CSc 453 — Compilers and Systems Software

7 : Top-Down Parsing II

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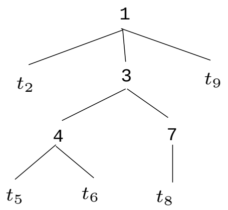
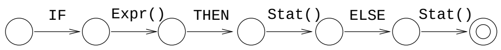
1 Top-Down Parsing

- The parse tree is constructed
  - From the top
  - From left to right.
- The terminals are seen in order of appearance in the token stream (t_2, t_5, t_6, t_8, t_9):



2 Building The LL(1) Parser

1. Remove left recursion.
2. Left factor the grammar.
3. Construct transition diagrams for each production:



   (a) Compute `FIRST(`A`)` for each grammar symbol A.
   (b) Compute `FOLLOW(`A`)` for each nonterminal A.

4. Simplify the transition diagrams.
5. Construct the recursive procedures.

3 FIRST Sets

- FIRST(α) is the set of terminals that begin strings derived from α.
- FIRST-sets help us solve problem 3.

$$
\begin{array}{rcl}
\text{prog} & \rightarrow & \text{decl} \mid \text{stat} \\
\text{stat} & \rightarrow & \underline{\texttt{if}} \cdots \mid \underline{\texttt{id()}} \mid \underline{\texttt{while}} \cdots \\
\text{decl} & \rightarrow & \underline{\texttt{int}}\ \underline{\texttt{id}} \mid \underline{\texttt{real}}\ \underline{\texttt{id}}
\end{array}
$$

$$
\begin{array}{rcl}
\texttt{FIRST}(\text{stat}) & = & \{\underline{\texttt{if}}, \underline{\texttt{id}}, \underline{\texttt{while}}\} \\
\texttt{FIRST}(\text{decl}) & = & \{\underline{\texttt{int}}, \underline{\texttt{real}}\}
\end{array}
$$

4 FIRST Sets...

```
PROCEDURE prog ();
  IF curr_tok ∈ {if, id, while} THEN stat();
  ELSIF curr_tok ∈ {int, real} THEN decl()
  ELSE syntax error ENDIF;
END;
PROCEDURE stat (); ··· END;
PROCEDURE decl (); ··· END;
```

$$
\begin{array}{rcl}
\texttt{FIRST}(\text{stat}) & = & \{\underline{\texttt{if}}, \underline{\texttt{id}}, \underline{\texttt{while}}\} \\
\texttt{FIRST}(\text{decl}) & = & \{\underline{\texttt{int}}, \underline{\texttt{real}}\}
\end{array}
$$

5 FIRST Sets...

- FIRST(α) is the set of terminals that begin strings derived from α, ie. FIRST(α) $= \{\underline{\text{a}} \mid \underline{\text{a}}$ a terminal, $\alpha \stackrel{*}{\Rightarrow} \underline{\text{a}}\beta\}$.

Example Grammar		
$E \rightarrow T\ E'$ $E' \rightarrow \underline{+}\ T\ E' \mid \epsilon$	$T \rightarrow F\ T'$ $T' \rightarrow \underline{*}\ F\ T' \mid \epsilon$	$F \rightarrow \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}}$

FIRST sets:

$$
\begin{array}{rcl}
\texttt{FIRST}(E) & = & \{\underline{(}, \underline{\texttt{id}}\} \\
\texttt{FIRST}(T) & = & \{\underline{(}, \underline{\texttt{id}}\} \\
\texttt{FIRST}(F) & = & \{\underline{(}, \underline{\texttt{id}}\}
\end{array}
\qquad
\begin{array}{rcl}
\texttt{FIRST}(E') & = & \{\underline{+}, \epsilon\} \\
\texttt{FIRST}(T') & = & \{\underline{*}, \epsilon\}
\end{array}
$$

6 Computing $\texttt{FIRST}(A)$

REPEAT until no more changes:
 1.IF A is a terminal THEN $\texttt{FIRST}(A) = \{A\}$.
 2.IF A is a nonterminal, and there
   is a production $A \rightarrow \epsilon$ THEN ϵ is in $\texttt{FIRST}(A)$.
 3.IF A is a nonterminal, and there
   is a production $A \rightarrow Y_1 \cdots Y_k$ THEN
   FOR i := 1 to $k-1$ DO
     IF $\epsilon \in \texttt{FIRST}(Y_1) \wedge \cdots \wedge \epsilon \in \texttt{FIRST}(Y_i)$
       and $\underline{\texttt{a}}\ (\neq \epsilon) \in \texttt{FIRST}(Y_{i+1})$ THEN $\underline{\texttt{a}}$ is in $\texttt{FIRST}(A)$;
   IF $\epsilon \in \texttt{FIRST}(Y_1) \wedge \cdots \wedge \epsilon \in \texttt{FIRST}(Y_k)$ THEN
     ϵ is in $\texttt{FIRST}(A)$;

