CSc 553 — Principles of Compilation

28 : Optimization III

Christian Collberg
Department of Computer Science
University of Arizona
collberg@gmail.com

April 7, 2011

1 Introduction

2 Computing Data-Flow Info.

- There are two principal methods of solving data-flow problems:
  1. Let `gen,kill,in,out` be AST attributes and the data-flow equations attribute evaluation rules. We'll look at this later.
  2. Treat data-flow equations as recurrences, and iterate over the set of equations until a solution is found.
- Sets are stored as bit-vectors, with one element for each possible object.

$$\begin{array}{rcl} \texttt{in}[\texttt{B1}] & = & \{d_3, d_5, d_7\} \\ & \equiv & \begin{array}{c|c|c|c|c|c|c|c} 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 \end{array} \end{array}$$

3 Data-flow Analysis by Iteration

4 Reaching Definitions — Equations

$$\begin{aligned}\texttt{out}[B] &= \texttt{gen}[B] \cup (\texttt{in}[B] - \texttt{kill}[B]) \\ \texttt{in}[B] &= \bigcup_{\texttt{preds} P \texttt{ of } B} \texttt{out}[P]\end{aligned}$$

- A definition $d : a := b + c$ reaches a use of a at point p, if the value given to a at d could be used at p.
- `gen[B]` is the set of definitions generated within `B`, that reach the end of `B`.
- `kill[B]` is the set of definitions outside `B`, killed by definitions within `B`.
- `in[B]` is the set of definitions valid at the entrance to `B`, `out[B]` is those valid at the exit of `B`.
- The equations for `in` and `out` are valid for each basic block.

5 Iterative Algorithms I

1. Compute `gen` and `kill` for each block.
2. Set up the $2n$ `in`- and `out`-equations for the n basic blocks, and set `in[B]=out[B]=`{} for each block `B`.
3. Repeat until no more changes:
   - For each block `B` eval. `in[B]` & `out[B]`.

Formal Algorithm:

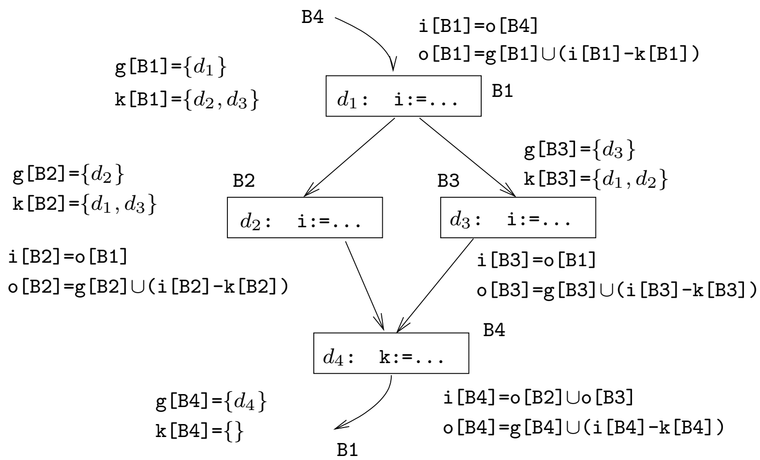
```
FOR each block B DO
   out[B] := in[B] := { };
END;
WHILE any out[B] has changed DO
   FOR each block B DO
      in[B] := ⋃_{preds P of B} out[P]
      out[B] := gen[B] ∪ (in[B] − kill[B])
   END;
END
```

6 Iterative Alg. Example I (a)

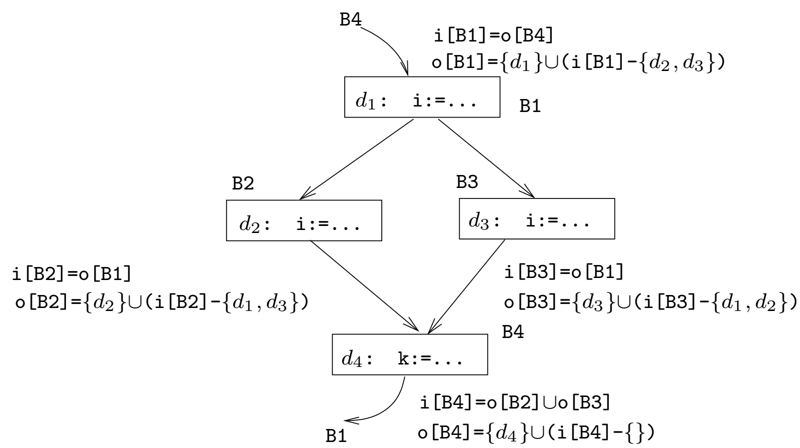
```
REPEAT
  d1: i := ...;
      IF ...THEN
  d2:   i := ...
      ELSE
  d3:   i := ...
      ENDIF;
  d4: k := ...
UNTIL ...;
```

