
CSc 520

Principles of Programming Languages

43 : Logic Programming — Prolog Unification

Christian Collberg

collberg+520@gmail.com

Department of Computer Science

University of Arizona

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Unification & Matching

- So far, when we've gone through examples, I have said simply that when trying to satisfy a goal, Prolog searches for a **matching** rule or fact.
- What does this mean, **to match**?
- Prolog's matching operator or **=**. It tries to make its left and right hand sides the same, by assigning values to variables.
- Also, there's an implicit **=** between arguments when we try to match a query

$?- f(x, y)$

to a rule

$f(A, B) :- \dots$

Matching Examples

The rule:

$\text{deriv}(U^C, X, C * U^L * DU) :-$
 $\text{number}(C), L \text{ is } C - 1,$
 $\text{deriv}(U, X, DU).$

?- $\text{deriv}(x^3, x, D).$
 $D = 1 * 3 * x^2$

The goal:

● x^3 matches U^C

● $x = U, C = 3$

● x matches X

● D matches $C * U^L * DU$

Matching Examples...

```
deriv(U+V, X, DU + DV) :-  
    deriv(U, X, DU),  
    deriv(V, X, DV).
```

```
?- deriv(x^3 + x^2 + 1, x, D).  
D = 1*3*x^2+1*2*x^1+0
```

- $x^3 + x^2 + 1$ matches $U + V$
 - $x^3 + x^2$ is bound to U
 - 1 is bound to V

Matching Algorithm

Can two terms A and F be “made identical,” by assigning values to their variables?

Two terms A and F match if

1. they are identical atoms
2. one or both are uninstantiated variables
3. they are terms $A = f_A(a_1, \dots, a_n)$ and $F = f_F(f_1, \dots, f_m)$, and
 - (a) the arities are the same ($n = m$)
 - (b) the functors are the same ($f_A = f_F$)
 - (c) the arguments match ($a_i \equiv f_i$)

Matching – Examples

A	F	$A \equiv F$	variable subst.
a	a	yes	
a	b	no	
$\sin(X)$	$\sin(a)$	yes	$\theta = \{X=a\}$
$\sin(a)$	$\sin(X)$	yes	$\theta = \{X=a\}$
$\cos(X)$	$\sin(a)$	no	
$\sin(X)$	$\sin(\cos(a))$	yes	$\theta = \{X=\cos(a)\}$

Matching – Examples...

A	F	$A \equiv F$	variable subst.
likes(c, X)	likes(a, X)	no	
likes(c, X)	likes(c, Y)	yes	$\theta = \{X=Y\}$
likes(X, X)	likes(c, Y)	yes	$\theta = \{X=c, X=Y\}$
likes(X, X)	likes(c, _)	yes	$\theta = \{X=c, X=_47\}$
likes(c, a(X))	likes(V, Z)	yes	$\theta = \{V=c, Z=a(X)\}$
likes(X, a(X))	likes(c, Z)	yes	$\theta = \{X=c, Z=a(X)\}$

Matching Consequences

Consequences of Prolog Matching:

- An uninstantiated variable will match any object.
- An integer or atom will match only itself.
- When two uninstantiated variables match, they *share*:
 - When one is instantiated, so is the other (with the same value).
- Backtracking undoes all variable bindings.

