CSc 520

Principles of Programming Languages

33 : Functional Programming

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Programming Paradigms

- During the next few weeks we are going to work with functional programming. Before I can explain to you what FP is, I thought I'd better put things into perspective by talking about other programming paradigms.
- Over the last 40 or so years, a number of programming paradigms (a programming paradigm is a way to think about programs and programming) have emerged.

Programming Paradigms. . .

A programming paradigm

- is a way to think about programs, programming, and problem solving,
- is supported by one or more programming languages.

Being familiar with several paradigms makes you a better programmer and problem solver. The most popular paradigms:

1. Imperative programming.
2. Functional programming.
3. Object-oriented programming.
4. Logic Programming.

When all you have is a hammer, everything looks like a nail.

Programming Paradigms...

Imperative Programming

- Programming with state.
- Also known as procedural programming. The first to emerge in the 1940s-50s. Still the way most people learn how to program.
- FORTRAN, Pascal, C, BASIC.

Functional Programming

- Programming with values.
- Arrived in the late 50s with the LISP language. LISP is still popular and widely used by AI people.
- LISP, Miranda, Haskell, Gofer.

Programming Paradigms. . .

Object-Oriented Programming

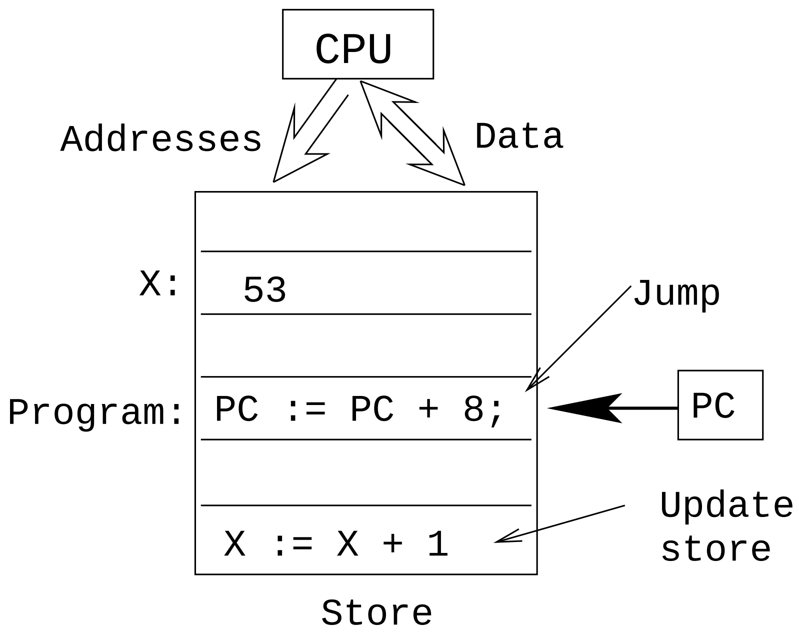
- Programming with objects that encapsulate data and operations.
- A variant of imperative programming first introduced with the Norwegian language Simula in the mid 60s.
- Simula, Eiffel, Modula-3, C++.

Logic Programming

- Programming with relations.
- Introduced in the early 70s. Based on predicate calculus. Prolog is popular with Computational Linguists.
- Prolog, Parlog.

Procedural Programming

We program an abstraction of the Von Neumann Machine, consisting of a store (memory), a program (kept in the store), A CPU and a program counter (PC):



Computing x:=x+1

1. Compute x's address, send it to the store, get x's value back.
2. Add 1 to x's value.
3. Send x's address and new value to the store for storage.
4. Increment PC.

Procedural Programming...

The programmer...

- uses control structures (IF, WHILE, ...) to alter the program counter (PC),
- uses assignment statements to alter the store.
- is in charge of memory management, i.e. declaring variables to hold values during the computation.

```
function fact (n:integer):integer;
var s,i : integer := 1;
begin
   while i<=n do s:=s*i; i:=i+1; end;
   return s;
end fact.
```

Procedural Programming. . .

Procedural programming is difficult because:

1. A procedural program can be in a large number of states. (Any combination of variable values and PC locations constitutes a possible state.) The programmer has to keep track of all of them.
2. Any global variable can be changed from any location in the program. (This is particularly true of languages like C & C++ [Why?]).
3. It is difficult to reason mathematically about a procedural program.

