

# The Tiled Bitmap Forensic Analysis Algorithm

Kyriacos E. Pavlou and Richard T. Snodgrass, *Senior Member, IEEE*

**Abstract**—Tampering of a database can be detected through the use of cryptographically strong hash functions. Subsequently, applied *forensic analysis* algorithms can help determine when, what, and perhaps ultimately who and why. This paper presents a novel forensic analysis algorithm, the *Tiled Bitmap Algorithm*, which is more efficient than prior forensic analysis algorithms. It introduces the notion of a *candidate set* (all possible locations of detected tampering(s)) and provides a complete characterization of the candidate set and its cardinality. An optimal algorithm for computing the candidate set is also presented. Finally, the implementation of the Tiled Bitmap Algorithm is discussed, along with a comparison to other forensic algorithms in terms of space/time complexity and cost. An example of candidate set generation and proofs of the theorems and lemmata and of algorithm correctness can be found in the appendix, which can be found on the Computer Society Digital Library at <http://doi.ieeecomputersociety.org/10.1109/TKDE.2009.121>.

**Index Terms**—Database management, security, integrity, and protection, temporal databases.

## 1 MOTIVATION

WIDESPREAD news coverage of collusion between auditors and the companies they audit (e.g., Enron and WorldCom) helped accelerate recent passage of federal laws (e.g., Health Insurance Portability and Accountability Act: HIPAA [12], Sarbanes-Oxley Act [13]) that mandate better controls on electronic data. *Compliant records* are those required by myriad laws and regulations to follow certain “processes by which they are created, stored, accessed, maintained, and retained” [2].

We previously proposed an innovative approach in which cryptographically strong one-way hash functions allow the detection of a *corruption event* (CE), which is any event that corrupts the data and compromises the database. The corruption event could be due to an adversary, including an auditor or an employee or even an unknown bug in the software (be it the DBMS or the file system or somewhere in the operating system), or a hardware failure, either in the processor or on the disk [10]. Tamper detection is accomplished by hashing data manipulated by transactions and periodically *validating* the audit log database to detect when it has been altered. Validation involves sending the hash value computed over all the database to an external *notarization service*, which will indicate whether that value matches one previously computed. Should tampering have occurred, the two hash values will not match.

At this point, all that is known is that at some time in the past, data somewhere in the database have been tampered. *Forensic analysis* is needed to ascertain *when* the tampering occurred, and *what* data were altered. Knowing the “when” and “what” can give indirect clues to the CIO and CSO that would perhaps allow them to ultimately determine who the adversary is and why the corruption was done. The identification of the adversary is not explicitly dealt with.

- The authors are with the Department of Computer Science, University of Arizona, PO Box 210077, 1040 E. Fourth St., Tucson, AZ 85721-0077. E-mail: {kpavlou, rts}@cs.arizona.edu.

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Validation provides a single bit of information: has the database been tampered with? To provide more information about when and what, we hash the data of various sequences of transactions during validation. The database transactions are hashed in commit order creating a *hash chain*. Then, during forensic analysis of a subsequent validation that detected tampering, those chains can be rehashed to provide a sequence of truth values (success or failure), which can be used to narrow down “what.”

We have elsewhere [7] proposed the Monochromatic, RGB, and Polychromatic forensic analysis algorithms. These algorithms differ in the amount of work necessary during normal processing (computing additional hash chains during periodic validation) and the precision of the when and what estimates produced by forensic analysis. Here, we introduce a more efficient algorithm, the Tiled Bitmap Algorithm.

We first present the threat model, then the Tiled Bitmap Algorithm by way of an example. This algorithm requires what we term as the *candidate set*. We then consider the more general problem of characterizing the candidate set, which can be utilized to produce two approaches for computing that set. This is followed by an evaluation of the implemented algorithm. We end with a discussion of previous work and a summary.

