CSc 553

Principles of Compilation

31 : Dominators and Natural Loops

Department of Computer Science
University of Arizona

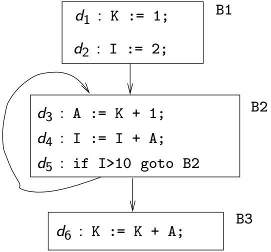
collberg@gmail.com

Introduction

Loop Invariants

- Let C be a computation in a loop body. C is **invariant** if it computes the same value during all iterations. C can sometimes be moved out of the loop.

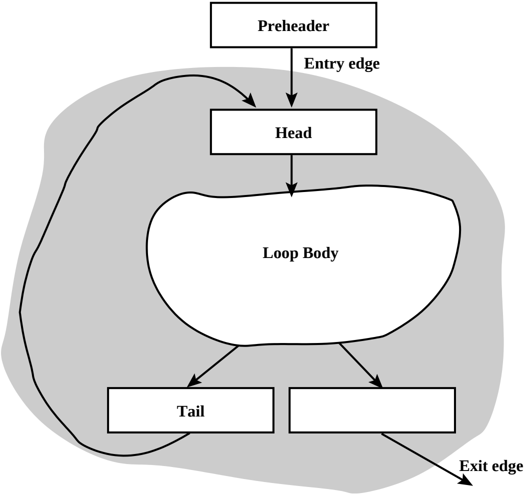
```
K := 1; I := 2;
REPEAT
  A := K + 1; I := I + A;
UNTIL I <= 10;
K := K + A;
```



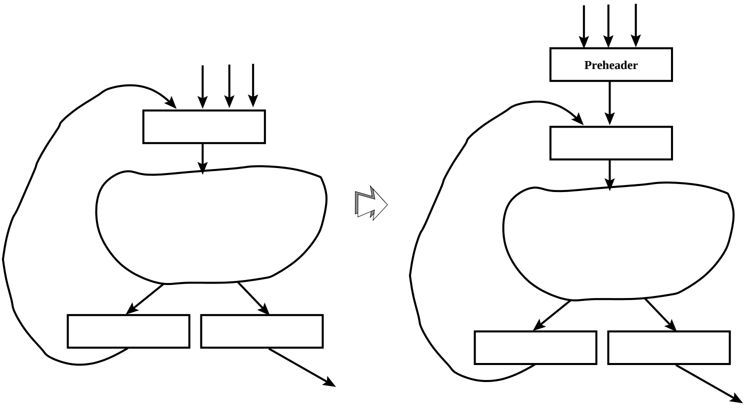
- How do we know what is a loop???

Loops

Loop Terminology



Preheaders



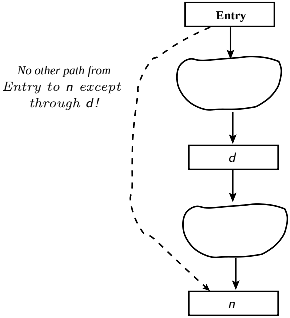
- A preheader is useful, for example if we want to move out loop-invariant computations.
- Not all loops have preheaders — but we can always add one.

Dominators

Dominators

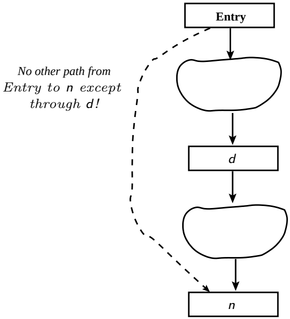
- To detect what the loops are in a program we first have to perform a *dominator analysis.*
- Definition:

  A node *d* dominates a node *n* if every path from the entry node to *n* must go through *d*.



Dominators

- Notation: $d \operatorname{dom} n$ — d strictly dominates n.
- Intuition: Given a node n, which blocks are guaranteed to have executed prior to executing n.



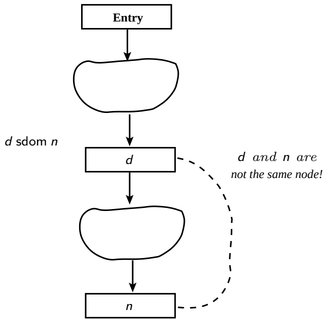
- Every node dominates itself: $d \operatorname{dom} d$.

Strict Dominator

- Definition:

  A node d *strictly dominates* a node n if d dominates n and $d \neq n$.

- Notation: d sdom n — d strictly dominates n.

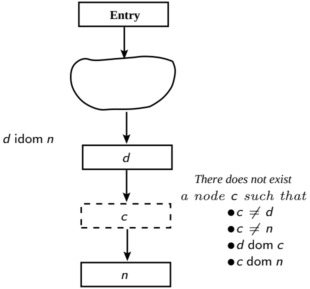


Immediate Dominator

- Definition:

  The immediate dominator d of a node n is the unique node that strictly dominates n but does not strictly dominate any other node that strictly dominates n.