Functional Programming

In contrast to procedural languages, functional programs don't concern themselves with state and memory locations. Instead, they work exclusively with values, and expressions and functions which compute values.

- Functional programming is not tied to the von Neumann machine.
- It is not necessary to know anything about the underlying hardware when writing a functional program, the way you do when writing an imperative program.
- Functional programs are more declarative than procedural ones; i.e. they describe what is to be computed rather than how it should be computed.

Functional Languages

Common characteristics of functional programming languages:

1. Simple and concise syntax and semantics.
2. Repetition is expressed as recursion rather than iteration.
3. Functions are first class objects. I.e. functions can be manipulated just as easily as integers, floats, etc. in other languages.
4. Data as functions. I.e. we can build a function on the fly and then execute it. (Some languages).

Functional Languages...

5. Higher-order functions. I.e. functions can take functions as arguments and return functions as results.
6. Lazy evaluation. Expressions are evaluated only when needed. This allows us to build infinite data structures, where only the parts we need are actually constructed. (Some languages).
7. Garbage Collection. Dynamic memory that is no longer needed is automatically reclaimed by the system. GC is also available in some imperative languages (Modula-3, Eiffel) but not in others (C, C++, Pascal).

Functional Languages. . .

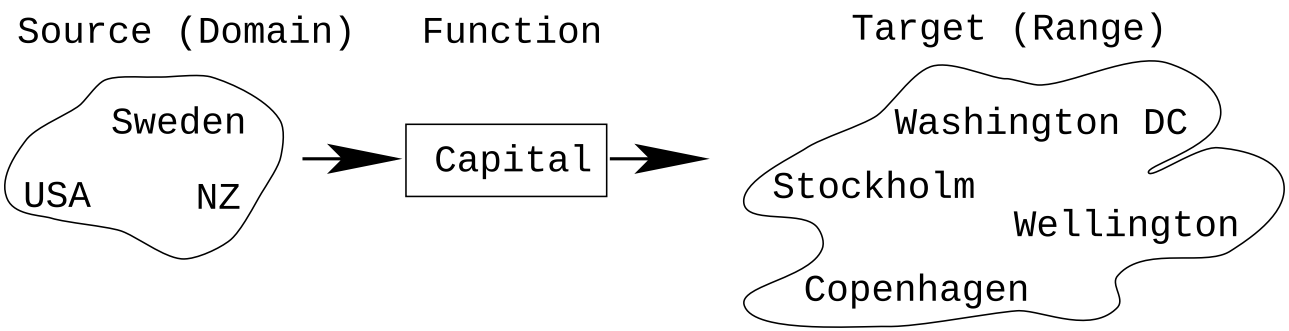
8. Polymorphic types. Functions can work on data of different types. (Some languages).
9. Functional programs can be more easily manipulated mathematically than procedural programs.

Pure vs. Impure FPL

- Some functional languages are pure, i.e. they contain no imperative features at all. Examples: Haskell, Miranda, Gofer.
- Impure languages may have assignment-statements, goto:s, while-loops, etc. Examples: LISP, ML, Scheme.

