Introduction

Compiler Phases

Semantic Analysis

- The parser returns an **abstract syntax tree (AST)**, a structured representation of the input program. All information present in the input program (except maybe for comments) is also present in the AST.
- Literals (integer/real/... constants) and identifiers are available as AST **input attributes**.
- During semantic analysis we add new attributes to the AST, and traverse the tree to evaluate these attributes and emit error messages.
- At compiler construction time we have to decide which attributes are needed, how they should be evaluated, and the **order** in which they should be evaluated.
Why Semantic Analysis?

1. Is the program statically correct? If not, report errors to user:
   - "undeclared variable"
   - "illegal procedure parameter"
   - "type incompatibility"

2. Make preparations for later compiler phases (code generation and optimization):
   - Compute types of variables.
   - Compute addresses of variables.
   - Store transfer modes of procedure parameters.
   - Compute labels for control structures (maybe).

Typical Semantic Errors

```
begin
end;
y := "x"
x, y : integer);

var k : P;
var z : R;
type R = array [9..7] of char;

end Y.
```

Static Semantic Rules

Static Semantics: ≈ type checking rules. The rules that are checked by the compiler before execution.

Dynamic Semantics: Rules that can only be checked when the program is run. Example: "pointer reference to NIL".

Context Conditions: Static semantic rules.

- Obviously, different languages have different static semantic rules. Ada, for example, allows null ranges (e.g. `array [9..7] of char`), while Modula-2 doesn’t.
- It’s our job as compiler writers to read the language definition and encode the rules in our semantic analyzer.
Kinds of Context Conditions

**Type Checks** We must check that every operator used in the program takes arguments of the correct type.

**Kind Checks** We must check that the right kind of identifier (procedure, variable, type, label, constant, exception) is used in the right place.

**Flow-of-control Checks** In Modula-2 an **EXIT** statement must only occur within a **LOOP**-statement:

```plaintext
LOOP IF · · · THEN EXIT ENDIF; END
```

**Uniqueness Checks** Sometimes a name must be defined exactly once. Example: variable declarations, case labels.

**Name Checks** Sometimes a name must occur more than once, e.g. at the beginning and end of a procedure.

Tree-Walk Evaluators

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**Attributes**

- The syntax analyzer produces an **Abstract Syntax Tree** (AST), a structured representation of the input program.
- Each node in the tree has a number of variables called **attributes**.
- We write a program that traverses the tree (one or more times) and assigns values to the attributes.

---

```
if node.type ≠ "integer" then
    print "Integer expected at " node.pos
```

---

**Context Conditions**

- The context conditions are encoded as tests on the values of attributes (node.type is the type attribute of node, node.pos the line number in the source code):

---

```
if node.type ≠ "integer" then
    print "Integer expected at " node.pos
```
**Tree-Walk Evaluation**

A tree-walker is a number of procedures that take a node as argument. They start by processing the root of the tree and then work their way down, **recursively**.

Often we will have one procedure for each major node-kind, i.e. one for declarations, one for statements, one for expressions. Notation:

- $n.\text{Kind}$ is $n$'s node type, for example IfStat, Assignment, etc.;
- $n.C$ is $n$'s child $C$, for example $n.\text{expr}$, $n.\text{left}$, etc.;
- $n.A$ is $n$'s attribute $A$, for example $n.\text{type}$, $n.\text{value}$, etc.

**Tree Traversal**
Each time we visit a node \( n \) we can
1. Evaluate some of \( n \)'s attributes.
2. Print a semantic error message.
3. Visit some of \( n \)'s children.

PROCEDURE Stat(n : Node);
    IF n.Kind = Assign THEN
        Expr(n.Des); Expr(n.Expr);
    ELSIF n.Kind = IfElse THEN
        Expr(n.Expr); Stat(n.Stat1); Stat(n.Stat2);
    ENDIF
END Stat;

PROCEDURE Expr(n : Node);
    IF n.Kind = BinOp THEN
        Expr(n.LOP);
        Expr(n.ROP);
    ELSIF n.Kind = Name THEN
        (* Process n.Name *)
    ELSIF n.Kind = IntCont THEN
        (* Process n.Value *)
    ENDIF
END Expr;

Constant Expressions

In many languages there are special constructs where only constant expressions may occur.

For example, in Modula-2 you can write

```plaintext
CONST C = 15;
TYPE A = ARRAY [5..C*6] OF CHAR;
```

but not

```plaintext
VAR C : INTEGER;
TYPE A = ARRAY [5..C] OF CHAR;
```

i.e. the upper bound of an array index must be constant (value known at compile time).
Constant Expressions...