- Start by setting up the $2n$ `in`- and `out`-equations (slide (b)).
- Simplify the example by inlining `gen` and `kill` into the equations for `in` and `out` (slide (c)).
- Visit each block in turn (we use numerical order, `B1, B2, B3, B4`) and evaluate `in` and `out` (slides (d)–(g)).

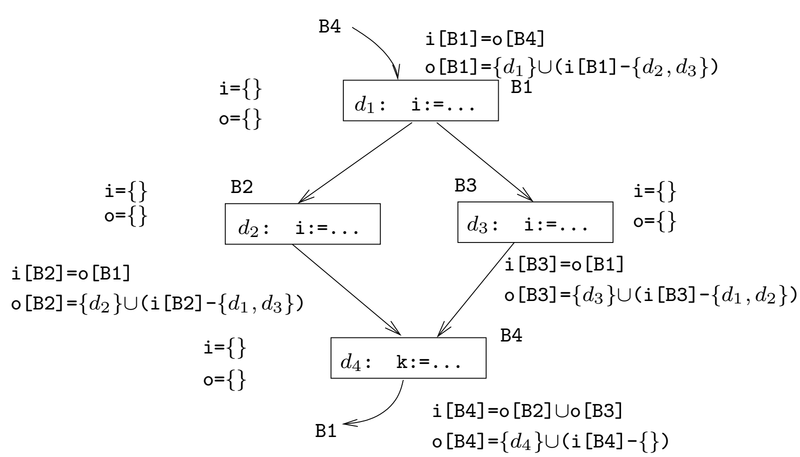
7 Iterative Alg. Example I (b)



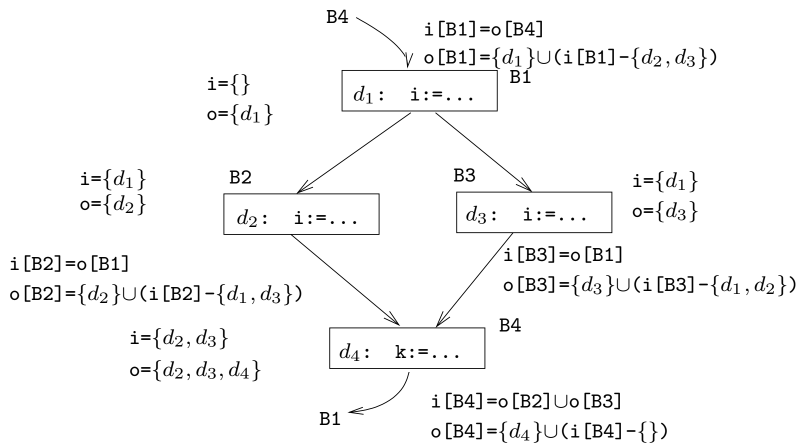
8 Iterative Alg. Example I (c) – Simplified



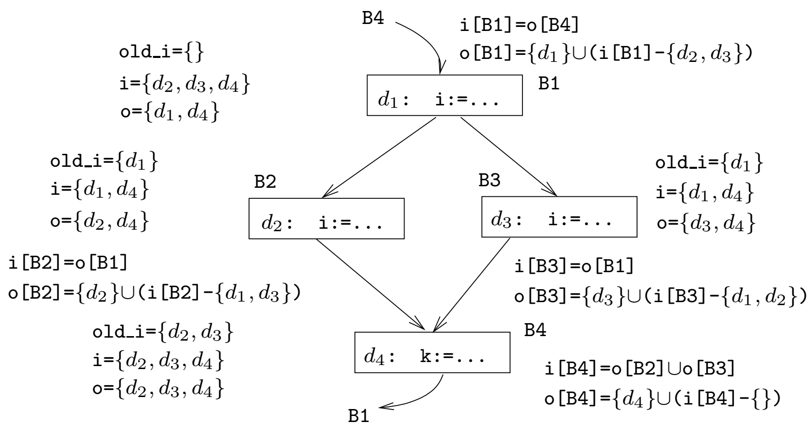
9 Iterative Alg. Example I (d) – Initialized