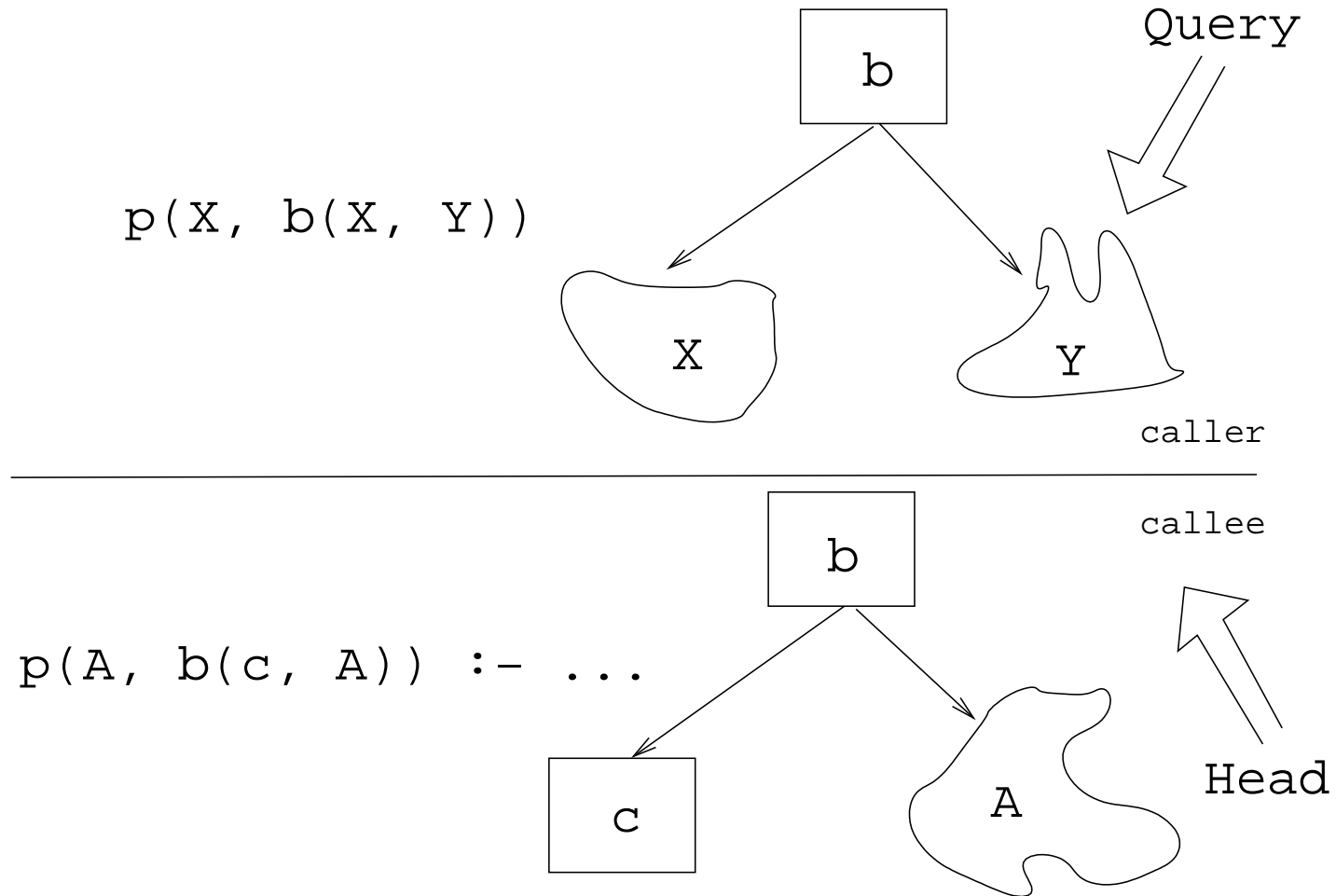
Matching Algorithm

```
FUNC Unify (A, F: term) :  BOOL;  
  IF Is_Var(F) THEN Instantiate F to A  
  ELSIF Is_Var(A) THEN Instantiate A to F  
  ELSIF Arity(F)  $\neq$  Arity(A) THEN RETURN FALSE  
  ELSIF Functor(F)  $\neq$  Functor(A) THEN RETURN FALSE  
  ELSE  
    FOR each argument  $i$  DO  
      IF NOT Unify(A( $i$ ), F( $i$ )) THEN  
        RETURN FALSE  
  RETURN TRUE;
```

Visualizing Matching

- From *Prolog for Programmers*, Kluzniak & Szpakowicz, page 18.
- Assume that during the course of a program we attempt to match the goal $p(X, b(X, Y))$ with a clause C , whose head is $p(X, b(X, Y))$.
- First we'll compare the arity and name of the functors. For both the goal and the clause they are 2 and p , respectively.