## 2 PARTIES INVOLVED AND THREAT MODEL

In this section, we introduce the parties involved and the underlying threat model.

The parties involved are:

- The DBMS.
- An external digital notarization service. This is a company which can digitally notarize documents and then validate their correctness.
- The validator. This is a DBMS application which periodically contacts the digital notarization service.
- The forensic analyzer. This is a DBMS application responsible for executing the chosen forensic analysis algorithm.

Few assumptions are made about the threat model. The system is assumed to be secure until an adversary gets

TABLE 1  
Summary of Notation Used

Symbol	Name	Definition
CE	Corruption event	An event that compromises the database
$I_V$	Validation interval	The time between two successive validations
$R_s$	Spatial detection resolution	Finest granularity of a CE's spatial bounds uncertainty
$\mathcal{T}_{FVF}$	Time of first validation failure	Time instant at which the CE is first detected

access, at which point he has access to everything: the DBMS, the operating system, the hardware, and the data in the database. We still assume that the notarization and validation services remain in a trusted computing base. This can be done by making them geographically and perhaps organizationally separate from the DBMS and the database [5], thereby effecting correct tamper detection even when the tampering is done by highly motivated insiders. (A recent FBI study indicates that almost half of attacks were by insiders [1].) To prevent spoofing between the DBMS and the validator, it is possible to use a combination of Trusted Platform Modules (TPMs), mutual authentication, and a secure communication channel. The specifics of this scheme are beyond the scope of this paper.

The basic mechanism described in the next section provides correct tamper detection. If an adversary modifies even a single byte of the data or its time stamp, the independent validator will detect a mismatch with the notarized document, thereby detecting the tampering. The adversary could simply reexecute the transactions, making whatever changes he wanted, and then replace the original database with his altered one. However, the notarized documents would not match in time. Avoiding tamper detection comes down to inverting the cryptographically strong one-way hash function. An extensive presentation of the approach, performance limitations, tamper detection, threat model, and other forensic analysis algorithms can be found elsewhere [8], [10].

### 3 AN EXAMPLE

Consider a database recording when privacy release authorizations were signed by a patient (in the US, all patients are now required by HIPAA [12] to sign such authorizations). For ease of discussion, we'll use a granularity of an hour. Dr. Dan inadvertently revealed confidential health information to an insurance company on hour 30, shortly before patient Pam actually signed the authorization (on hour 31). Dr. Dan later realized his mistake, which is an offense under HIPAA and can have significant legal implications. So, on hour 51, Dr. Dan colludes with his friend the database administrator to alter the database to backdate that authorization from hour 31 to hour 28. The database now implies that authorization had been received on hour 28, just before the confidential information was transferred on hour 30: everything looks fine.

In order to ensure HIPAA compliance, the health care company that Dr. Dan works for uses a database management

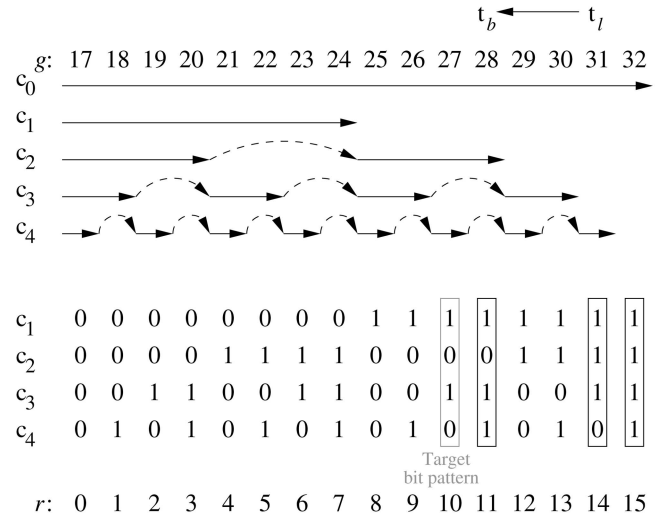


Fig. 1. The hash chains of a single (second) tile.  $t_i$  is the actual time of authorization, while  $t_b$  is the backdated authorization time. The rectangles mark the elements of the candidate set.