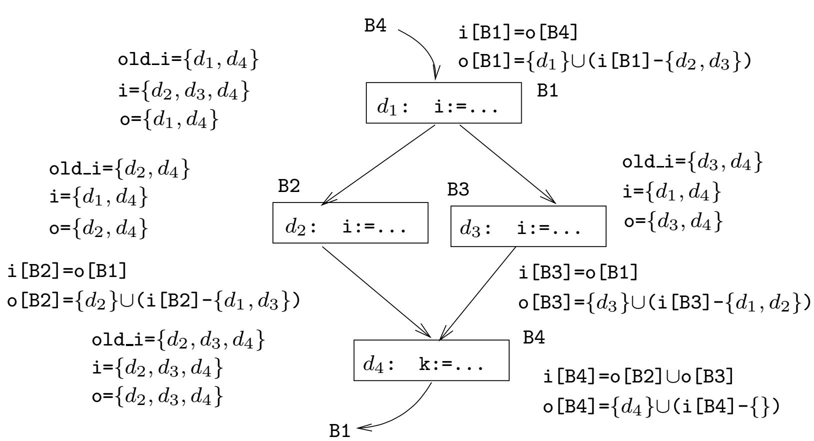
10 Iterative Alg. Example I (e) – 1st Iteration



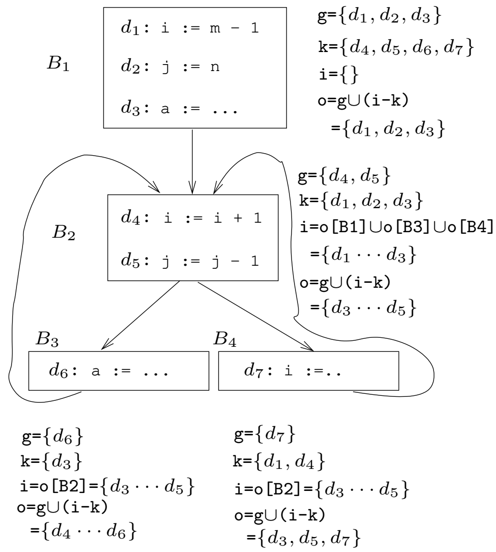
11 Iterative Alg. Example I (f) – 2nd Iteration



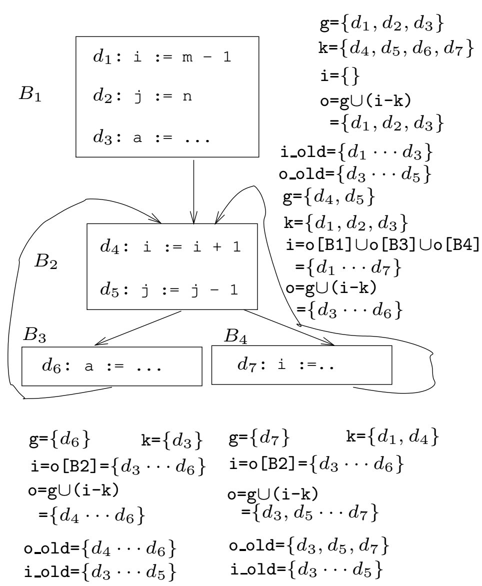
12 Iterative Alg. Example I (g) – 3rd Iteration



13 Example II (a) – 1st Iteration



14 Example II (b) – 2nd Iteration



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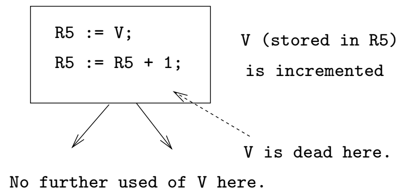
Live Variable Analysis

16 Live Variable Analysis I

- For each definition/use of a variable `V`, **Global Live Variable Analysis** answers the question

"Could the value of V computed/used here be used further on in the program?"

- If a variable V is stored in a register R5 and V is dead at the end of the block, then we don't have to store R5 back into V.
- Assignments to dead variables can be removed.

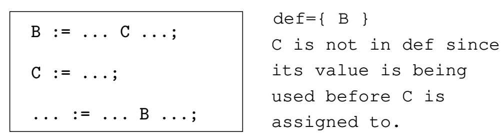


17 Live Variable Analysis II

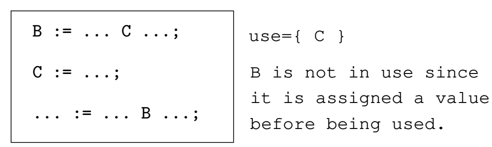
`in[B]` Variables live on entrance to `B`.