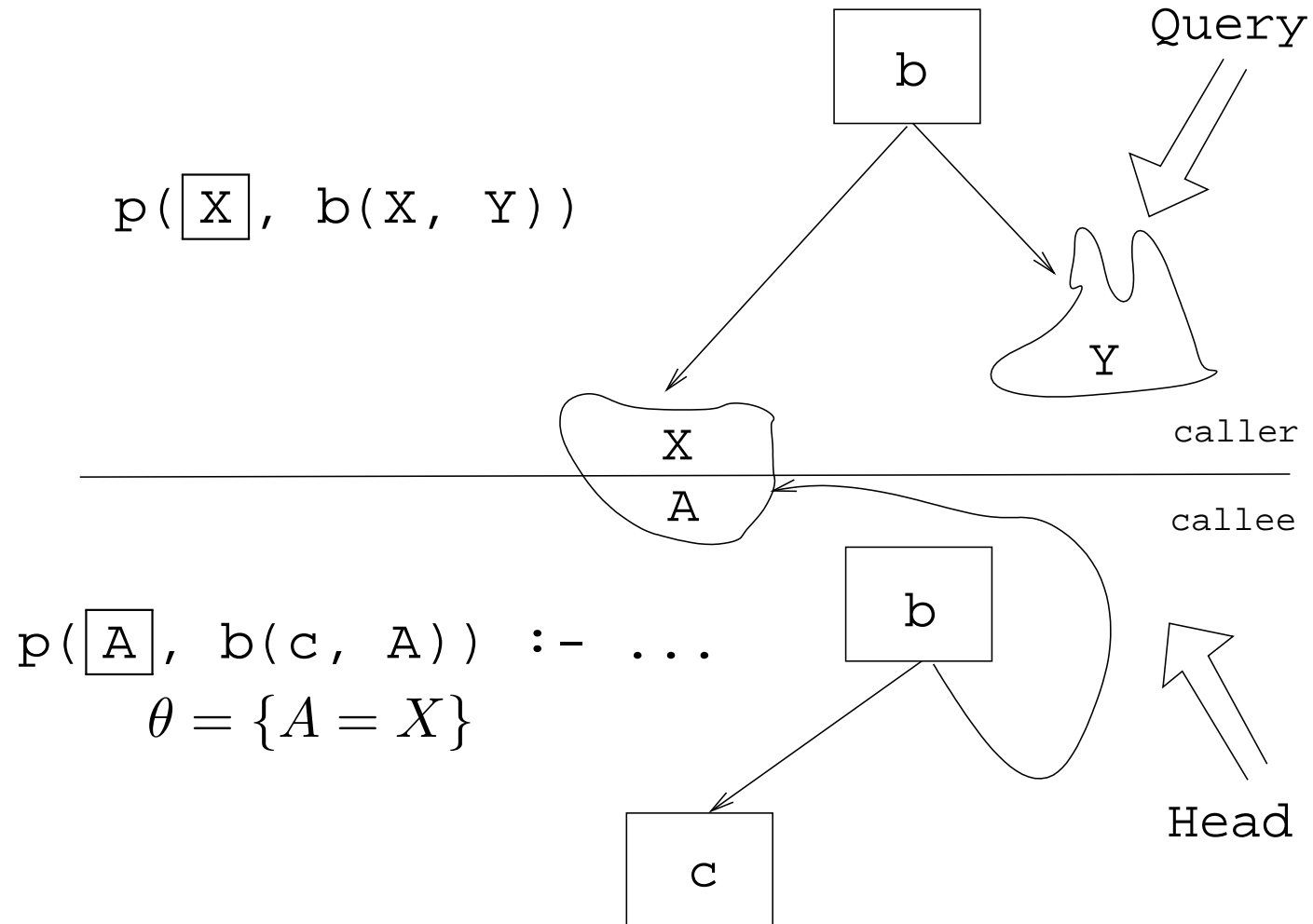
Visualizing Matching...



Visualizing Matching...

- The second step is to try to unify the first argument of the goal (x) with the first argument of the clause head (A).
- They are both variables, so that works OK.
- From now on A and x will be treated as identical (they are in the list of variable substitutions θ).

