# CSc 553

# Principles of Compilation

## 31 : Dominators and Natural Loops

Department of Computer Science
University of Arizona

collberg@gmail.com

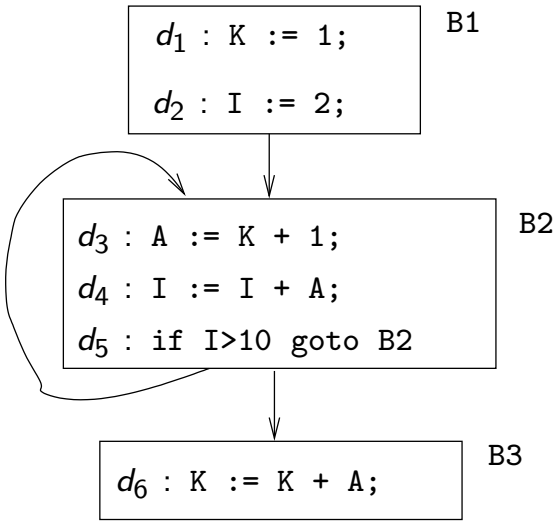
Copyright © 2011 Christian Collberg

# Introduction

# Loop Invariants

- Let $C$ be a computation in a loop body. $C$ is **invariant** if it computes the same value during all iterations. $C$ can sometimes be moved out of the loop.

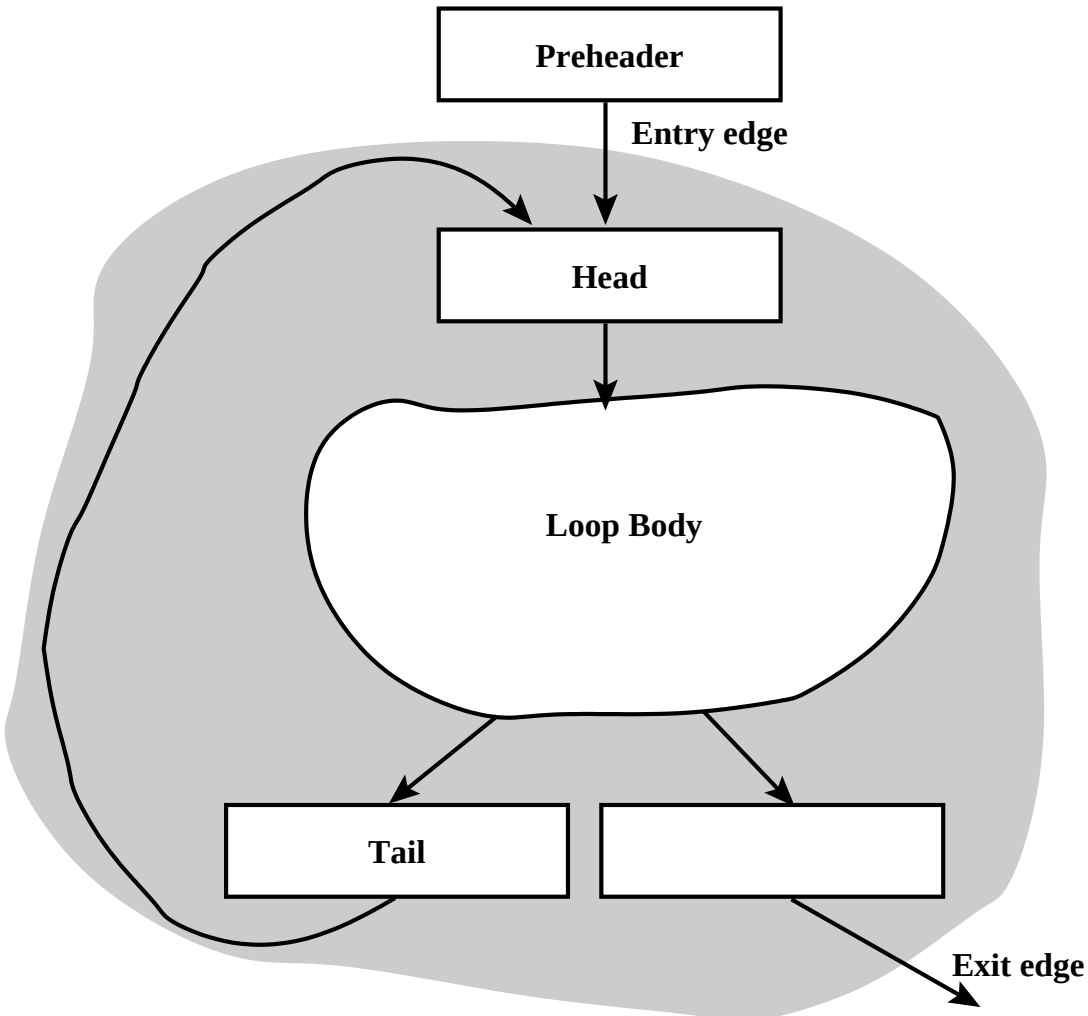
```
K := 1; I := 2;
REPEAT
   A := K + 1; I := I + A;
UNTIL I <= 10;
K := K + A;
```



