begin{array}{rcl}
\text{prog} & \rightarrow & \text{decl} \mid \text{stat} \\
\text{stat} & \rightarrow & \underline{\texttt{if}} \cdots \mid \underline{\texttt{id()}} \mid \underline{\texttt{while}} \cdots \\
\text{decl} & \rightarrow & \underline{\texttt{int}}\ \underline{\texttt{id}} \mid \underline{\texttt{real}}\ \underline{\texttt{id}}
\end{array}
$$

```
PROCEDURE prog ();
   IF curr_tok ∈ {if, id, while} THEN stat();
   ELSIF curr_tok ∈ {int, real} THEN decl()
   ELSE syntax error ENDIF;
END;
PROCEDURE stat (); ··· END;
PROCEDURE decl (); ··· END;
```

Left Recursion Removal

Left recursion must be removed from the grammar, by turning it into right recursion:

$$A \rightarrow A\alpha \mid \beta \quad \Rightarrow \quad \begin{array}{l} A \rightarrow \beta R \\ R \rightarrow \alpha R \mid \epsilon \end{array}$$

Example:

$$\begin{array}{rcl} \text{expr} & \rightarrow & \text{expr} \pm \text{term} \mid \text{term} \\ & \Downarrow & \\ \text{expr} & \rightarrow & \text{term } R \\ R & \rightarrow & \pm \text{ term } R \mid \epsilon \end{array}$$

Left Recursion Removal...

- After left recursion removal, our expression grammar

$$\begin{array}{rcl} E & \rightarrow & E \pm T \mid T \\ T & \rightarrow & T * F \mid F \\ F & \rightarrow & (E) \mid \texttt{id} \end{array}$$

turns into

$$\begin{array}{rcl} E & \rightarrow & T\ E' \\ E' & \rightarrow & \pm\ T\ E' \mid \epsilon \\ T & \rightarrow & F\ T' \\ T' & \rightarrow & *\ F\ T' \mid \epsilon \\ F & \rightarrow & (\ E\ ) \mid \texttt{id} \end{array}$$

Left Factoring

A top-down parser that reads input from left-to-right, can't choose between productions $E \rightarrow abF$ and $E \rightarrow abcF$. These must be left factored.

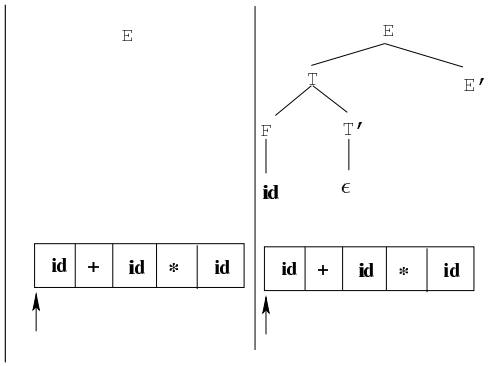
$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \quad \Rightarrow \quad \begin{array}{l} A \rightarrow \alpha A' \\ A' \rightarrow \beta_1 \mid \beta_2 \end{array}$$

Example:

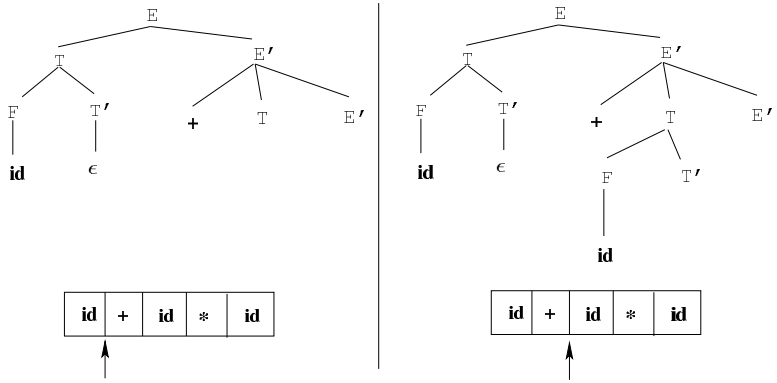
$$\begin{array}{rcl} S & \rightarrow & \texttt{if}\ E\ \texttt{then}\ S\ \texttt{else}\ S \mid \\ & & \texttt{if}\ E\ \texttt{then}\ S \end{array} \quad \Rightarrow \quad \begin{array}{rcl} S & \rightarrow & \texttt{if}\ E\ \texttt{then}\ S\ S' \\ S' & \rightarrow & \texttt{else}\ S \mid \epsilon \end{array}$$

Top-Down Expression Parser

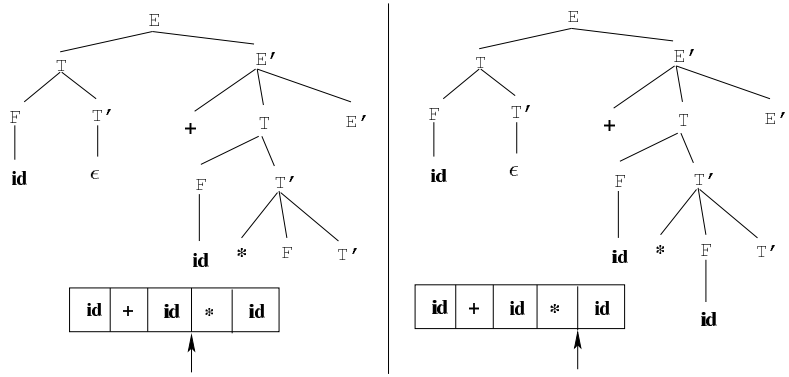
$$\begin{array}{rcl} E & \rightarrow & T\ E' \\ E' & \rightarrow & \pm\ T\ E' \mid \epsilon \\ T & \rightarrow & F\ T' \\ T' & \rightarrow & *\ F\ T' \mid \epsilon \\ F & \rightarrow & (\ E\ ) \mid \texttt{id} \end{array}$$



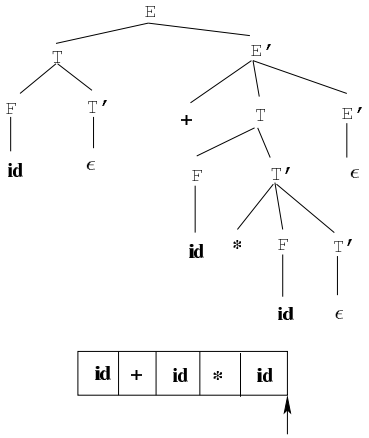
Top-Down Expression Parser...



Top-Down Expression Parser...



Top-Down Expression Parser...



Readings and References

- Read Louden, pp. 143–196.
- Or, the Dragon Book:
  Top-Down Parsing 181–190
  Error Recovery 192–195
  Recursive Descent Parsing 40–55, 75–76