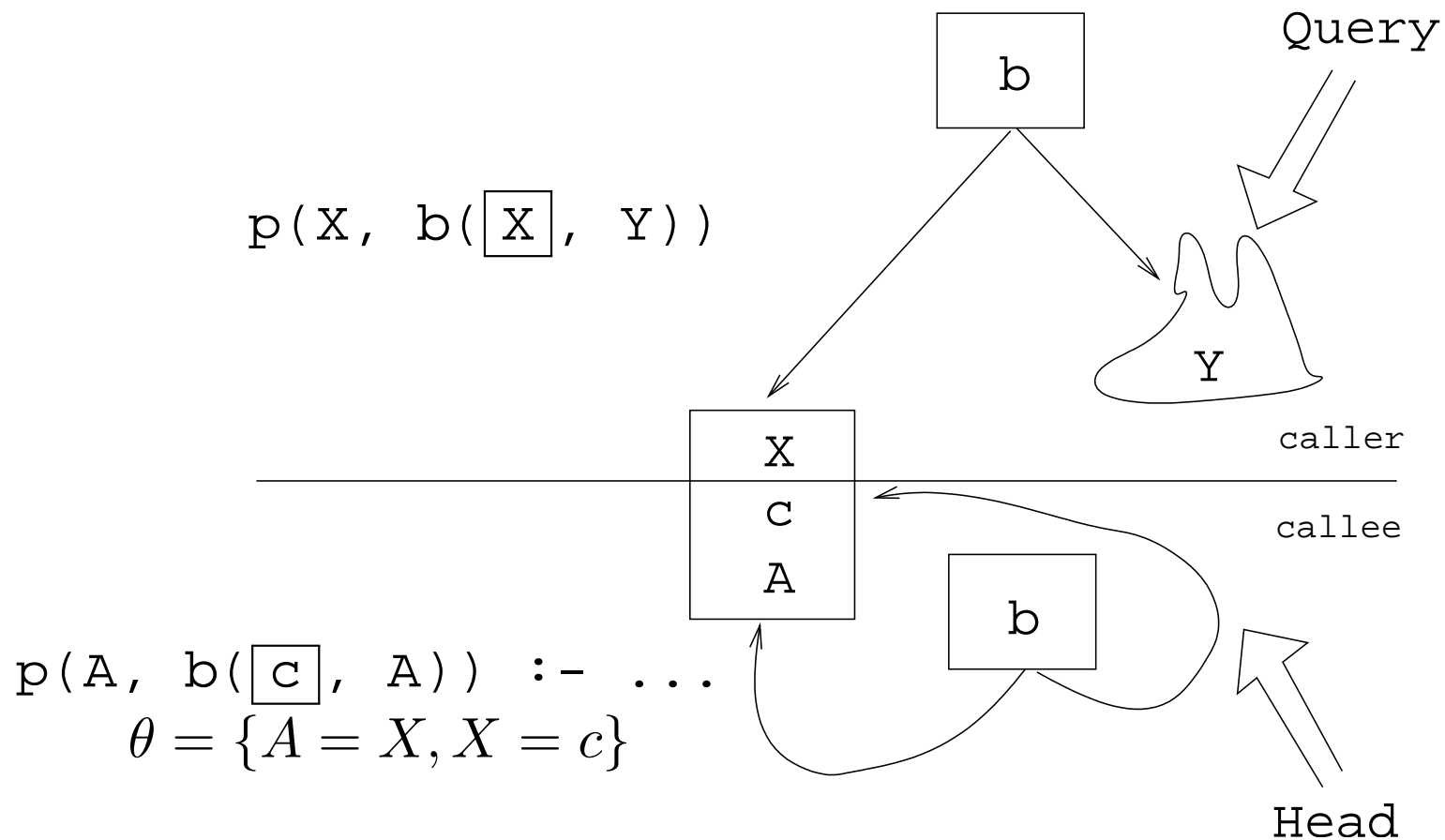
Visualizing Matching...



Visualizing Matching...

- Next we try to match the second argument of the goal $(b(X, Y))$ with the second argument of the clause head $(b(c, A))$.
- The arities and the functors are the same, so we go on to try to match the arguments.
- The first argument in the goal is x , which is matched by the first argument in the clause head (c). I.e., x and c are now treated as identical.