system incorporating tamper detection and forensic analysis. Each transaction is hashed when it is committed and linked to the previous transaction. Every 16 hours the system runs the validator, which rehashes all the transactions and compares the value with the previously notarized and stored hash value. The time interval between two successive validations is termed the *validation interval*, or  $I_V$  (see Table 1). The validator also computes partial chains that will later be useful in forensic analysis. Specifically, it computes the five hash chains shown in Fig. 1, hash chain  $c_0$  through hash chain  $c_4$ , over the previous 16 hours, storing five hash values in a secure database available only to an external digital notarization service. Each 16-hour collection of partial hash chains is termed a *tile*.

When the validator runs at hour 64, it detects the tampering. The forensic analysis algorithm springs into action. It first reports to the compliance service that the database was tampered sometime within the last 16 hours, between hour 49 and hour 64. That helps bound the "when" of the tampering. The algorithm then recomputes some of the partial hash chains on the tampered data and sends the new hash values to the notarization service, which responds with "success" (the old and new values match) or "failure," for each hash chain queried.

Specifically, the algorithm linearly scans all the tiles in the database to identify in which tile(s) the tampering occurred. The "success" and "failure" response of the  $c_0$  chain of each tile will, in this case, narrow down the tampering to the tile covering hours 17–32. Note that each tile that includes a tampering can be independently analyzed, and a corruption across tiles, say changing a time stamp from hour 31 (in the second tile) to hour 7 (in the first tile) can be analyzed by examining each tile independently. In Dr. Dan's case, the validation of the  $c_0$  chain will report a "failure," for only the 17–32 tile. This tells the algorithm that the "what" of the tampering was data stored between hours 17 and 32, a 16-hour period. However, we would like to narrow down the tampering to a much finer granularity: that of a single hour, or at least

down to a few hours. (It turns out that depending on when the corruption occurred, sometimes we can do very well and sometimes less well.)

In Fig. 1, Dr. Dan's backdating of the authorization from hour 31 to hour 28 is shown at the top, as a left-pointing arrow. The first hash chain,  $c_0$ , is also shown at the top.

The algorithm now recomputes the other four partial hash chains for this tile,  $c_1$ - $c_4$ . Four partial hash chains are used to get down to an hour granularity, given that each tile is 16 hours, which is the validation interval  $I_V$ .

The finest spatial granularity of the corrupted data would be an explicit attribute of a tuple or a particular time stamp attribute. However, this proves to be costly and hence we define  $R_s$ , the finest granularity chosen to express the uncertainty of the spatial bounds of a corruption event.  $R_s$  is called the *spatial detection resolution*.

The database administrator specifies both  $I_V$  and  $R_s$ , in this case, 16 hours and 1 hour, respectively. An  $R_s$  of 1 hour implies four other chains, excluding  $c_0$ , are needed in a tile (since  $\lg(I_V) = \log_2(16) = 4$ ). If we wanted a finer granularity of, say, 15 minutes ( $1/4$  hour), we would need an additional two chains (i.e.,  $\lg(16 \div \frac{1}{4}) = \lg(64) = 6$ ).

Hash chain  $c_1$  covers the first eight hours of the tile. Hash chain  $c_2$  covers the first four hours, then skips four hours, then covers hours 8–11. Similarly,  $c_3$  covers four two-hour periods, with embedded skips, and  $c_4$  covers every other hour. (Hash chain linking is discussed in more detail elsewhere [10].)