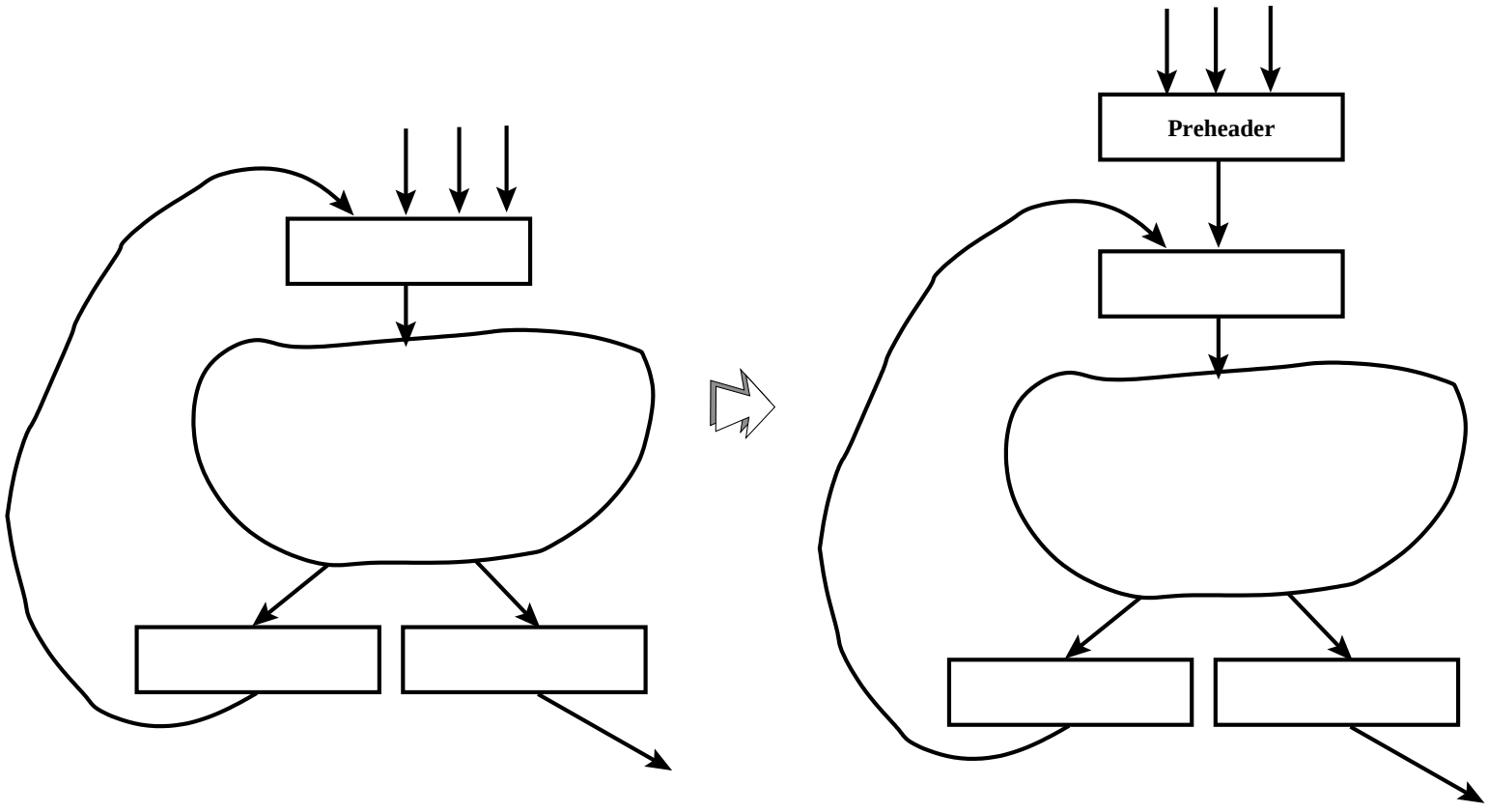

- How do we know what is a loop???

# Loops

# Loop Terminology



# Preheaders



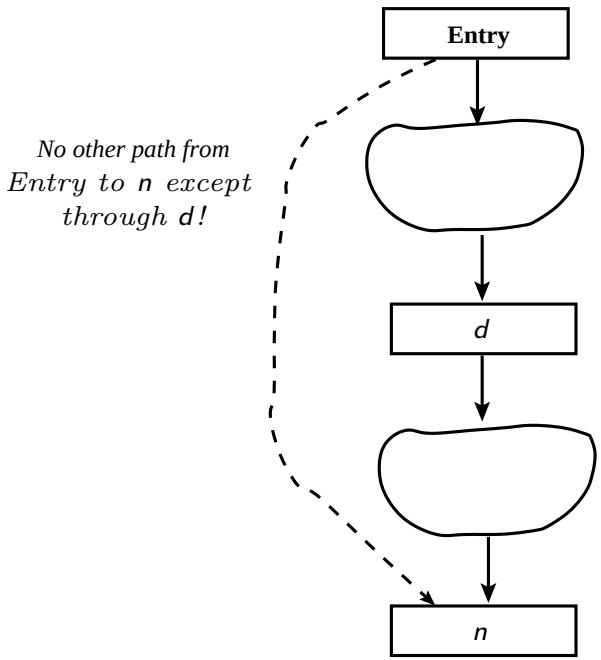

- A preheader is useful, for example if we want to move out loop-invariant computations.
- Not all loops have preheaders — but we can always add one.