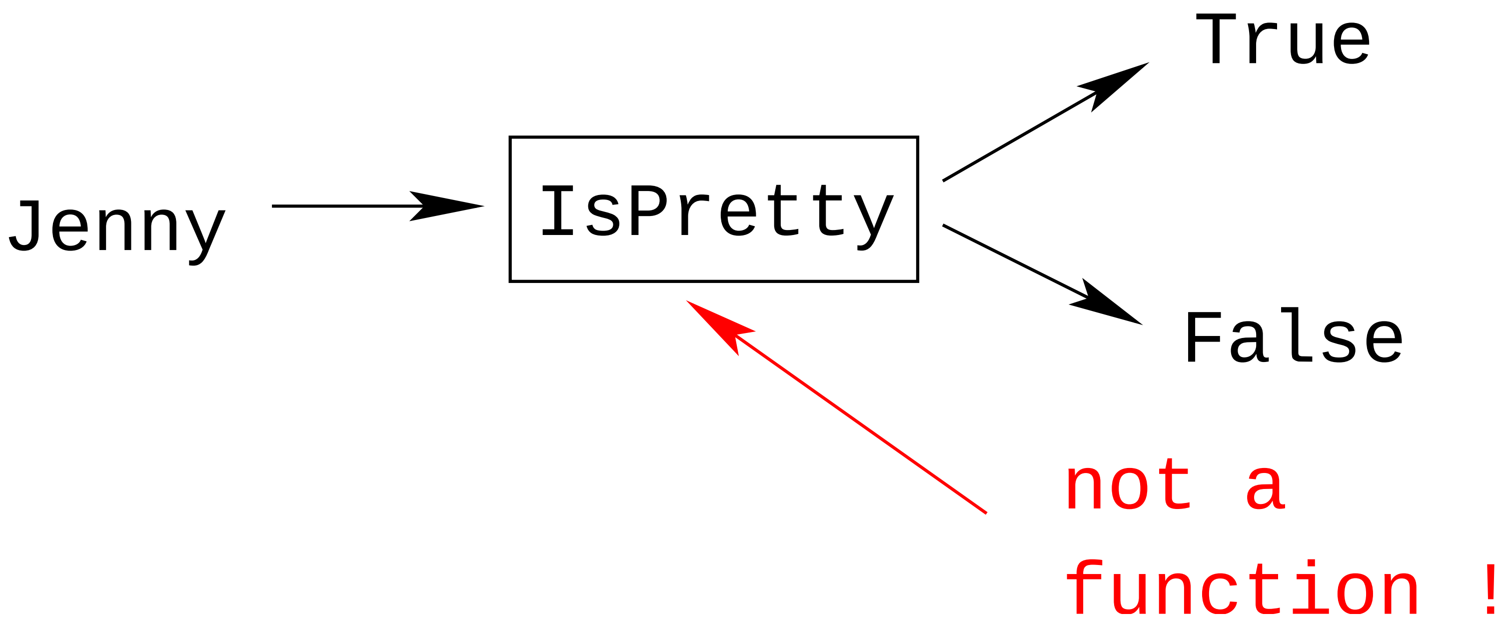
What is a function?

- A function maps argument values (inputs) to result values (outputs).
- A function takes argument values from a source set (or domain).
- A function produces result values that lie in a target set (or range).



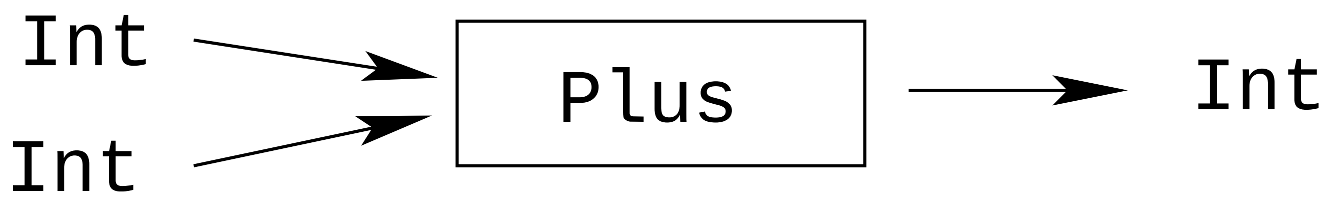
More on functions

- A function must not map an input value to more than one output value. Example:



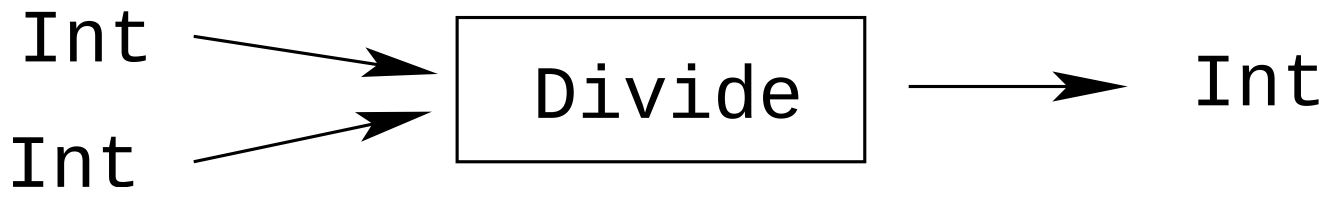
More on functions…

- If a function F maps every element in the domain to some element in the range, then F is total. I.e. a total function is defined for all arguments.