`out[B]` Variables live on exit from `B`.

`def[B]` Variables assigned values in `B` before the variable is used:



`use[B]` Variables whose values are used before being assigned to:



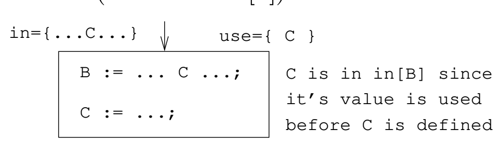
18 Live Variable Analysis III

Data-Flow (Equations):

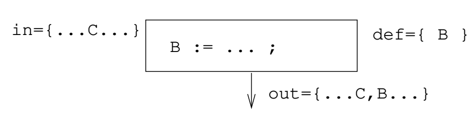
$$\texttt{in}[\mathrm{B}] \quad = \quad \texttt{use}[\mathrm{B}] \cup (\texttt{out}[\mathrm{B}] - \texttt{def}[\mathrm{B}])$$

Data-Flow (English):

- V is live at the entrance to `B` if
  1. it is being used before it's defined (i.e. $V \in \texttt{use}[\mathrm{B}]$)



  2. it is live at the exit of the block, and it is not defined within the block (i.e. $V \in \texttt{out}[\mathrm{B}]$ and $V \notin \texttt{def}[\mathrm{B}]$)



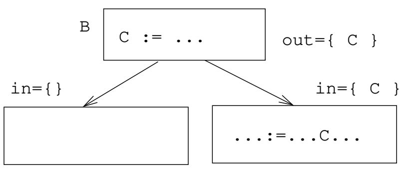
19 Live Variable Analysis IV

Data-Flow (Equations):

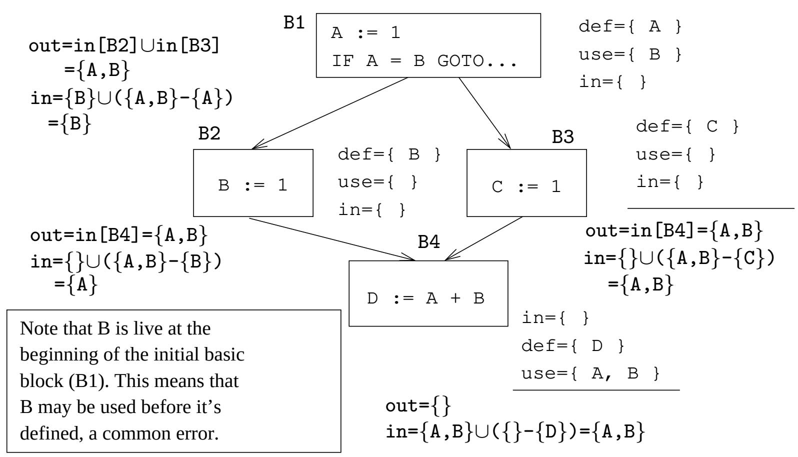
$$\texttt{out}[\mathrm{B}] = \bigcup_{\substack{\texttt{successors} \\ S \texttt{ of } B}} \texttt{in}[\mathrm{S}]$$

Data-Flow (English):

- A variable V is live coming out of `B` if it is live going into any one of `B`'s successors.



20 Live Variable Analysis V – Example



21 Live Variables VI – Example

`out[B4]={}` since `out` is the union of all of `B4`'s successor's `in`, and `B4` doesn't have any successors.

`in[B4]={}` because both `A & B` are live coming in to `B4`, i.e. their values will be used before they are assigned new values.

`out[B3]=in[B4]={A,B}` because the values of `A` and `B` will be used in `B3`'s successor block, `B4`. Note that since `C` $\notin$ `out[B3]` `C`'s value is *dead* and the assignment `C := 1` can be removed.

`out[B1]={A}∪{A,B}={A,B}` since if we take the left branch (through `B2`) `A` will be used further on, and if we take the right branch (through `B3`) both `A` and `B` will have a future use.

`in[B1]={B}` since `B`'s value is used but not defined in `B`.

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Classifying Data-Flow Problems

23 Classification — Forward vs. Backward Flow

- Data-flow problems can be classified according to the direction of flow:

  Forward-flow problems: Data flows from the initial block to the end block.