# Dominators

# Dominators

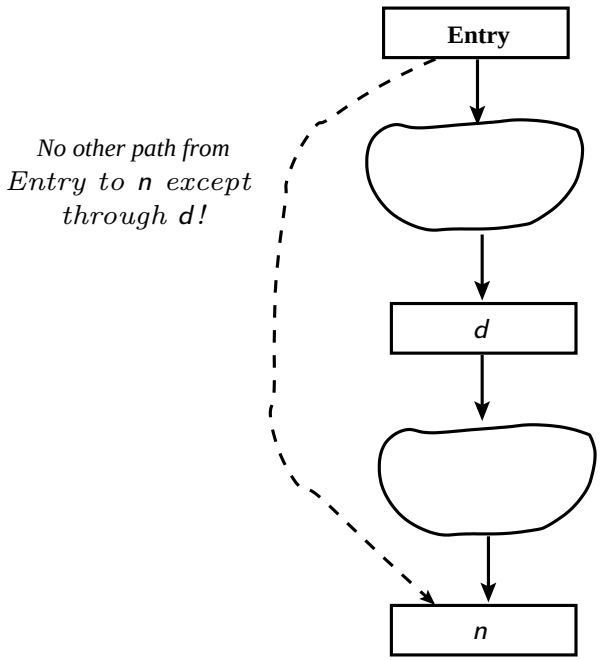
- To detect what the loops are in a program we first have to perform a *dominator analysis.*
- Definition:

  A node *d* dominates a node *n* if every path from the entry node to *n* must go through *d*.



# Dominators

- Notation: $d$ dom $n$ — $d$ strictly dominates $n$.
- Intuition: Given a node $n$, which blocks are guaranteed to have executed prior to executing $n$.



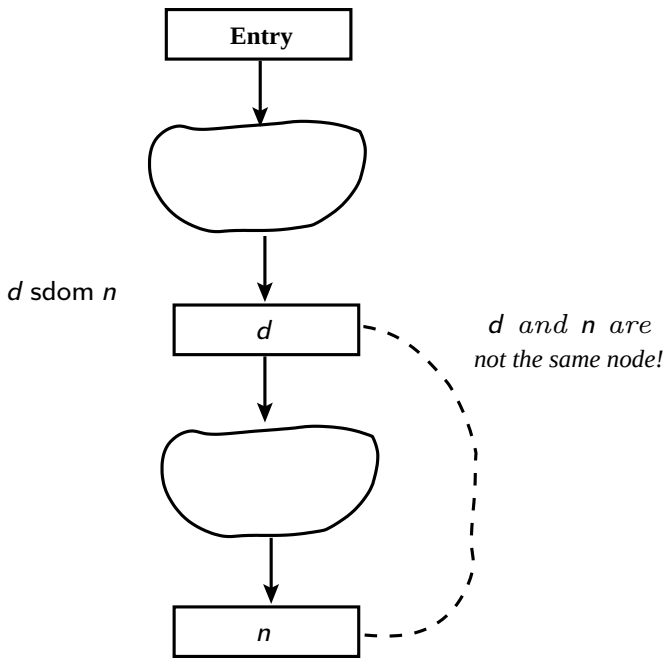

- Every node dominates itself: $d$ dom $d$.

# Strict Dominator

- Definition:

  A node $d$ *strictly dominates* a node $n$ if $d$ dominates $n$ and $d \neq n$.

- Notation: $d$ sdom $n$ — $d$ strictly dominates $n$.

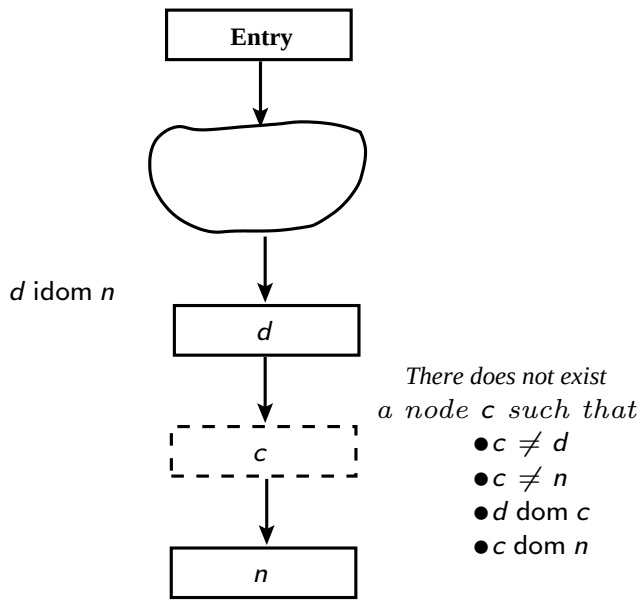


# Immediate Dominator

- Definition:

  The immediate dominator $d$ of a node $n$ is the unique node that strictly dominates $n$ but does not strictly dominate any other node that strictly dominates $n$.

- Entry nodes don't have an immediate dominator.
- Notation: $d$ idom $n$ — $d$ immediately dominates $n$.