- Constant declarations can depend on other constant declarations:
  
  ```
  CONST C1 = 15;
  CONST C2 = C1 * 6;
  TYPE A = ARRAY [5..C2] OF CHAR;
  ```

- Write a tree-walk evaluator that evaluates constant integer expressions.

  `IntConst` has an input attribute `Value`. We mark input attributes with a `⇐` in the abstract syntax.

  Each node is given an attribute `Val`. `Val` moves up the tree, so we mark it with a `⇑` in the abstract syntax.

```plaintext
PROCEDURE Expr (n: Node);
  IF n.Kind = Add THEN
    Expr(n.LOP); Expr(n.ROP);
    n.Val := n.LOP.Val + n.ROP.Val;
  ELSIF n.Kind = Mul THEN
    Expr(n.LOP); Expr(n.ROP);
    n.Val := n.LOP.Val * n.ROP.Val;
  ELSIF n.Kind = IntConst THEN
    n.Val := n.Value;
  ENDIF
END;
```

- `n.LOP.Val` has been evaluated after `Expr(n.LOP)` has returned.
- `n.LOP.Val` is the value of `n`'s left child's `Val` attribute.

---

**Concrete Syntax:**

```plaintext
Expr ::= Add | Mul | IntConst
Add ::= Expr + Expr
Mul ::= Expr * Expr
IntConst ::= number
```

**Abstract Syntax:**

```plaintext
Expr ::= Add | Mul | IntConst
Add ::= LOP:Expr ROP:Expr⇑Val:INTEGER
Mul ::= LOP:Expr ROP:Expr⇑Val:INTEGER
IntConst ::= ⇐Value:INTEGER⇑Val:INTEGER
```

---

The AST after parsing:

- `IntConst` Value=5
- `IntConst` Value=7
- `IntConst` Value=9

The AST after the tree-walk:

- `IntConst` Value=5
- `IntConst` Value=7
- `IntConst` Value=9
- `Add` Value=12
- `Add` Value=36

---

Constant Expressions...
Constant Declarations

Let’s extend this exercise to handle Modula-2 style constant declarations:

```
CONST C1 = 15;
CONST C2 = C1 * 6;
TYPE A = ARRAY [5..C2] OF CHAR;
```

We assume there is a magic function `Lookup(ID)` that returns `TRUE` if `ID` is a constant identifier, and a function `GetValue(ID)` which returns the value of this constant.

Concrete Syntax:

```
ConstDecl ::= CONST Ident = Expr
Expr ::= Expr + Expr | Ident | IntConst
IntConst ::= number
Ident ::= name
```

Abstract Syntax:

```
ConstDecl ::= ID:Ident EXPR:Expr
Expr ::= Add | IntConst | Ident
Add ::= LOP:Expr ROP:Expr \uparrow \text{Val:INTEGER} \uparrow \text{IsConst:BOOLEAN}
IntConst ::= \leftarrow \text{Value:INTEGER} \uparrow \text{Val:INTEGER} \uparrow \text{IsConst:BOOLEAN}
Ident ::= \leftarrow \text{ID:String} \uparrow \text{IsConst:BOOLEAN}
```

PROCEDURE ConstDecl (n: Node);
Expr(n.EXPR);
IF NOT n.EXPR.IsConst THEN
    PRINT "Constant expression expected."
ENDIF

PROCEDURE Expr (n: Node);
IF n.Kind = Add THEN
    Expr(n.LOP); Expr(n.ROP);
n.Val := n.LOP.Val + n.ROP.Val;
n.IsConst := n.LOP.IsConst AND n.ROP.IsConst;
ELSIF n.Kind = IntConst THEN
ELSIF n.Kind = Ident THEN
    n.IsConst := Lookup(n.ID); n.Val := GetValue(n.ID);
ENDIF

Source

```
VAR x:INTEGER;
ID="C"
CONST C = 45+X;
Value=45
```

```
``
Typechecking

Write a tree-walker that type checks assignments in Pascal:

```pascal
var i : integer; var r : real; var c : char;
begin
  i := 34;
  i := i + 2;
  r := 3.4;
  r := 3.4 + i; (* OK, automatic conversion. *)
  i := r; (* Illegal. *)
  i := c; (* Illegal. *)
end.
```

Assume a function `lookup` that returns the type of an identifier.

Concrete Syntax:

- `Assign ::= Expr := Expr`
- `Expr ::= Expr + Expr | name | integer | real | char`

Abstract Syntax:

- `Assign ::= Left:Expr Right:Expr`
- `Expr ::= Add | Name | IntConst | RealConst | CharConst`
- `Add ::= LOP:Expr ROP:Expr ↑Type:STRING`
- `Name ::= ⇐Name:STRING ↑Type:STRING`
- `IntConst ::= ⇐Value:INTEGER ↑Type:STRING`
- `RealConst ::= ⇐Value:REAL ↑Type:STRING`
- `CharConst ::= ⇐Value:CHAR ↑Type:STRING`

```pascal
PROCEDURE Assign (n: Node);
  Expr(n.Left); Expr(n.Right);
  IF NOT(n.Left.Type = n.Right.Type OR
    (n.Left.Type="REAL" AND n.Right.Type="INT"))
    THEN PRINT n.Left.Pos ":Type mismatch" ENDIF

PROCEDURE Expr (n: Node);
  IF n.Kind = Add THEN BinArith(n);
  ELSIF n.Kind = Name THEN n.Type := lookup(n.Name);
  ELSIF n.Kind = IntConst THEN n.Type := "INT";
  ELSIF n.Kind = CharConst THEN n.Type := "CHAR";
  ELSIF n.Kind = RealConst THEN n.Type := "REAL";
  ENDIF
```
PROCEDURE BinArith (n: Node);
Expr(n.LOP); Expr(n.ROP);
IF n.LOP.Type = "INT" AND n.ROP.Type = "INT" THEN
n.Type := "INT"
ELSIF (n.LOP.Type = "INT" OR n.LOP.Type = "REAL") AND
(n.ROP.Type = "INT" OR n.ROP.Type = "REAL") THEN
n.Type := "REAL"
ELSIF n.LOP.Type = "ERROR" OR n.ROP.Type = "ERROR" THEN
n.Type := "ERROR"
ELSE
PRINT n.Pos ":Illegal operation";
n.Type := "ERROR"
ENDIF

Type Checking Assignments...

- n.LOP.Type & n.ROP.Type are available once we've returned from Expr(n.LOP); Expr(n.ROP).
- We use the special type value "ERROR" to avoid printing an error message more than once for each expression.
- Note the difference between type equivalence and assignability:
  1. Type equivalence is used e.g. with binary operators such as + and <. In Pascal, integer & real are equivalent.
  2. In Pascal, an integer can be assigned to a real, but not vice versa.
- In Modula-2, integers and reals are neither type equivalent nor assignable.
Synthesized Attributes

Synthesized attributes move values *up* the tree (from the leaves towards the root). The value of a synthesized attribute $A$ at a node $n$ is determined from the values of $n$'s children:

$$n.A := f(n.Ch_1.A_1, n.Ch_2.A_2)$$

**PROCEDURE** ConstExpr (n:Node);
ConstExpr(n.LOP);
ConstExpr(n.ROP);
n.Val :=
n.LOP.Val +
n.ROP.Val;

**LOOP – EXIT**

In Modula-2, the `EXIT` statement can only occur within a `LOOP` statement:

```
BEGIN
  LOOP
    IF ... THEN
      WHILE ... DO
        EXIT;  \Leftarrow OK!
      END
    END
  END
END
EXIT \Leftarrow Illegal!
```

However, in Modula-2, you can terminate a `DO` loop and exit the enclosing `LOOP` statement with the `EXIT` statement:

```
BEGIN
  LOOP
    IF ... THEN
      WHILE ... DO
        EXIT;
      END
    END
  END
END
```

This is a legal construct in Modula-2.
In the previous type checking example we assumed there was a function lookup that would find the type of an identifier.

The problem is that there may be several uses of the same name in a program, and each may have a different type:

```plaintext
char x = 'c';
int x = 10;
float x = 10.0;
printf("%.1f", x); // Which x?
```

We'll be using environment attributes to disambiguate identifier references.

Write a tree-walk evaluator that type checks Pascal assignment statements.

Let declared variables be stored in an environment attribute, a set of tuples of type `EnvT=Name ↦→ Type`.

Let there be a function `lookup(E, V)` that returns the type of an variable `V` in an environment `E`.

<table>
<thead>
<tr>
<th>Assign</th>
<th>Des:Exp Expr:Expr (\downarrow)Env:EnvT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expr</td>
<td>Add</td>
</tr>
<tr>
<td>Add</td>
<td>LOP:ConstExpr ROP:ConstExpr (\uparrow)Type:String (\downarrow)Env:EnvT</td>
</tr>
<tr>
<td>Name</td>
<td>(\leftarrow)Id:String (\uparrow)Type:String (\downarrow)Env:EnvT</td>
</tr>
<tr>
<td>IntConst</td>
<td>(\leftarrow)Value:INTEGER (\uparrow)Type:String (\downarrow)Env:EnvT</td>
</tr>
<tr>
<td>RealConst</td>
<td>(\leftarrow)Value:REAL (\uparrow)Type:String (\downarrow)Env:EnvT</td>
</tr>
</tbody>
</table>
PROCEDURE Assign (n: Node);
    n.Des.Env := n.Env;
    n.Expr.Env := n.Env;
    Expr(n.Des); Expr(n.Expr);
    IF n.Des.Type \neq n.Expr.Type THEN
        PRINT n.Expr.Pos "::Type mismatch"
    ENDIF
END;

PROCEDURE Expr (n: Node);
    IF n.Kind = Add THEN BinArith(n);
    ELSIF n.Kind = Name THEN
        IF member(n.Env, n.Id) THEN
            n.Type := lookup(n.Env, n.Id);
        ELSE
            PRINT "Ident not declared"
            n.Type := "ERROR"
        ENDIF;
        ELSIF n.Kind = IntConst THEN
            n.Type := "INT";
        ELSIF n.Kind = RealConst THEN
            n.Type := "REAL";
    ENDIF;
END;

PROCEDURE BinArith (n: Node);
    n.LOP.Env := n.Env;
    Expr(n.LOP);
    n.ROP.Env := n.Env;
    Expr(n.ROP);
    IF n.LOP.Type = "INT" AND n.ROP.Type = "INT" THEN
    n.Type := "INT"
    ELSIF (n.LOP.Type = "INT" OR n.LOP.Type = "REAL") AND
    (n.ROP.Type = "INT" OR n.ROP.Type = "REAL") THEN
    n.Type := "REAL"
    ELSE
    PRINT n.Pos ":Illegal operation";
    n.Type := "ERROR"
    ENDIF
Inherited Attributes

- **Inherited attributes** move values *down* the tree (from the root towards the leaves). They inform the nodes of a subtree of the environment (context) in which they occur.

- The value of an inherited attribute $A$ at a node $n$ is determined from the attributes of $n$'s parent $p$: $n.A := f(p.A_1, p.A_2)$