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$E \rightarrow T\ E'$ $E' \rightarrow \underline{+}\ T\ E' \mid \epsilon$	$T \rightarrow F\ T'$ $T' \rightarrow \underline{*}\ F\ T' \mid \epsilon$	$F \rightarrow \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}}$

1. $\texttt{FIRST}(\underline{+}) = \{\underline{+}\}$, $\texttt{FIRST}(\underline{*}) = \{\underline{*}\}$, etc., because of point 1.
2. $\epsilon \in \texttt{FIRST}(E')$, $\epsilon \in \texttt{FIRST}(T')$, because of point 2.
3. $\underline{(}, \underline{\texttt{id}} \in \texttt{FIRST}(F)$, because of point 3.
4. $\underline{(}, \underline{\texttt{id}} \in \texttt{FIRST}(T)$, because $\underline{(}, \underline{\texttt{id}} \in \texttt{FIRST}(F)$, and $T \rightarrow F\ T'$.
5. $\underline{(}, \underline{\texttt{id}} \in \texttt{FIRST}(E)$, because $\underline{(}, \underline{\texttt{id}} \in \texttt{FIRST}(T)$, and $E \rightarrow T\ E'$.
6. $\underline{+} \in \texttt{FIRST}(E')$, $\underline{*} \in \texttt{FIRST}(T')$, because $E' \rightarrow \underline{+}\ T\ E'$ and $T' \rightarrow \underline{*}\ F\ T'$.

8 FOLLOW Sets

- We let \$ symbolize *end-of-input*.
- $\texttt{FOLLOW}(A)$ is the set of terminals that can follow right after the nonterminal A in some sentential form. $\texttt{FOLLOW}(A) = \{\underline{\texttt{a}} \mid \underline{\texttt{a}}$ a terminal, $S \stackrel{*}{\Rightarrow} \alpha A \underline{\texttt{a}} \beta\}$.
- $\$ \in \texttt{FOLLOW}(A)$ if A is the rightmost symbol in a sentential form, i.e. $S \stackrel{*}{\Rightarrow} \alpha A$.

9 FOLLOW Sets...

$E \rightarrow T\ E'$ $E' \rightarrow \underline{+}\ T\ E' \mid \epsilon$	$T \rightarrow F\ T'$ $T' \rightarrow \underline{*}\ F\ T' \mid \epsilon$	$F \rightarrow \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}}$

$$\begin{aligned} \texttt{FOLLOW}(E) &= \{\underline{)}, \underline{\$}\} \\ \texttt{FOLLOW}(T) &= \{\underline{+}, \underline{)}, \underline{\$}\} \\ \texttt{FOLLOW}(F) &= \{\underline{+}, \underline{*}, \underline{)}, \underline{\$}\} \end{aligned}$$

$$\begin{aligned} \texttt{FOLLOW}(F) &= \\ \texttt{FOLLOW}(E') &= \\ \texttt{FOLLOW}(T') &= \end{aligned}$$

10 FOLLOW Sets...

$E \rightarrow T\,E'$ $E' \rightarrow \underline{+}\,T\,E' \mid \epsilon$	$T \rightarrow F\,T'$ $T' \rightarrow \underline{*}\,F\,T' \mid \epsilon$	$F \rightarrow \underline{(}\,E\,\underline{)} \mid \underline{\texttt{id}}$

- $\underline{)}$ is in FOLLOW(E), because
$$E \Rightarrow TE' \Rightarrow FT'E' \Rightarrow \underline{(}E\boxed{\underline{)}}T'E'.$$

- $\underline{+}$ is in FOLLOW(T), because
$$E \Rightarrow TE' \Rightarrow T\boxed{\underline{+}}TE'.$$

11 FOLLOW Sets...

$E \rightarrow T\,E'$ $E' \rightarrow \underline{+}\,T\,E' \mid \epsilon$	$T \rightarrow F\,T'$ $T' \rightarrow \underline{*}\,F\,T' \mid \epsilon$	$F \rightarrow \underline{(}\,E\,\underline{)} \mid \underline{\texttt{id}}$

- $\underline{*}$ is in FOLLOW(F), because
$$E \Rightarrow TE' \Rightarrow FT'E' \Rightarrow F\boxed{\underline{*}}FT'E'.$$