# Post dominator

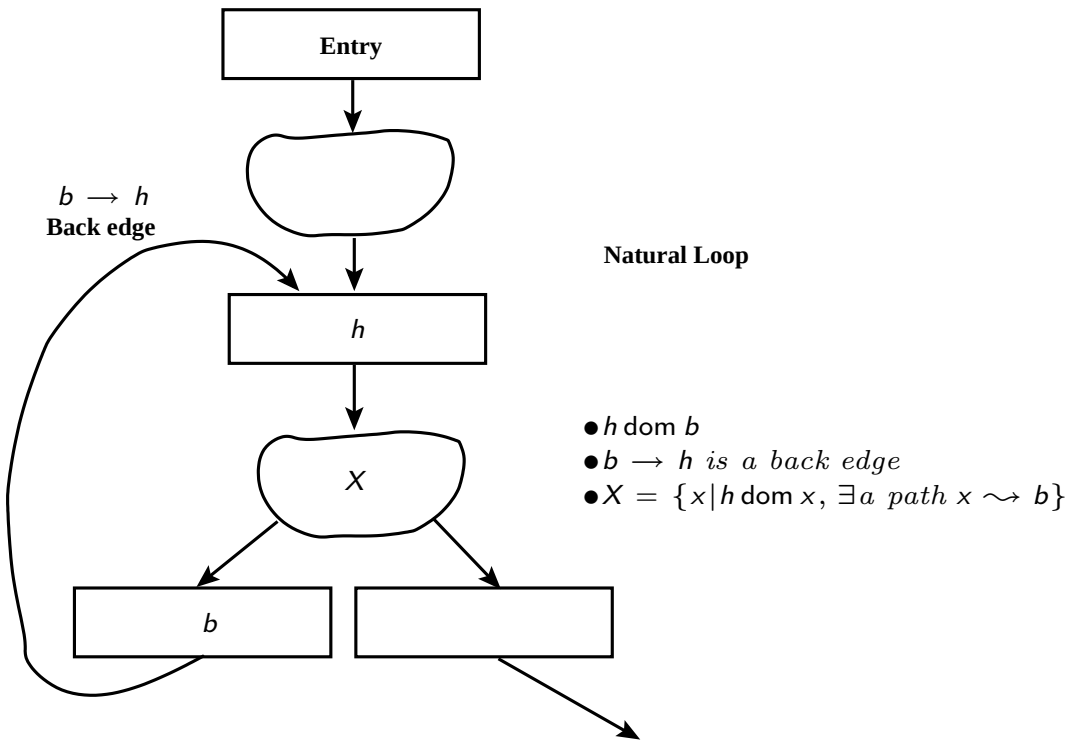
A node $d$ post dominates a node $n$ if every path from $n$ to the exit node must go through $d$.

- Notation: $d$ pdom $n$ — $d$ post dominates $n$.
- Intuition: Given a node $n$, which blocks are guaranteed to execute *after* executing $n$.

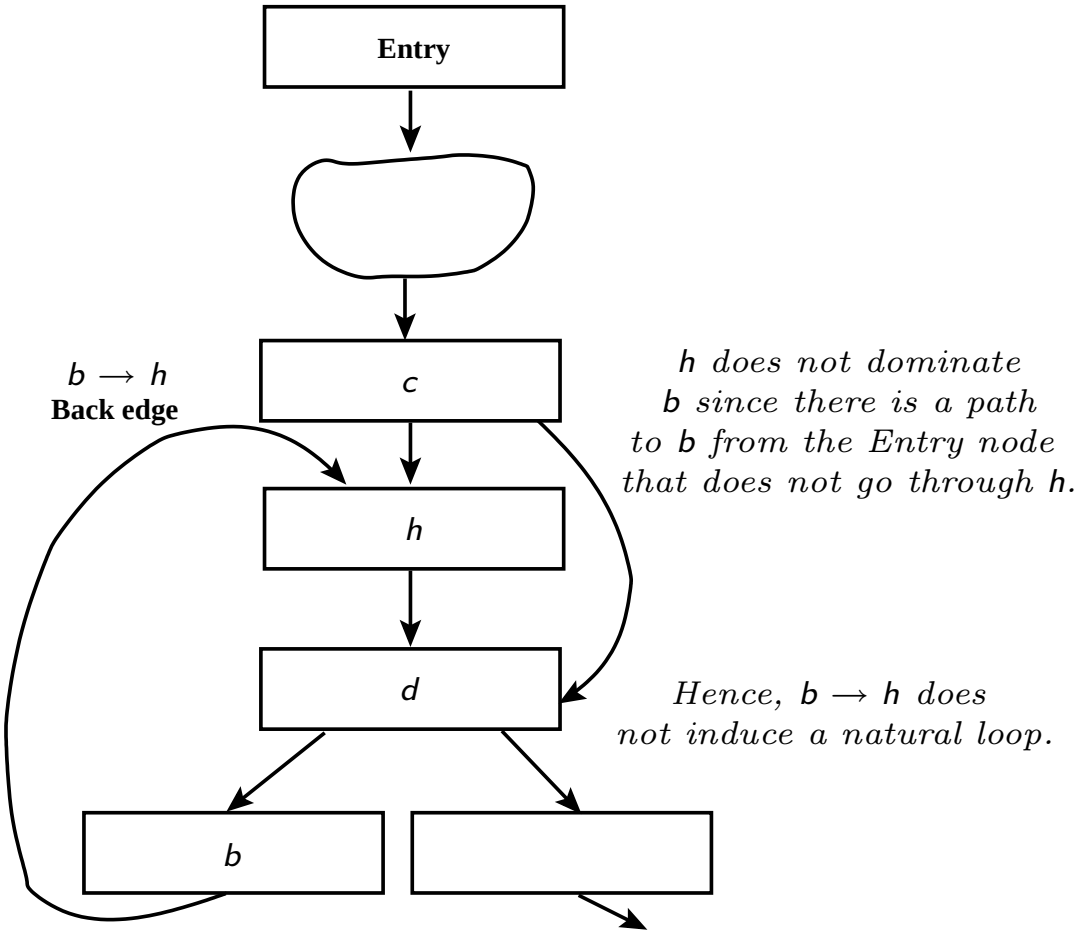
# Natural Loop

- Definition:

  A back edge $b \longrightarrow h$, where $h$ dom $b$, induces a *natural loop* consisting of all nodes $x$, where $h$ dom $x$ and there there is a path from $x$ to $b$ not containing $b$.