```plaintext
PROCEDURE BinArith (n: Node);
n.LOP.Env := n.Env;
Expr(n.LOP);
n.ROP.Env := n.Env;
Expr(n.ROP);
```

History of Attribute Grammars

- What you have seen so far of attribute evaluation was known to the programming language community already in the early 1960’s. It was also clear at the time that synthesized attributes alone were not enough to specify the semantics of the languages that were of concern at the time (Algol 60).

- Something more powerful was needed, and it was not clear to anyone exactly what that was.

- The person who finally came up with the answer was Donald Knuth (of Stanford University), one of the best known researchers in computer science. The following excerpts are taken from a talk he gave to a conference on attribute grammars.
In the Beginning... I

“Much of my story takes place in 1967, by which time a great many computer programs had been written all over the world. [...] One of the puzzling questions under extensive investigation at the time was the problem of programming language semantics: How should we define the meaning of statements in algorithmic languages?

[...] I was ACM Lecturer that year [...]. My first stop was Cornell, where I spent the first weekend staying at Peter Wegner’s home in Ithaca, New York. I went with Peter to synagogue on Saturday, he went with me to a church on Sunday. We hiked outside the city in a beautiful river valley that contained many frozen water falls. But mostly we talked Computer Science.

In the Beginning... II

Peter asked me what I thought about formal semantics [...]. [...] my answer was that the best way I knew to define semantics was to use attributes whose values could be defined on a parse tree from bottom to top. [...] We also needed to include some complicated ad hoc methods, in order to get context-dependent information into the tree.

So Peter asked, “Why can’t attributes be defined from the top down as well as from the bottom up?”

A shocking idea! Of course I instinctively replied that it was impossible to go both bottom up and top-down. But after some discussion I realized that his suggestion wasn’t so preposterous after all, if circular definitions could somehow be avoided.

In the Beginning... III

Although attribute grammars remained at the back of my mind for several months, my next chance to think seriously about them didn’t come until I was away from home again — this time at a SIAM conference in Santa Barbara, California, at the end of November. Although the conference lists me as one of the participants, the truth is that I spent most of the whole time sitting on the beach outside the conference hotel writing a paper about “semantics of context free languages” (Mathematical Systems Theory, Vol 2 (1968), pp.127–145). [...] I spent the first day working on a test for circularity; after rejecting three obviously false starts, I thought I had found a correct algorithm, and didn’t try to too hard to find fault with it.

In the Beginning... IV

[1970] I spent three of four pleasant days sitting under an oak tree near Lake Langunita [Stanford], writing “Examples of formal semantics” (Lecture Notes in Mathematics 188, (1971), pp. 95-96). It is clear from reading [this paper] that I was still unaware of the serious error in the circularity test [...] I returned the galley proofs [...] to the printer on July 28; then on August 6, I received a letter from Stein Krogdahl in Norway, containing an elegantly presented counterexample to my circularity algorithm. (His letter had come by surface mail, taking six weeks to reach me, otherwise I could have alluded to the problem in [the paper].)

In 1977 I began to work on a language for computer typesetting called \TeX, and you might ask why I didn’t use an attribute grammar to define the semantics of \TeX. Good question.”

The Genesis of Attribute Grammars, Donald E. Knuth, Stanford University. LNCS 461, Attribute Grammars and their Applications.
Donald Knuth was born on January 10, 1938 in Milwaukee, Wisconsin. He studied mathematics as an undergraduate at Case Institute of Technology, where he also wrote software at the Computing Center. The Case faculty took the unprecedented step of awarding him a Master’s degree together with the B.S. he received in 1960. After graduate studies at California Institute of Technology, he received a Ph.D. in Mathematics in 1963 and then remained on the mathematics faculty. Throughout this period he continued to be involved with software development, serving as consultant to Burroughs Corporation from 1960–1968 and as editor of Programming Languages for ACM publications from 1964–1967.

He joined Stanford University as Professor of Computer Science in 1968, and was appointed to Stanford’s first endowed chair in computer science nine years later. As a university professor he introduced a variety of new courses into the curriculum, notably Data Structures and Concrete Mathematics. In 1993 he became Professor Emeritus of The Art of Computer Programming. He has supervised the dissertations of 28 students.

Knuth began in 1962 to prepare textbooks about programming techniques, and this work evolved into a projected seven-volume series entitled *The Art of Computer Programming*. Volumes 1–3 appeared in 1968, 1969, and 1973, and he is now working full time on the remaining volumes. Approximately one million copies have already been printed, including translations into six languages. He took ten years off from this project to work on digital typography, developing the TeX system for document preparation and the METAFONT system for alphabet design. Noteworthy byproducts of those activities were the WEB and CWEB languages for structured documentation, and the accompanying methodology of Literate Programming. TeX is now used to produce most of the world’s scientific literature in physics and mathematics.

His research papers have been instrumental in establishing several subareas of computer science and software engineering: LR(k) parsing; attribute grammars; the Knuth–Bendix algorithm for axiomatic reasoning; empirical studies of user programs and profiles; analysis of algorithms. In general, his works have been directed towards the search for a proper balance between theory and practice.
