1 Comparison Functions

- Boolean functions (by convention) end with a ?.
- We can discriminate between different kinds of numbers:

```
> (complex? 3+4i)
#t
> (complex? 3)
#t
> (real? 3)
#t
> (real? -2.5+0.0i)
#t
> (rational? 6/10)
#t
```

2 Comparison Functions...

```
> (rational? 6/3)
#t
> (integer? 3+0i)
#t
> (integer? 3.0)
#t
> (integer? 8/4)
#t
```

3 Tests on Numbers

- Several of the comparison functions can take multiple arguments.
- `(<= 4 5 6 7 9 234)` returns true since the numbers are monotonically increasing.
> ( < 4 5)
true
> ( < 4 5 6 7 9 234)
true
> (> 5 2 1 3) 
false
> (= 1 1 1 1 1)
true
> ( <= 1 2 2 2 3)
true

4 Tests on Numbers...

> (>= 5 5)
true
> (zero? 5)
false
> (positive? 5)
true
> (negative? 5)
false
> (odd? 5)
true
> (even? 5)
false

5 Conditionals — If

- If the test-expression evaluates to #f (False) return the value of the then-expression, otherwise return the value of the else-expression:

  (if test-expression
   then-expression
   else-expression)

- Up to language level “Advanced Student” if-expressions must have two parts.

- Set the language level to Standard (R5RS) to get the standard Scheme behavior, where the else-expression is optional.
6  Conditionals — If...

> (define x 5)
> (if (= x 5) 2 4)
2
> (if (< x 3)
    (display "hello")
    (display "bye"))
bye
> (display
    (if (< x 3) "hello" "bye"))
bye

7  If it’s not False (#f), it’s True (#t)

• Any value that is not false, is interpreted as true.
• NOTE: In DrScheme this depends on which language level you set. Up to “Advanced Student”, the test-expression of an if must be either #t or #f.
• Set the language level to Standard (R5RS) to get the standard Scheme behavior:

> (if 5 "hello" "bye")
"hello"
> (if #f "hello" "bye")
"bye"
> (if #f "hello")
> (if #t "hello")
"hello"

8  Boolean Operators

• and and or can take multiple arguments.
• and returns true if none of its arguments evaluate to False.
• or returns true if any of its arguments evaluates to True.

> (and (< 3 5) (odd? 5) (inexact? (cos 32)))
#t
> (or (even? 5) (zero? (- 5 5)))
#t
> (not 5)
#f
> (not #t)
#f

9  Boolean Operators...

• In general, any value that is not #f is considered true.
• and and or evaluate their arguments from left to right, and stop as soon as they know the final result.
• The last value evaluated is the one returned.

```lisp
> (and "hello")
"hello"
> (and "hello" "world")
"world"
> (or "hello" "world")
"hello"
```

10 Defining Boolean Functions

• We can define our own boolean functions:

```lisp
(define (big-number? n)
  (> n 10000000))
```

```lisp
> (big-number? 5)
#f
> (big-number? 384783274832748327)
#t
```

11 Conditionals — cond

• cond is a generalization of if:

```lisp
(cond
  (cond-expression1 result-expression1)
  (cond-expression2 result-expression2)
  ...
  (else else-expression))
```

• Each cond-expressionᵢ is evaluated in turn, until one evaluates to not False.

```lisp
> (cond
    ((< 2 3) 4)
    ((= 2 3) 5)
    (else 6))
4
```

12 Conditionals — cond...

• To make this a bit more readable, we use square brackets around the cond-clauses:

```lisp
(cond
  [cond-expr₁ result-expr₁]
  [cond-expr₂ result-expr₂]
  ...
  [else else-expression])
```
13 Conditionals — case

- case is like Java/C’s switch statement:

\[
\text{(case key}
\begin{array}{l}
[(\text{expr}_1 \ \text{expr}_2 \ \ldots) \ \text{result-expr}_1] \\
[\text{else else-expr})]
\end{array}
\]

- The key is evaluated once, and compared against each cond-expr in turn, and the corresponding result-expr is returned.

\[
> \text{(case 5 [(2 3) "hello"] [(4 5) "bye"])}
\]

"bye"

14 Conditionals — case...

(define (classify n)
  (case n
    [(2 4 8 16 32) "small power of 2"]
    [(2 3 5 7 11) "small prime number"]
    [else "some other number"]
  )
)