- $\underline{)}$ is in FOLLOW(F), because
$$E \Rightarrow TE' \Rightarrow FT'E' \Rightarrow \underline{(}E\underline{)}T'E' \Rightarrow$$
$$\underline{(}TE'\underline{)}T'E' \Rightarrow \underline{(}FT'E'\underline{)}T'E' \Rightarrow$$
$$\underline{(}FE'\underline{)}T'E' \Rightarrow \underline{(}F\boxed{\underline{)}}T'E'.$$

12 Computing FOLLOW Sets

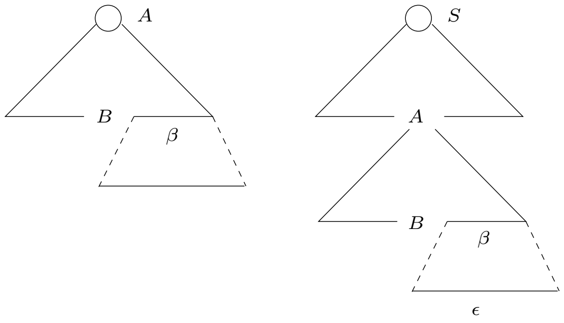
- Let S be the start symbol and $\underline{\$}$ the *end-of-file* marker.

```
REPEAT until no more changes:
 1. Add $ to FOLLOW(S).

 2. IF there is a production A → αBβ THEN
    Add everything in FIRST(β) (except ε) to FOLLOW(B).

 3. IF there is a production A → αB OR
      a production A → αBβ where ε ∈ FIRST(β) THEN
      Add everything in FOLLOW(A) to FOLLOW(B).
```

13 Computing `FOLLOW` Sets...



$\mathtt{FIRST}(\beta) \subseteq \mathtt{FOLLOW}(B)$ $\mathtt{FOLLOW}(A) \subseteq \mathtt{FOLLOW}(B)$

14 LL(1) Grammars

- A grammar is LL(1) if we can construct a recursive descent parser that handles it (without using backtracking).
- LL(1) stands for
  - The input is scanned from $\boxed{\text{L}}$eft-to-right.
  - The parse produces a $\boxed{\text{L}}$eftmost derivation.
  - We have $\boxed{1}$-token lookahead.

15 LL(1) Grammars...

- Formal Definition:

  A grammar is LL(1) iff for any two productions

  $$A \rightarrow \alpha \mid \beta$$

  the following conditions hold

  1. $\mathtt{FIRST}(\alpha) \cap \mathtt{FIRST}(\beta) = \emptyset$
  2. If $\beta \stackrel{*}{\Rightarrow} \epsilon$ then $\mathtt{FIRST}(\alpha) \cap \mathtt{FOLLOW}(A) = \emptyset$

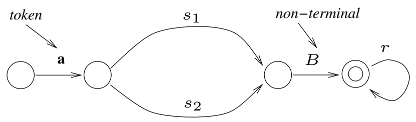
16 Recursive Descent Parsers

`FOR each non-terminal` A `DO`
   `create initial` $\bigcirc$ `and final` $\circledcirc$ `states;`

`FOR each production` $A \rightarrow X_1 \cdots X_n$ `DO`
   `create a path` A`'s initial to` A`'s final node`
   `with edges labeled` $X_1 \cdots X_n$`:`
$$\bigcirc \xrightarrow{X_1} \bigcirc \xrightarrow{X_2} \bigcirc \xrightarrow{\cdots} \bigcirc \xrightarrow{X_n} \circledcirc \;;$$
`Simplify the diagrams;`

`FOR each transition diagram` P `DO`
   `Create a procedure` P `that ''traverses'' the diagram`
   `guided by the input.`

17 Recursive Descent Parsers...




```
PROCEDURE P();
   IF curr_tok = a THEN curr_tok := next_token()
   ELSE syntax_error; ENDIF
   IF curr_tok ∈ FIRST(s1) THEN code for parsing s1
   ELSIF curr_tok ∈ FIRST(s2) THEN code for parsing s2
   ELSE syntax_error; ENDIF
   B();
   WHILE curr_tok ∈ FIRST(r) DO code for parsing r ENDDO
END P;
```

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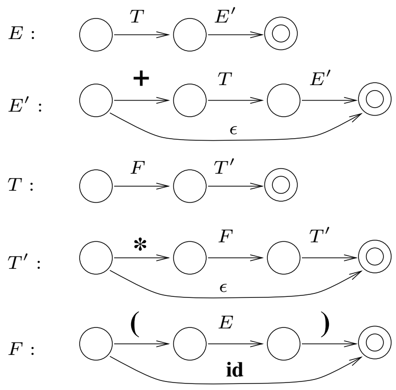
$E \rightarrow T\ E'$ $E' \rightarrow \underline{+}\ T\ E' \mid \epsilon$	$T \rightarrow F\ T'$ $T' \rightarrow \underline{*}\ F\ T' \mid \epsilon$	$F \rightarrow \underline{(}\ E\ \underline{)} \mid \underline{\mathtt{id}}$