Professor Knuth received the ACM Turing Award in 1974 [...]
Professor Knuth lives on the Stanford campus with his wife, Jill. They have two children, John and Jennifer. Music is his main avocation.
Professor Knuth has an asteroid named after him:
http://neo.jpl.nasa.gov/cgi-bin/db?name=21656
Professor Knuth’s home page: http://www-cs-faculty.stanford.edu/~knuth

Readings and References
Read Louden:
Abstract Syntax: 109–114
Attribute grammars: 257–270
or read the Dragon book:
Abstract Syntax: 49
Type Checking: 343–345
AST Construction: 287–290
Syntax-Directed Definitions: 280–283
Recursive Evaluators: 316–319

Summary
We use the description of the abstract syntax as a description of the structure of abstract syntax trees.
In other words, we use context free grammars for parsing, and to describe the data structure (the AST) produced by the parser.
There exist tools that take an abstract grammar as input and produce a AST-manipulation module (with routines for construction, traversal, and input/output of trees) as output.
To perform semantic analysis we
1. Build an abstract syntax tree during parsing.
2. Decorate the AST with input attributes (literals and identifiers found in the source).
3. Add attributes needed during semantic analysis.
4. Traverse the tree (one or more times) to evaluate the attributes and emit error messages.

Designators are the kinds of expressions that denote writable locations (i.e. L-values). They are common on the left hand sides of assignment statements but also occur as actual reference parameters in procedure calls.

The Concrete Syntax describes the physical layout of the language, the Abstract Syntax describes the logical structure of the language.

A language’s Static Semantics gives the rules that a “correct” program has to obey. Static semantic rules are most often (but not always) enforced at compile-time. The Dynamic Semantics describes the “meaning” of a program, how it will behave at run-time.

Synthesized attributes get their values from their children only. They move up the tree. Inherited attributes get their values from their parent only. They move down the tree.

The rôle of the parser (in a multi-pass analysis compiler) is to construct an abstract syntax tree.

We can’t always determine a visit sequence (the order in which the AST nodes are visited) that will evaluate all attributes in one pass. Then several traversals will be necessary.

We always have to convince ourselves that we have devised a non-circular attribute evaluation scheme. We cannot have two attributes \(A_1\) & \(A_2\) such that \(A_1\) must be evaluated before \(A_2\) and vice versa.

Should we know how to convert from concrete to abstract syntax for the exam. If so, can you indicate where I might be able to find more information on how to do this.

Don’t really know what you’re asking. Converting from concrete to abstract syntax is what the parser does. As it is parsing the input it builds the abstract syntax tree; with a bottom-up parser this is almost trivial.

I have read some of the text book, but I didn’t find what I was looking for (I think I’m looking for some sort of algorithm, or set of rules that I can use to make the conversion, like for removing left recursion, and common left factors). There is no need to do anything like that to the abstract grammar since it is not used for parsing. The abstract grammar will often be ambiguous, left-recursive, etc, and that’s quite all right. The abstract grammar just describes the structure of the AST nodes, that’s all.
Don't let the word "abstract" in "Abstract Syntax Tree" confuse you. There isn't anything abstract about it at all; in fact, it is about as concrete as you can get. The idea is that performing semantic analysis on or generating code from an input program in source form (a text file) is much too hard. Therefore we build an internal representation (a data structure) of the input program during parsing, and then work on this structure. The structure happens to be a tree, because programs are naturally tree-shaped.

Homework

Homework I

Give an abstract syntax specification of Pascal and Modula-2 for-loops.

Pascal's concrete syntax:

\[
\text{ForStat ::= for \ ident := \ expr \ to \ expr \ do \ Stat} 
\]

Modula-2's concrete syntax:

\[
\text{ForStat ::= for \ ident := \ expr \ downto \ expr \ do \ Stat} 
\]

The optional BY-part is an integer constant expression which gives the amount to add to the iteration variable each time we go around the loop. If omitted, the increment defaults to 1.

Homework II

Give an abstract syntax for Modula-2's CASE-statement, and construct the AST for the example below.

\[
\text{CASE \ i \ OF} 
\begin{align*}
4 \ .. \ 7 & : \ j := 77; \\
2, 6 \ .. \ 12 & : \ j := 99; \\
\text{ELSE} & \quad \text{j := 0;}
\end{align*}
\text{END;}
\]

Concrete Syntax:

\[
\text{CaseStat ::= \ CASE \ Expr \ OF \ CaseList [ELSE \ StatSeq] \ END} 
\]

\[
\text{CaseList ::= CaseLabelList : StatSeq | CaseList | \epsilon} 
\]

\[
\text{CaseLabelList ::= CaseLabel \ | \ CaseLabelList | CaseLabel} 
\]

\[
\text{CaseLabel ::= ConstExpr \ [.. \ ConstExpr]} 
\]
Write a Modula-2 type checker. M2 has two mutually assignable but inequivalent integer types: INTEGER and CARDINAL (unsigned). Integer literals $\geq 0$ are either INTEGERS or CARDINALS. Integers and reals are neither assignable nor equivalent. TRUNC and FLOAT convert between the two.

Assign ::= Left:Expr Right:Expr
Expr ::= Add | Name | Trunc | Float | IntConst | RealConst
Add ::= LOP:Expr ROP:Expr
Trunc ::= LOP:Expr
Float ::= LOP:Expr
Name ::= ⇐Id:String
IntConst ::= ⇐Value:INTEGER
RealConst ::= ⇐Value:REAL

Write a concrete grammar that describes the syntax we have been using to describe our abstract grammars.

The concrete grammar should describe

Rules LHS ::= RHS
Choice LHS ::= CH1 | CH2 | · · ·
Children LHS ::= Name:Child
Input Attributes LHS ::= ⇐Attr:Type
Synthesized Attributes LHS ::= ↑Attr:Type
Inherited Attributes LHS ::= ↓Attr:Type