> (classify 4)
"small power of 2"
> (classify 3)
"small prime number"
> (classify 2)
"small power of 2"
> (classify 32476)
"some other number"

15 Sequencing

- To do more than one thing in sequence, use begin:

\[
\begin{array}{l}
\text{(begin arg}_1 \ \text{arg}_2 \ \ldots)\\
\end{array}
\]
> (begin
   (display "the meaning of life=")
   (display (* 6 7))
   (newline)
) the meaning of life=42

16 Examples — !\(n\)

- Write the factorial function !\(n\):
  \[
  (define (! n)
    (cond
      [(zero? n) 1]
      [else (* n (! (- n 1)))])
  )
  \]

  > (! 5)
  120

17 Examples — \(\binom{n}{r}\)

- Write the \(\binom{n}{r}\) function in Scheme:
  \[
  \binom{n}{r} = \frac{n!}{r! * (n-r)!}
  \]

  - Use the factorial function from the last slide.
    \[
    (define (choose n r)
      (/ (! n) (* (! r) (! (- n r))))
    )
    \]

    > (choose 5 2)
    10

18 Examples — (sum m n)

- Write a function (sum m n) that returns the sum of the integers between m and n, inclusive.
  \[
  (define (sum m n)
    (cond
      [(= m n) m]
      [else (+ m (sum (+ 1 m) n))])
  )
  \]

  > (sum 1 2)
  3
  > (sum 1 4)
  10
19 Examples — Ackermann’s function

- Implement Ackermann’s function:

\[
A(i, j) = \begin{cases} 
2j & \text{for } j \geq 1 \\
A(i-1, j) & \text{for } i \geq 1, j \geq 2 \\
A(i-1, A(i, j-1)) & \text{for } i, j \geq 2
\end{cases}
\]

\[
\text{(define (A i j)}
\text{  (cond}
\text{    [(and (= i 1) (>= j 1)) (* 2 j)]}
\text{    [(and (>= i 2) (= j 1)) (A (- i 1) 2)]}
\text{    [(and (>= i 2) (>= j 2))}
\text{      (A (- i 1) (A i (- j 1)))]}
\text{    ]})
\text{)}
\]

20 Examples — Ackermann’s function...

- Ackermann’s function grows very quickly:

\[
\begin{align*}
> & \ (A \ 1 \ 1) \\
& \quad 2 \\
> & \ (A \ 3 \ 2) \\
& \quad 512 \\
> & \ (A \ 3 \ 3) \\
& \quad 1561585986519419914804999641169225 \\
& \quad 4958731641184786755447122887443528 \\
& \quad 606147093563037485963338068553800 \\
& \quad 6371637297210170750776562389313989 \\
& \quad 2867298012168192
\end{align*}
\]

21 Scheme so Far

- Unlike languages like Java and C which are statically typed (we describe in the program text what type each variable is) Scheme is dynamically typed. We can test at runtime what particular type of number an atom is:

- (complex? arg), (real? arg)
- (rational? arg), (integer? arg)

- Tests on numbers:

- (< arg1, arg2), (> arg1, arg2)
- (= arg1, arg2), (<= arg1, arg2)
- (>= arg1, arg2), (zero? arg)
- (positive? arg), (negative? arg)
- (odd? arg), (even? arg)
22 **Scheme so Far...**

- Unlike many other languages like Java which are *statement-oriented*, Scheme is *expression-oriented*. That is, every construct (even if, cond, etc) return a value. The if-expression returns the value of the then-expr or the else-expr:

\[
\text{(if test-expr then-expr else-expr)}
\]

depending on the value of the test-expr.

23 **Scheme so Far...**

- The cond-expression evaluates its guards until one evaluates to non-false. The corresponding value is returned:

\[
\text{(cond}
\text{ (guard}_1\text{ value}_1)\text{)}
\text{(guard}_2\text{ value}_2)\text{)
...}
\text{(else else-expr))}
\]

24 **Scheme so Far...**

- The case-expression evaluates key, finds the first matching expression, and returns the corresponding result:

\[
\text{(case key}
\text{ [(expr}_1\text{ expr}_2\text{ ...) result-expr}_1\text{]}
\text{ [(expr}_1\text{ expr}_1\text{ ...) result-expr}_2\text{]}
...\text{(else else-expr))}
\]

25 **Scheme so Far...**

- and and or take multiple arguments, evaluate their results left-to-right until the outcome can be determined (for or when the first non-false, for and when the first false is found), and returns the last value evaluated.