# Example — Not a Natural Loop



# Computing Dominators

# Dataflow Equations

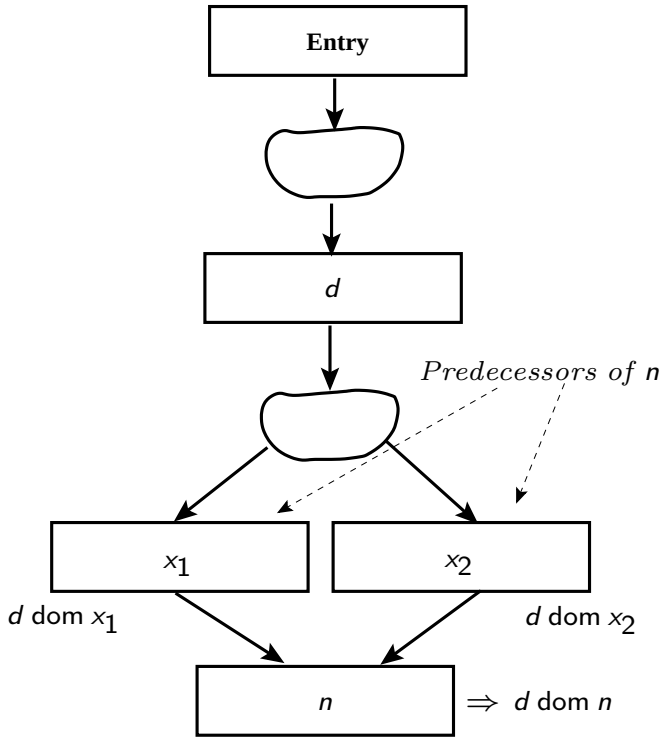
- The dominators of a node $n$ are given by

$$\begin{aligned} \mathtt{dom}(\text{entry node}) &= \{\text{entry node}\} \\ \mathtt{dom}(n) &= \{n\} \cup \Big( \bigcap_{\mathtt{preds}\ p\ \mathtt{of}\ n} \mathtt{dom}(p) \Big) \end{aligned}$$

- The dominator of the entry node is the entry node itself.
- The set of dominators for a node $n$ is the intersection of the set of dominators for all predecessors of $n$.
- $n$ is also in the set of dominators for $n$.

# Dataflow Equations — Intuition

$$\texttt{dom}(n) \;=\; \{n\} \cup \Big( \bigcap_{\texttt{preds } p \texttt{ of } n} \texttt{dom}(p) \Big)$$




- If $d$ dominates all predecessors of $n$, then it also dominates $n$

# Algorithm

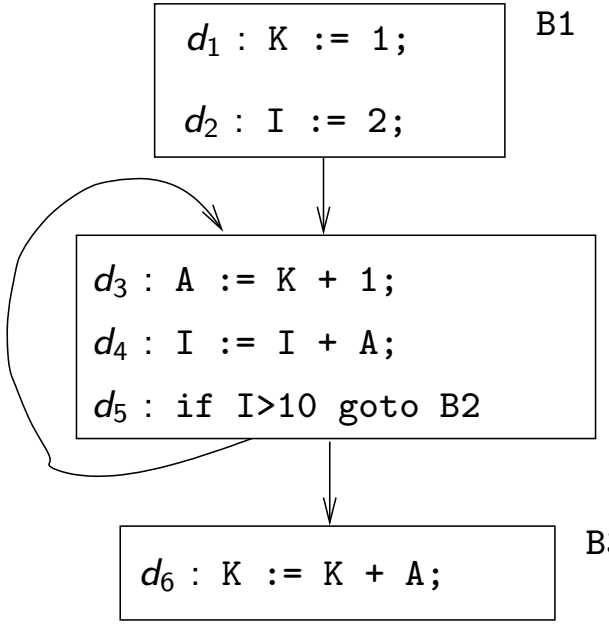
- $N$ is the set of all nodes.
- $n_0$ is the entry node.

$$
\begin{array}{l}
\texttt{dom}(n_0) \texttt{ := } \{n_0\}\texttt{;} \\
\texttt{FOR EACH } n \in N - \{n_0\} \texttt{ DO} \\
\quad \texttt{dom}(n) \texttt{ := } N\texttt{;} \\
\texttt{WHILE CHANGES IN ANY dom}(n) \texttt{ DO} \\
\quad \texttt{FOR EACH } n \in N - \{n_0\} \texttt{ DO} \\
\qquad \texttt{dom}(n) \texttt{ := } \{n\} \cup (\bigcap_{\texttt{preds } p \texttt{ of } n} \texttt{dom}(p))
\end{array}
$$