  - `Out`-sets are computed from `In`-sets within basic blocks,
  - `In`-sets are computed from `Out`-sets across basic blocks.

  Backward-flow problems: Data flows from the end block to the initial block.

  - `In`-sets are computed from `Out`-sets within basic blocks,
  - `Out`-sets are computed from `In`-sets across basic blocks.

24 Forward vs. Backward Flow — Equations

	Forward-Flow	Backward-Flow
Any Path	$\mathtt{o}_B = \mathtt{g}_B \cup (\mathtt{i}_B - \mathtt{k}_B)$ $\mathtt{i}_B = \bigcup_{b \in P(B)} \mathtt{o}_b$	$\mathtt{i}_B = \mathtt{g}_B \cup (\mathtt{o}_B - \mathtt{k}_B)$ $\mathtt{o}_B = \bigcup_{b \in S(B)} \mathtt{i}_b$
All Paths	$\mathtt{o}_B = \mathtt{g}_B \cup (\mathtt{i}_B - \mathtt{k}_B)$ $\mathtt{i}_B = \bigcap_{b \in P(B)} \mathtt{o}_b$	$\mathtt{i}_B = \mathtt{g}_B \cup (\mathtt{o}_B - \mathtt{k}_B)$ $\mathtt{o}_B = \bigcap_{b \in S(B)} \mathtt{i}_b$

- $P(B)$ = Predecessors of B, $S(B)$ = Successors of B.
- $\mathtt{i}_B = \mathtt{in}_B$, $\mathtt{o}_B = \mathtt{out}_B$, $\mathtt{g}_B = \mathtt{gen}_B$, $\mathtt{k}_B = \mathtt{kill}_B$, $\mathtt{o}_b = \mathtt{out}_b$, $\mathtt{i}_b = \mathtt{in}_b$.

25 Classification — Any- vs. All-Path

- We classify data-flow problems by the way they combine incoming information:

  Any-path problems: All values coming in to a block are valid. Use $\bigcup$.

  All-path problems: Only values coming in to a block through *every* path are valid. Use $\bigcap$.

	Forward-Flow	Backward-Flow
Any Path	Reaching Definitions Uninitialized Variables	Live Variables Du-chains
All Paths	Available Expressions Copy Propagation	Very Busy Expressions

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Summary

27 Summary I

- Read the Dragon Book: 608–611, 614–615, 618, 620–622, 624–633.
- With B blocks & bit-vectors of length V, iterative data-flow analysis is $\mathcal{O}(B^2 \times V)$ in the worst case.

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Homework

29 Homework I

- Show each step of the iterative reaching definitions algorithm applied to the procedure body below:

```
K := 1; I := 2;
REPEAT
   IF I = 4 THEN
      A := K + 1;
   ELSE
      A := K + 2;
      I := I + A;
   ENDIF;
UNTIL I <= 10;
K := K + A;
```

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Exam Problems

31 Exam Problem I (a) [07.430 ’95]

- An expression E is *very busy* if – regardless of which path we take through the flow graph – E's value will be used before it is killed. Example (`A+3` is *very busy*):

```
(1)  BEGIN
(2)     IF expr THEN
(3)        V := A + 3;
(4)        R := K + 3;
(5)     ELSE
(6)        Z := A + 3;
(7)        K := 5;
(8)        L := K + 3;
(9)     END;
(10) END
```

32 Exam Problem I (b) [07.430 ’95]

Data-Flow Equations:

- The data-flow equations for computing very busy expressions are:

$$\begin{aligned} \texttt{in}[B] &= \texttt{used}[B] \cup (\texttt{out}[B] - \texttt{killed}[B]) \\ \texttt{out}[B] &= \bigcap_{\substack{\texttt{successors} \\ S \texttt{ of } B}} \texttt{in}[S] \end{aligned}$$

Problems:

1. Give an iterative pseudo-code routine for computing `in` and `out`.
2. Is *very-busy expressions* a forward-flow or a backward-flow problem?
3. Show the workings of the algorithm on the procedure body in the next slide:

33 Exam Problem I (c) [07.430 '95]

```
BEGIN
   X := 5;
   Y := 10;
   IF e1 THEN
      IF e2 THEN
         A := X * Y;
      ELSE
         B := 3;
         V := X * Y;
         X := 1;
      END;
   ELSE
      Y := 2;
      A := X * Y;
   END
END
```