More on functions...

- A function that is undefined for some inputs, is called partial.



- `Divide` is partial since $\frac{?}{0} = ?$ is undefined.

Specifying functions

A function can be specified extensionally or intentionally.

Extensionally:

- Enumerate the elements of the (often infinite) set of pairs "`(argument, result)`" or "`Argument` $\mapsto$ `Result`."
- The extensional view emphasizes the external behavior (or specification), i.e. what the function does, rather than how it does it.

```
double = {···, (1,2), (5,10), ···}
even = {···, (0,True), (1,False), ···}
double = {···, 1↦2, 5↦10, ···}
isHandsome={Chris↦True,Hugh↦False}
```

Specifying functions...

Intensionally:

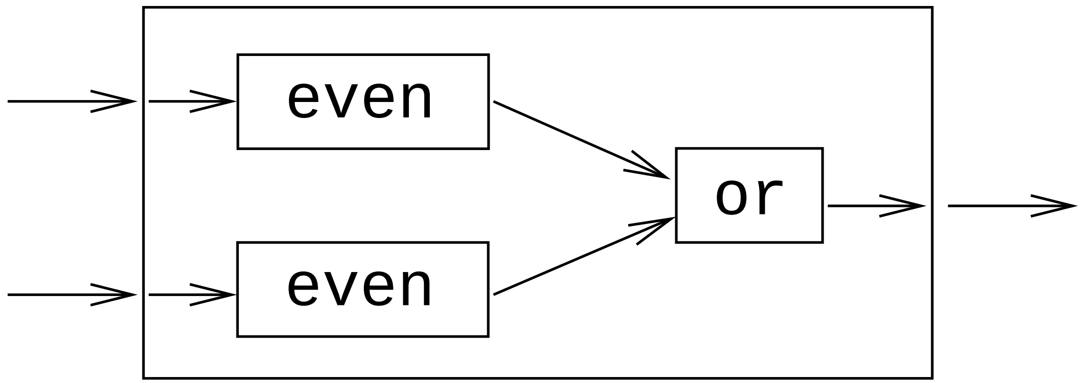
- Give a rule (i.e. algorithm) that computes the result from the arguments.
- The intentional view emphasizes the process (or algorithm) that is used to compute the result from the arguments.

```
double x = 2 * x
even x = x mod 2 == 0
isHandsome x = if isBald x
               then True
               else False
```

Specifying functions. . .

Graphically:

- The graphical view is a notational variant of the intentional view.



eithereven

Function Application

- The most important operation in a functional program is function application, i.e. applying an input argument to the function, and retrieving the result:

```
double x = 2 * x
even x = x mod 2 == 0

double 5 ⇒ 10
even 6 ⇒ True
```

Function Composition

- Function composition makes the result of one function application the input to another application:

```
double x = 2 * x
even x = x mod 2 == 0

even (double 5) ⇒ even 10 ⇒ True
```

Function Definition — Example

Example: How many numbers are there between m and n, inclusive?

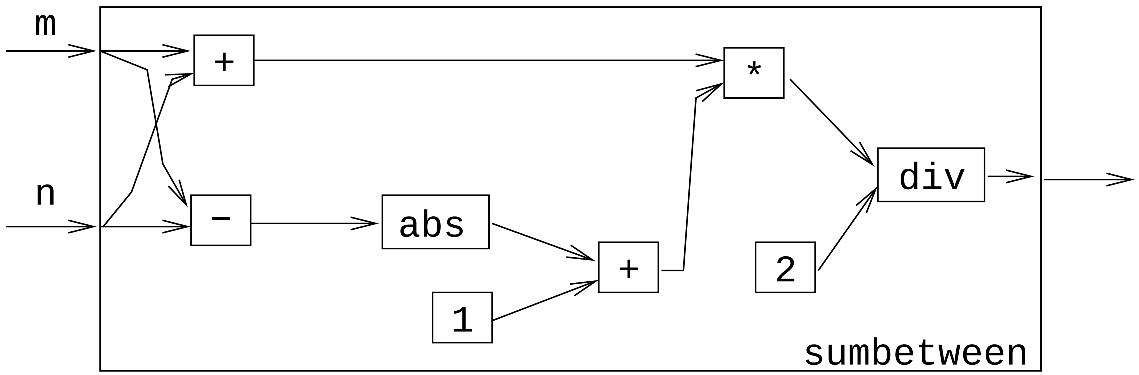
Extensional Definition:

`sumbetween m n =` $\{\cdots \quad (1,1) \mapsto 1, (1,2) \mapsto 2, \cdots, (2,10) \mapsto 9\}$

Intentional Definition:

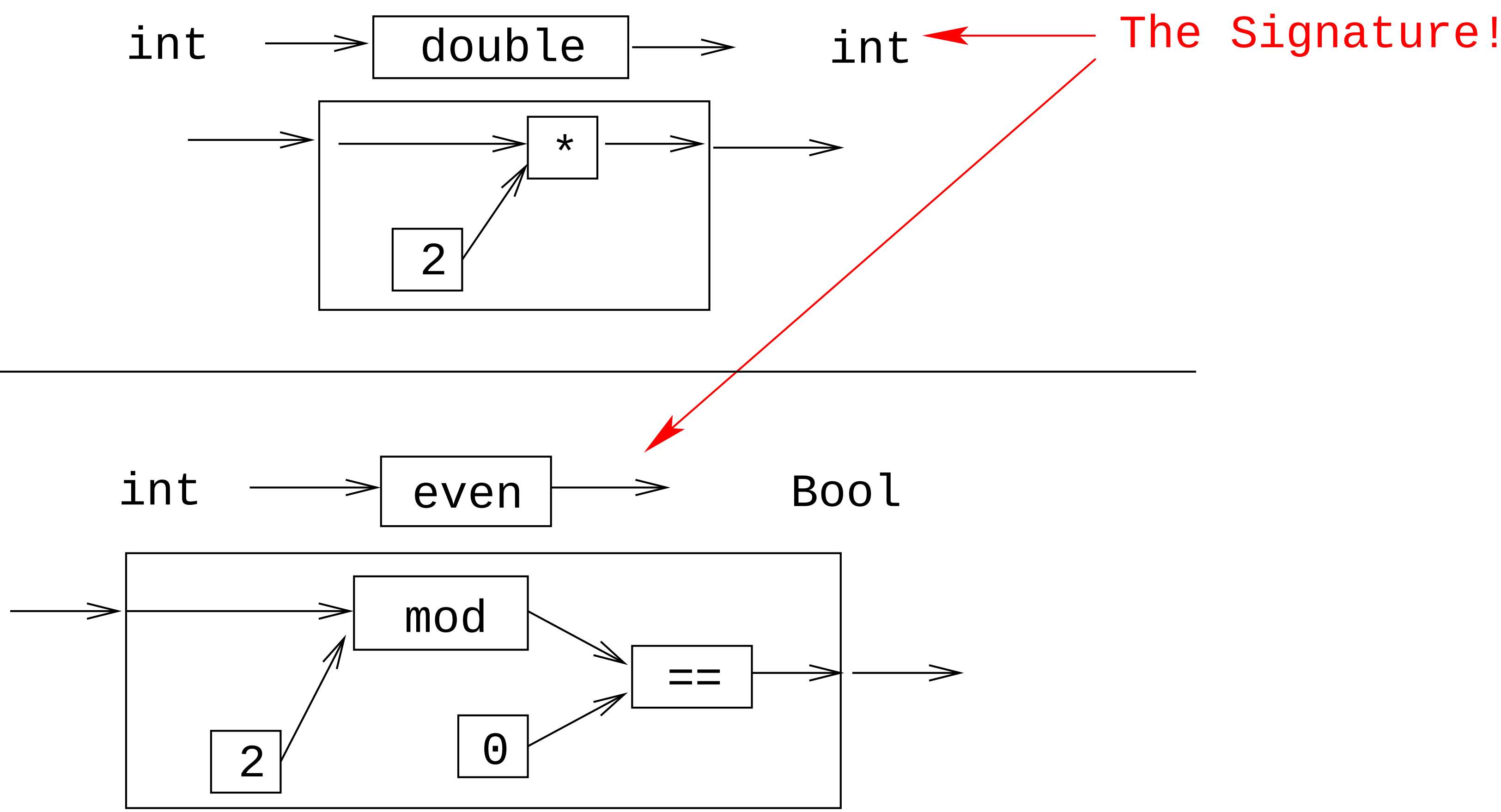
```
sumbetween m n = ((m + n) * (abs (m-n) + 1)) div 2
```

Graphical Definition:



Function Signatures

To define a function we must specify the types of the input and output sets (domain and range, i.e. the function's signature), and an algorithm that maps inputs to outputs.