# Example 1

# Example 1 — Initialization

```
K:=1;
I:=2;
REPEAT
   A:=K+1;
   I:=I+A;
UNTIL I<=10;
K:=K+A;
```

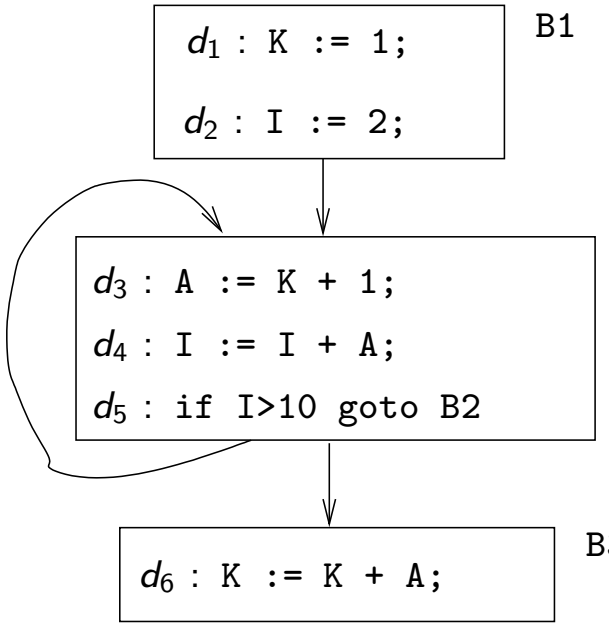



dom=$\{B_1\}$

dom=$\{B_1, B_2, B_3\}$

dom=$\{B_1, B_2, B_3\}$

# Example 1 — First Iteration




$$\text{dom}=\{B_1\}$$

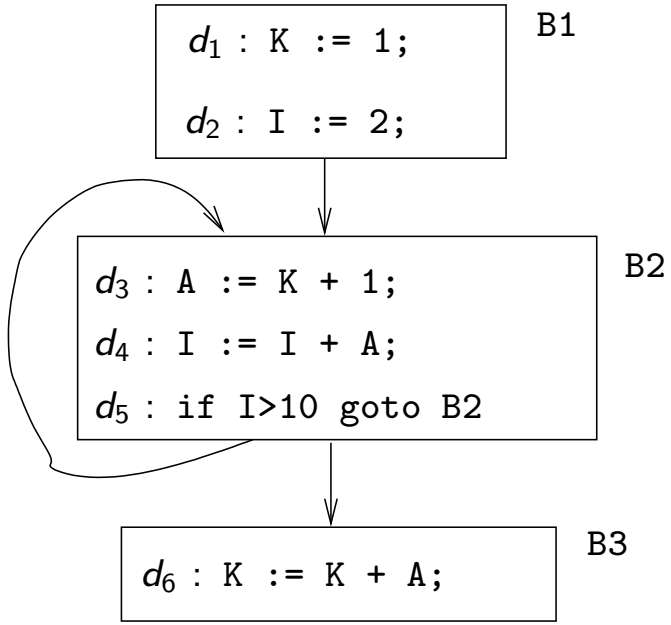
$$\begin{aligned}\text{dom}&=\{B_2\}\cup(\{B_1\}\cap\{B_1,B_2,B_3\})\\&=\{B_1,B_2\}\end{aligned}$$

$$\begin{aligned}\text{dom}&=\{B_3\}\cup(\{B_1,B_2\})\\&=\{B_1,B_2,B_3\}\end{aligned}$$

# Example 1 — Final Result

- A back edge $b \longrightarrow h$, where $h$ dom $b$, induces a *natural loop* consisting of all nodes $x$, where $h$ dom $x$ and there there is a path from $x$ to $b$ not containing $b$.

