CSc 520 — Principles of Programming Languages

40 : Scheme — Metacircular Interpretation

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1 Introduction

- In this lecture I'm going to show how you can define Scheme by writing a *metacircular interpreter* for the language, i.e. an interpreter for Scheme written in Scheme.
- Before we can do that, we first need to learn a few more this about the language

2 Let Expressions

- A *let-expression* binds names to values:

```
(let ((name1 value1) (name2 value2) ...)
     expression)
```

- The first argument to `let` is a list of `(name value)` pairs. The second argument is the expression to evaluate.

```
¿ (let ((a 3) (b 4) (square (lambda (x)(* x x)))
          (plus +))
       (sqrt (plus (square a) (square b))))
5.0
```

3 Let Expressions...

- `Let`-expressions can be nested:

```
> (let ((x 5) (c 4))
    (let ((v (* 4 x))
          (t (* 2 c)))
       (+ v t)))
28
```

4 Imperative Features

- Scheme is an *impure* functional language.
- I.e., Scheme has *imperative* features.
- I.e., in Scheme it is possible to program with *side-effects*.

`(set! var value)` Change the value of `var` to `value`.

`(set-car! var value)` Change the `car`-field of the cons-cell `var` to `value`.

`(set-cdr! var value)` Change the `cdr`-field of the cons-cell `var` to `value`.

5 Imperative Features...

- Example:

```
> (let ((x 2) (l '(a b)))
    (set! x 3)
    (set-car! l '(c d))
    (set-cdr! l '(e))
    (display x) (newline)
    (display l) (newline))
3
((c d) e)
```

6 Dotted Pairs

- S-expressions are constructed using *dotted pairs*.
- It is implemented as a `struct` (called a *cons-cell*) consisting of two fields (the size of a machine word) called `car` and `cdr`.
- We can manipulate these fields directly:

```
> '(1 . 2)
(1 . 2)
> (cons "stacy's" "mom")
("stacy's" . "mom")
> '(1 . (2 . 3))
(1 2 . 3)
> (cons 1 2)
(1 . 2)
```

7 Dotted Pairs...

- When the second part of a dottend pair (the `cdr`-field) is a list, and the innermost `cdr`-field is the empty list, we get a "normal" Scheme list:

```
> '(1 . ())
(1)
> '(1 . (2 . ()))
(1 2)
> '(1 . (2 3))
(1 2 3)
```

8 Dotted Pairs...

- We can use `set-car!` and `set-cdr!` to manipulate the fields of a `cons`-cell directly:

```
> (define x '(1 . 2))
> (set-car! x 'a)
> x
(a . 2)
> (set-cdr! x '(2 3))
> x
(a 2 3)
```

9 Dotted Pairs...

- `(cons A B)` can be thought of as first creating a `cons`-cell on the heap (using `malloc`, for example), and then setting the `car` and `cdr` fields to `A` and `B`, respectively:

```
> (define x (cons 0 0))
> x
(0 . 0)
> (set-car! x '1)
> (set-cdr! x '())
> x
(1)
```

10 Loops

Scheme's "for-loop" `do` takes these arguments:

1. A list of triples (*var init update*) which declares a variable *var*, with an initial value *init*, and which gets updated using the expression *update*, on each iteration;
2. A pair (*termination_cond return_value*) which gives the termination condition and return value of the loop; and
3. a loop body:

```
(do ((var1 init1 update1)
     (var12 init2 update2)
          ...
     )
     (termination_cond return_value)
   loop_body
)
```

11 Loops...

- Sum the numbers 1 to 4, printing out intermediate results:

```
> (do ((i 1 (+ i 1))
    (sum 0 (+ sum i)))
    ((= i 5) sum)
    (display sum)
    (newline)
)
0
1
3
6

10
```

12 Association Lists

- *Association lists* are simply lists of *key-value* pairs that can be searched sequentially:

```
> (assoc 'bob '((bob 22) (joe 32) (bob 3)))
(bob 22)
```

- The list is searchedy the list from beginning to end, returning the first pair with a matching key:

  `(assoc` *`key alist`*`)` Search for *key*; compare using `equal?`.

  `(assq` *`key alist`*`)` Search for *key*; compare using `eq?`.

  `(assv` *`key alist`*`)` Search for *key*; compare using `eqv?`.

13 Association Lists...

```
> (define e '((a 1) (b 2) (c 3)))
> (assq 'a e)
(a 1)
> (assq 'b e)
(b 2)
> (assq 'd e)
#f
> (assq (list 'a) '(((a)) ((b)) ((c))))
#f
> (assoc (list 'a) '(((a)) ((b)) ((c))))
((a))
> (assv 5 '((2 3) (5 7) (11 13)))
(5 7)
```

14 Association Lists...

- We can actually have more than one value:

```
> (assoc 'bob '((bob 5 male)
                (jane 32 'female)))
(bob 5 male)
```

15 Apply

- *Apply* returns the result of applying its first argument to its second argument.

```
> (apply + '(6 7))
13
> (apply max '(2 5 1 7))
7
```

16 Eval

- *(eval* arg*)* evaluates its argument.

```
> (eval '(+ 4 5))
9
> (eval '(cons 'a '(b c))) (a b c)
```