Referential Transparency

- The most important concept of functional programming is referential transparency. Consider the expression

$$(2 * 3) + 5 * (2 * 3)$$

- $(2 * 3)$ occurs twice in the expression, but it has the same meaning (6) both times.
- RT means that the value of a particular expression (or sub-expression) is always the same, regardless of where it occurs.
- This concept occurs naturally in mathematics, but is broken by imperative programming languages.
- RT makes functional programs easier to reason about mathematically.

Referential Transparency...

- RT is a fancy term for a very simple concept.
- What makes FP particularly simple, direct, and expressive, is that there is only one mechanism (function application) for communicating between subprograms.
- In imperative programs we can communicate either through procedure arguments or updates of global variables. This is hard to reason about mathematically.
- A notation (programming language) where the value of an expression depends only on the values of the sub-expressions is called referentially transparent.

Referential Transparency. . .

- Pure functional programming languages are referentially transparent.
- This means that it is easy to find the meaning (value) of an expression.
- We can evaluate it by substitution. I.e. we can replace a function application by the function definition itself.

Referential Transparency...

Evaluate $\boxed{\texttt{even (double 5)}}$:

```
double x = 2 * x
even x = x mod 2 == 0
```

```
even (double 5) ⇒
        even (2 * 5) ⇒
        even 10 ⇒
        10 mod 2 == 0 ⇒
        0 == 0 ⇒ True
```

Referential Transparency...

So, isn't Pascal referentially transparent??? Well, sometimes, yes, but not always. If a Pascal function f has side-effects (updating a global variable, doing input or output), then $f(3) + f(3)$ may not be the same as $2 * f(3)$. I.e. The second $f(3)$ has a different meaning than the first one.

```
var G : integer;
function f (n:integer) :  integer;
begin G:=G+1; f:=G+n; end;
```

```
begin
   G := 0;
   print f(3)+f(3);
   {prints 4+5=9}
end.
```

```
begin
   G := 0;
   print 2*f(3);
   {prints 2*4=8}
end.
```

Referential Transparency. . .

Furthermore, in many imperative languages the order in which the arguments to a binary operator are evaluated are undefined.

```
var G : integer;
function f (n:integer) :  integer;
begin G:=G+1; f:=G+n; end;
```

```
Left f(3) evaluated first.
begin
   G := 0;
   print f(3)-f(4);
   prints 4-6=-2
end.
```

```
Right f(3) evaluated first.
begin
   G := 0;
   print f(3)-f(4);
   prints 5-5=0
end.
```

Referential Transparency...

This cannot happen in a pure functional language.

1. Expressions and sub-expressions always have the same value, regardless of the environment in which they're evaluated.
2. The order in which sub-expressions are evaluated doesn't effect the final result.
3. Functions have no side-effects.
4. There are no global variables.

Referential Transparency...

5. Variables are similar to variables in mathematics: they hold a value, but they can't be updated.
6. Variables aren't (updatable) containers the way they are imperative languages.
7. Hence, functional languages are much more like mathematics than imperative languages. Functional programs can be treated as mathematical text, and manipulated using common algebraic laws.

Readings and References

- Read Scott, pp. 589–593.

Homework

- Here is a mathematical definition of the combinatorial function $\binom{n}{r}$ "n choose r", which computes the number of ways to pick r objects from n:

$$\binom{n}{r} = \frac{n!}{r! * (n-r)!}$$

- Give an extensional, intentional, and graphical definition of the combinatorial function, using the notations suggested in this lecture.
- You may want to start by defining an auxiliary function to compute the factorial function, $n! = 1 * 2 * \cdots * n$.