- Entry nodes don't have an immediate dominator.
- Notation: d idom n — d immediately dominates n.



Post dominator

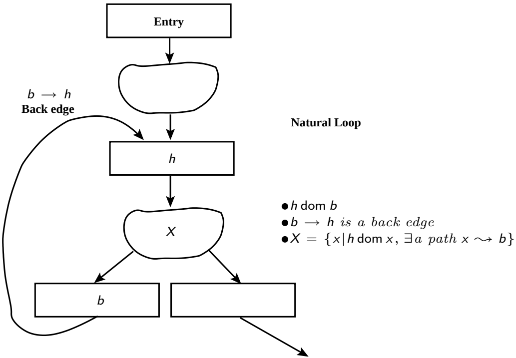
A node d post dominates a node n if every path from n to the exit node must go through d.

- Notation: $d \, \text{pdom} \, n$ — d post dominates n.
- Intuition: Given a node n, which blocks are guaranteed to execute *after* executing n.

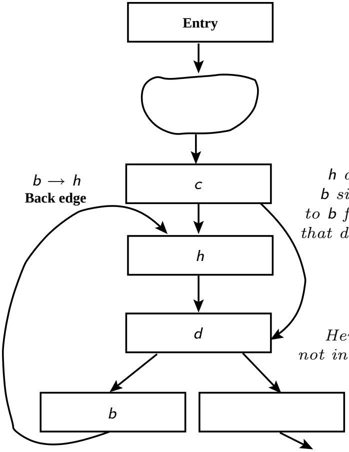
Natural Loop

- Definition:

  A back edge $b \longrightarrow h$, where $h \operatorname{dom} b$, induces a *natural loop* consisting of all nodes x, where $h \operatorname{dom} x$ and there there is a path from x to b not containing b.



Example — Not a Natural Loop



h does not dominate b since there is a path to b from the Entry node that does not go through h.

Hence, $b \longrightarrow h$ does not induce a natural loop.

Computing Dominators

Dataflow Equations

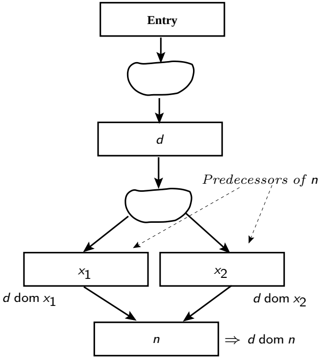
- The dominators of a node n are given by

$$\begin{aligned} \texttt{dom}(\text{entry node}) &= \{\text{entry node}\} \\ \texttt{dom}(n) &= \{n\} \cup \Big(\bigcap_{\texttt{preds}\ p\ \texttt{of}\ n} \texttt{dom}(p) \Big) \end{aligned}$$

- The dominator of the entry node is the entry node itself.
- The set of dominators for a node n is the intersection of the set of dominators for all predecessors of n.
- n is also in the set of dominators for n.

Dataflow Equations — Intuition

$$\texttt{dom}(n) \;=\; \{n\} \cup \Big(\bigcap_{\texttt{preds } p \texttt{ of } n} \texttt{dom}(p)\Big)$$



- If d dominates all predecessors of n, then it also dominates n

Algorithm

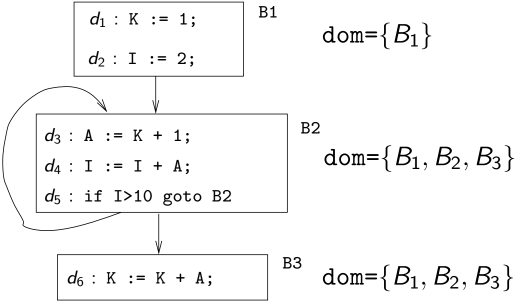
- N is the set of all nodes.
- n_0 is the entry node.

```
dom(n0) := {n0};
FOR EACH n ∈ N − {n0} DO
    dom(n) := N;
WHILE CHANGES IN ANY dom(n) DO
    FOR EACH n ∈ N − {n0} DO
        dom(n) := {n} ∪ (⋂_{preds p of n} dom(p))
```

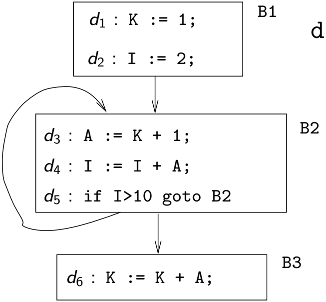
Example 1

Example 1 — Initialization

```
K:=1;
I:=2;
REPEAT
   A:=K+1;
   I:=I+A;
UNTIL I<=10;
K:=K+A;
```