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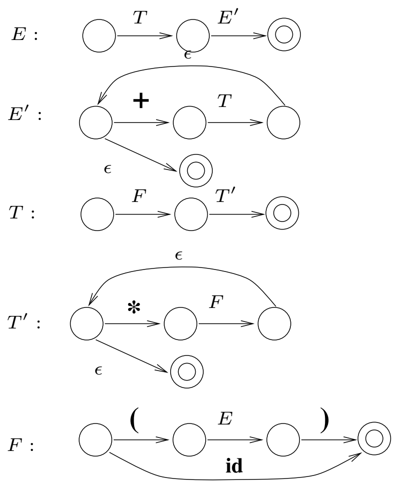

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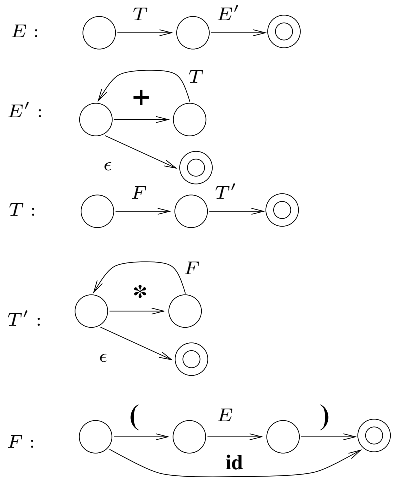

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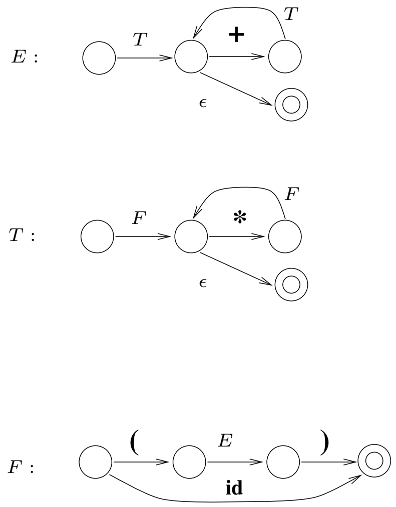

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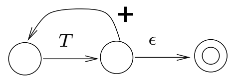
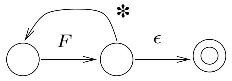
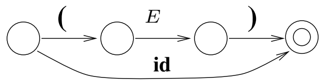
$E:$



$T:$



$F:$



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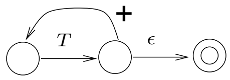
$E:$




```
PROCEDURE E();
   LOOP
      T();
      IF cur_tok = + THEN
         cur_tok := next_token()
      ELSE EXIT ENDIF
   ENDLOOP;
```

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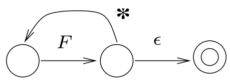
$T:$




```
PROCEDURE T();
   LOOP
      F();
      IF cur_tok = * THEN
         cur_tok := next_token()
      ELSE EXIT ENDIF
   ENDLOOP;
```

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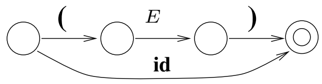
$F:$



```
PROCEDURE F();
   IF cur_tok = ( THEN
      cur_tok := next_token();
      E();
      IF cur_tok = ) THEN
         cur_tok := next_token()
      ELSE syntax error ENDIF
   ELSIF cur_tok = id THEN
      cur_tok := next_token();
   ELSE syntax error ENDIF
```

26 Parsing Expressions

- Here's the parser coded in Java.

```
class ExprH {
   static String[] input = {"i","+","i","*","i","$"};
   static int current = 0;

    static boolean lookahead(String S) {
       return input[current].equals(S);
    }

    static void match(String S) throws Exception {
        if   (input[current].equals(S)) current++;
        else throw new Exception();
    }
```

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```
    static void E() throws Exception {
        T(); while(lookahead("+")) {match("+"); T();}
    }

    static void T() throws Exception {
        F(); while(lookahead("*")) {match("*"); F();}
    }

    static void F() throws Exception {
        if (lookahead("(")) {match("("); E(); match(")");
        } else match("i");
    }

    public static void main(String[] args) {
       try {E(); System.out.println("true");
       } catch (Exception e)
           {System.out.println("false");}
    }
```

28 Readings and References

- Read Louden, pp. 143–196.
- Or, the Dragon Book:

  Top-Down Parsing 181–190

  Error Recovery 192–195

  Recursive Descent Parsing 40–55, 75–76