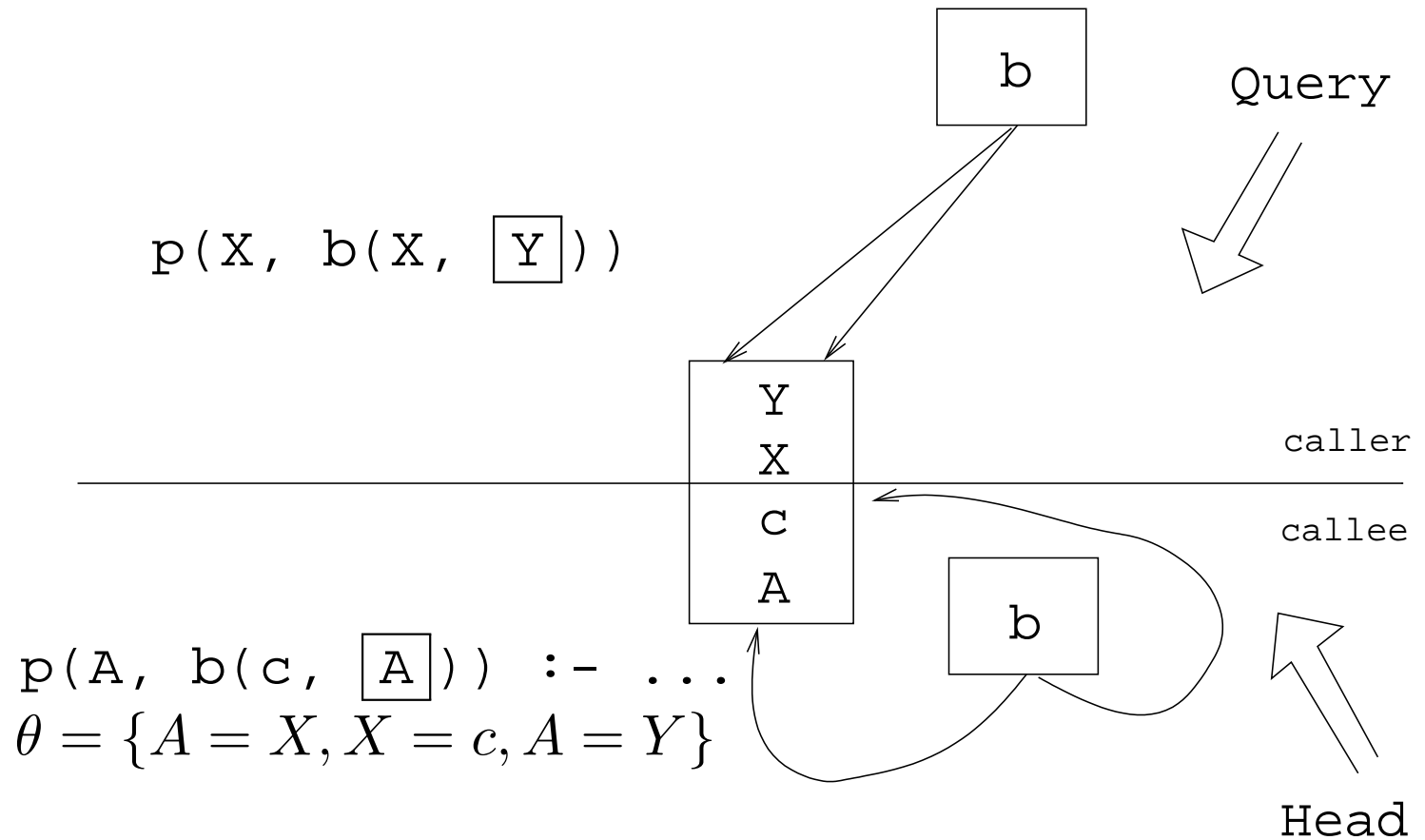
Visualizing Matching...



Visualizing Matching...

- Finally, we match A and Y . Since $A=X$ and $X=C$, this means that $Y=C$ as well.

Visualizing Matching...



Readings and References

- Read Clocksin-Mellish, Sections 2.4, 2.6.3.

Prolog So Far...

- A term is either a
 - a constant (an atom or integer)
 - a variable
 - a structure
- Two terms *match* if
 - there exists a variable substitution θ which makes the terms identical.
- Once a variable becomes instantiated, it stays instantiated.
- Backtracking *undoes* variable instantiations.
- Prolog searches the database sequentially (from top to bottom) until a matching clause is found.