Example 1 — First Iteration



B1: $\text{dom}=\{B_1\}$

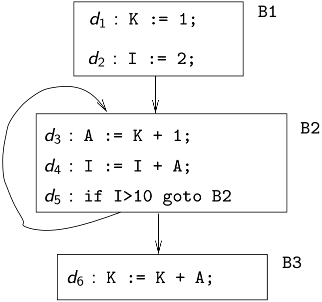
B2: $$\text{dom}=\{B_2\} \cup (\{B_1\} \cap \{B_1, B_2, B_3\}) = \{B_1, B_2\}$$

B3: $$\text{dom}=\{B_3\} \cup (\{B_1, B_2\}) = \{B_1, B_2, B_3\}$$

Example 1 — Final Result

- A back edge $b \rightarrow h$, where $h\,\mathrm{dom}\,b$, induces a *natural loop* consisting of all nodes x, where $h\,\mathrm{dom}\,x$ and there there is a path from x to b not containing b.

B_2 dom B_2



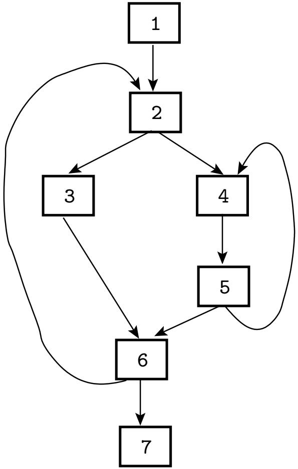
dom=$\{B_1\}$

dom=$\{B_1, B_2\}$

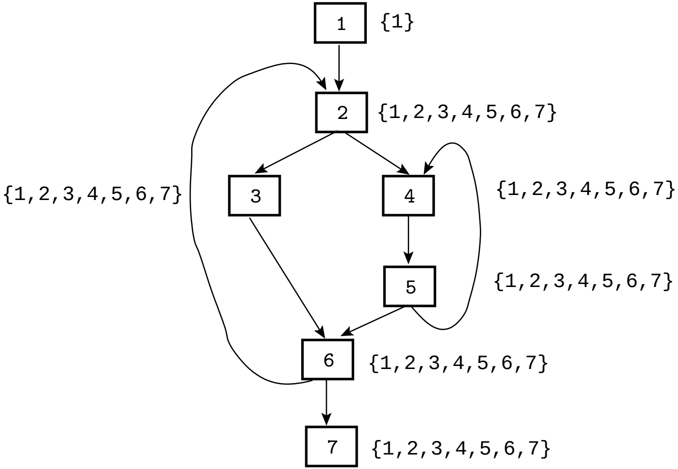
dom=$\{B_1, B_2, B_3\}$

Example 2

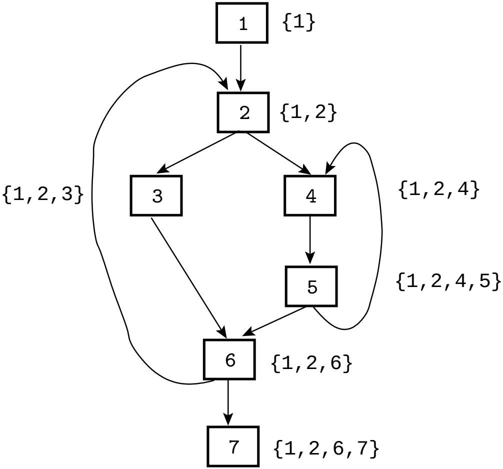
Example 2



Example 2 — Initialization

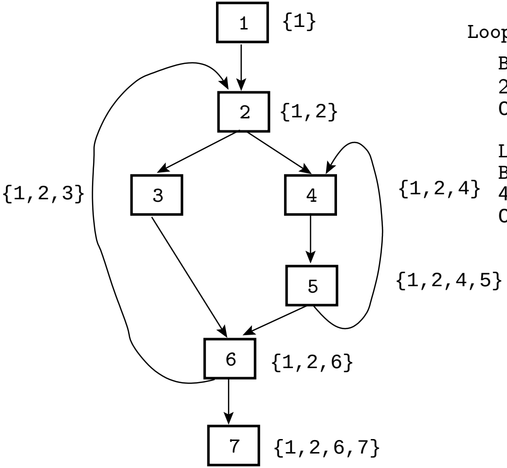


Example 2 — First iteration



Example 2 — Identifying loops

Back edge $b \longrightarrow h$, $h\,\mathrm{dom}\,b$, induces a loop with all nodes x, where $h\,\mathrm{dom}\,x$ and there there is a path $x \rightsquigarrow b$ not containing b.



Loop 1:

Back edge: $6 \longrightarrow 2$
$2\,\mathrm{dom}\,6$
Contains: {2,3,4,5,6}

Loop 2:
Back edge: $5 \longrightarrow 4$
$4\,\mathrm{dom}\,5$
Contains: {4,5}

Summary

Summary

- Each node dominates itself.
- If x dominates y, and y dominates z, then x dominates z.
- If x dominates z and y dominates z, then either x dominates y or y dominates x.