17 Eval...

- `eval` and `quote` are each other's inverses:

```
> (eval ''(+ 4 5))
(+ 4 5)
> (eval (eval ''(+ 4 5)))
9
> (eval (eval (eval '''(+ 4 5))))
9
```

18 Programs as Data

- Scheme is *homoiconic*, self-representing, i.e. programs and data are both represented the same (as S-expressions).
- This allows us to write programs that generate programs - useful in AI, for example.

```
> (define x 'car)
> (define y ''(a b c))
> (define p (list x y))
> p
(car '(a b c))
> (eval p)
a
```

19 Evaluation Order

- So far, we have said that to evaluate an expression `(op arg1 arg2 arg3)` we first evaluate the arguments, then apply the operator `op` to the resulting values.
- This is known as *applicative-order* evaluation.
- Example:

```
(define (double x) (* x x))

> (double (* 3 4))
    ⇒ (double 12)
    ⇒ (+ 12 12)
    ⇒ 24
```

20 Evaluation Order...

- This is not the only possible order of evaluation
- In *normal-order* evaluation parameters to a function are always passed unevaluated.
- This sometimes leads to extra work:

```
(define (double x) (* x x))

> (double (* 3 4))
    ⇒ (+ (* 3 4) (* 3 4)))
    ⇒ (+ 12 (* 3 4))
    ⇒ (+ 12 12)
    ⇒ 24
```

21 Evaluation Order...

- Applicative-order can sometimes also lead to more work than normal-order:

```
(define (switch x a b c)
   (cond
      ((< x 0) a)
      ((= x 0) b)
      ((> x 0) c)))

> (switch -1 (+ 1 2) (+ 2 3) (+ 3 4))
```

- Here, applicative-order evaluates all the arguments, although only one value will ever be needed.

22 Evaluation Order...

- Ordinary Scheme functions (such as `+`, `car`, etc) use applicative-order evaluation.
- Some *special forms* (`cond`, `if`, etc) must use normal order since they need to consume their arguments unevaluated:

```
> (if #t (display 5) (display 6))
5
> (cond (#f (display 5))
         (#f (display 6))
         (#t (display 7)))
7
```

23 A Metacircular Interpreter

- One way to define the semantics of a language (the effects that programs written in the language will have), is to write a *metacircular interpreter*.
- I.e, we define the language by writing an interpreter for it, in the language itself.
- A metacircular interpreter for Scheme consists of two mutually recursive functions, `mEval` and `mApply`:

```
(define (mEval Expr)
   ...
)
(define (mApply Op Args)
   ...
)
```

24 A Metacircular Interpreter...

- We want to be able to call our interpreter like this:

```
> (mEval (+ 1 2))
3
> (mEval (+ 1 (* 3 4)))
13
> (mEval (quote (2 3)))
(2 3)
> (mEval (car (quote (1 2))))
1
```

25 A Metacircular Interpreter...

```
> (mEval (cdr (quote (1 2))))
(2)
> (mEval (cons (quote 5) (quote (1 2))))
(5 1 2)
> (mEval (null?  (quote (1 2))))
#f
> (mEval (null?  (quote ())))
#t
> (mEval (if (eq?  1 1) 5 6))
5
```

26 A Metacircular Interpreter...

- `mEval` handles *primitive special forms* (`lambda`, `if`, `const`, `define`, `quote`, etc), itself.
- Note that, for these forms, we must use normal-order evaluation.
- For other expressions, `mEval` evaluates all arguments and calls `mApply` to perform the required operation:

27 A Metacircular Interpreter...

```
(define (mEval Expr)
   (cond
      [(null?  Expr) '()]
      [(number?  Expr) Expr]
      [(eq?  (car Expr) 'if)
         (mEvalIf (cadr Expr)
                  (caddr Expr)
                  (cadddr Expr))]
      [(eq?  (car Expr) 'quote) (cadr Expr)]
      [else (mApply (car Expr)
                  (mEvalList (cdr Expr)))]
   )
)
```

28 A Metacircular Interpreter...

- `mApply` checks if the operation is one of the builtin primitive ones, and if so performs the required operation:

```
(define (mApply Op Args)
   (case Op
      [(car) (caar Args)]
      [(cdr) (cdar Args)]
      [(cons) (cons (car Args) (cadr Args))]
      [(eq?)  (eq?  (car Args) (cadr Args))]
      [(null?)  (null?  (car Args))]
      [(+) (+ (car Args) (cadr Args))]
      [(*) (* (car Args) (cadr Args))]
   )
)
```

29 A Metacircular Interpreter...

- Some auxiliary functions:

```
(define (mEvalIf b t e)
   (if (mEval b) (mEval t) (mEval e))
)

(define (mEvalList List)
   (cond
      [(null?  List) '()]
      [else (cons (mEval (car List))
            (mEvalList (cdr List)))]
   )
)
```

30 A Metacircular Interpreter...

- Note that this little interpreter lacks many of Scheme's functions.

- We don't have symbols, `lambda`, `define`.
- We can't define or invoke user-defined functions.
- There are no way to define or lookup variables, local or global. To do that, `mEval` and `mApply` pass around *environments* (*association lists*) of variable/value pairs.

31 Readings and References

- **Read Scott, pp. 592–606, 609-610**