$B_2$ dom $B_2$



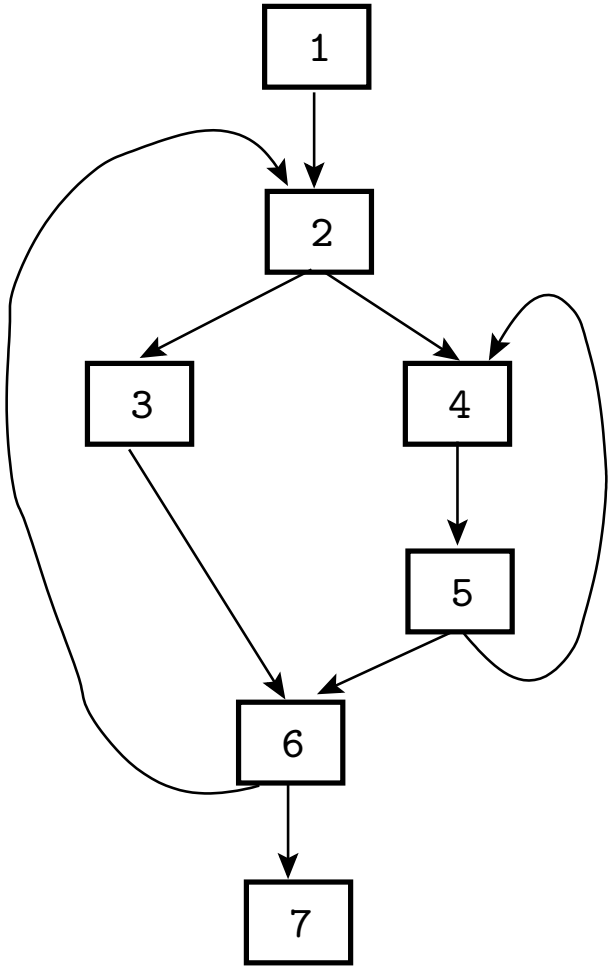

dom=$\{B_1\}$

dom=$\{B_1, B_2\}$

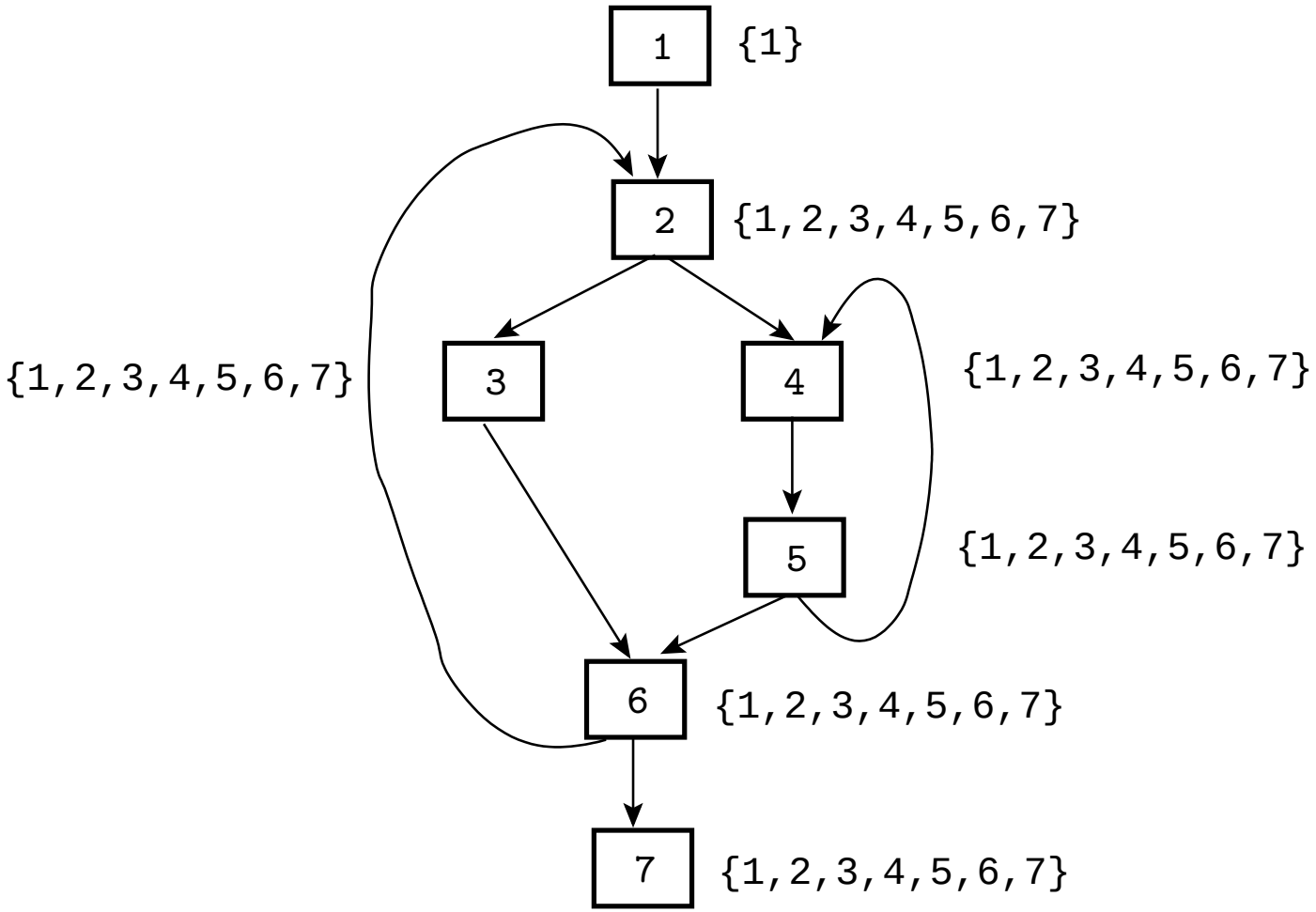
dom=$\{B_1, B_2, B_3\}$

# Example 2

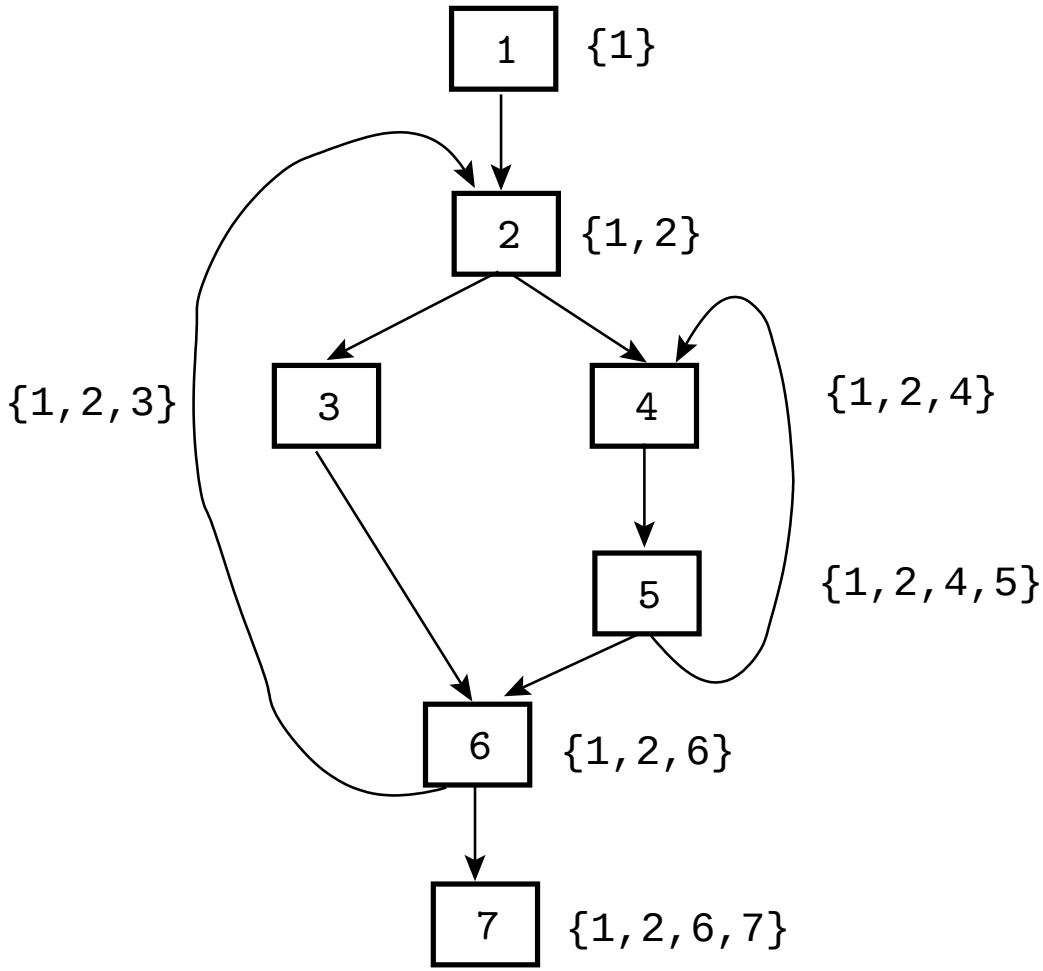
# Example 2



# Example 2 — Initialization

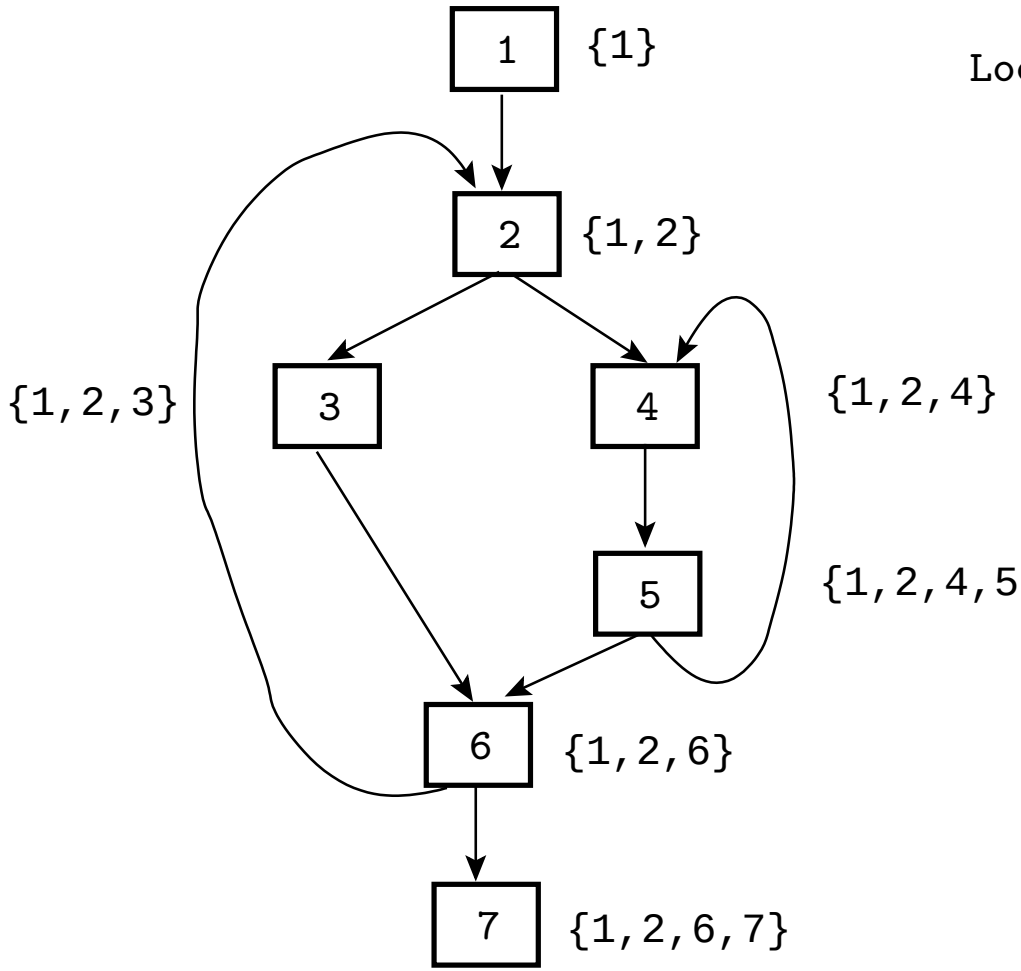


# Example 2 — First iteration



# Example 2 — Identifying loops

Back edge $b \to h$, $h \,\mathrm{dom}\, b$, induces a loop with all nodes $x$, where $h \,\mathrm{dom}\, x$ and there there is a path $x \rightsquigarrow b$ not containing $b$.




Loop 1:

Back edge: $6 \to 2$
$2 \,\mathrm{dom}\, 6$
Contains: {2,3,4,5,6}

Loop 2:
Back edge: $5 \to 4$
$4 \,\mathrm{dom}\, 5$
Contains: {4,5}

# Summary

# Summary

- Each node dominates itself.
- If $x$ dominates $y$, and $y$ dominates $z$, then $x$ dominates $z$.
- If $x$ dominates $z$ and $y$ dominates $z$, then either $x$ dominates $y$ or $y$ dominates $x$.