Changing the time stamp on an authorization is equivalent to removing that authorization from all hash chains that cover the original time and adding that authorization to all hash chains that cover the inserted time. Examination of Fig. 1 will explain why hour 28, in which the authorization was added by Dr. Dan, appears in hash chains  $c_0$  and  $c_2$ . Hour 31, from which the authorization was removed by Dr. Dan, appears in hash chains  $c_0$  and  $c_4$ . Hence,  $c_1$  and  $c_3$  report "success" and  $c_0$ ,  $c_2$ , and  $c_4$  report "fail."

We can assemble the success and failure results for the four hash chains  $c_1$ - $c_4$  into a 4-bit binary number, with failure denoted with "0" and success with "1." The number that results from this particular backdating from hour 31 to hour 28 is 1010. We term this value the *target binary number* or *target*. The target is the input to the forensic analysis. Our task is to take this binary number, the target, and figure out what could have happened.

The truth values shown at the bottom of the figure indicate the target string that would result had the corruption event tampered with data stored at the indicated hour. For example, changing the data of a tuple that were originally stored in the first hour of this interval would have rendered all of the chains as failure, resulting in a value of 0000.

Recall that our corruption event occurred at hour 51, changing a time stamp from 31 to 28, the hash chains provide a target of 1010. What could such a target indicate? It could indicate the corruption of data during a single hour, or any combination of time stamp and/or data during two or more separate hours such that the resulting target after validation is equal to 1010. For example, one possibility is that only the data in hour 27 ( $r = 10$ ) was modified. Another is that the time stamp was moved from

31 ( $r = 14 = 1110_2$ ) to 28 ( $r = 11 = 1011_2$ ), again yielding a target 1010. This is in fact what happened. A different possibility is that the data in hours 28 and 31 were corrupted independently. A fourth possibility is that the time was moved from 28 to 31. Other possibilities are a change from hour 27 to hour 31, a change from hour 27 to hour 32, a change from hour 27 to hour 28, or a change in the other direction. All these possibilities and many others result in a target of 1010. Precisely because this list of possibilities can get quite long, we introduce in the next section the notion of candidate set which retains comprehensiveness but is a lot simpler.

There are two special cases worth discussing separately. If target bit pattern is 1111 then it is not the case that no corruption has happened. To begin with, we are *certain* that there is corruption in this tile because  $c_0$  reported "failure." The only thing that pattern 1111 implies is that no corruption has occurred in the time granules covered by hash chains  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$ . The only granule *not* covered by these last four hash chains is the last granule (15) so we can conclude (by eliminating all other possibilities) that the corruption must have been located in time granule 15. This is the only case where we are certain that there are no false positives.

On the other hand, if the target bit pattern is 0000 then a corruption event can be anywhere in the tile's 16 time granules. Even though this does not affect correctness, this is the worst case scenario where we could potentially have the maximum number of false positives.

## 4 THE TILED BITMAP ALGORITHM

We formally define the problem as follows: *Problem Definition: The task is to compute from a single target all the possible corruption events, which we term the candidate set.*

For the example in the previous section, the candidate set would comprise the hours {27, 28, 31, 32}. We now present an algorithm to do this.

In the algorithm shown in Fig. 2,  $I_V$  is the number of hours between validations (in the example,  $I_V = 16$ ). We use a helper function called `val_check`. This function takes a hash chain as a parameter and returns the boolean result of the validation of that chain.

The partial hash chains within a tile are denoted by  $c_0(\tau), c_1(\tau), \dots, c_{\lg(I_V)}(\tau)$ , with  $c_i(\tau)$  denoting the  $i$ th hash chain of the tile which starts at time instant  $\tau$ . On line 4, the algorithm iterates through the different tiles and checks if the longest partial chain  $c_0(\tau)$  evaluates to FALSE. If not, it moves on to the next tile. If the chain evaluates to FALSE (line 5), the algorithm iterates through the rest of the partial chains in the tile (line 7) and "concatenates" the result of each validation to form the target number (line 8). Then, the `candidateSet` function is called (line 9) to compute all the candidate set elements from the target number according to the user-specified parameter  $k$  discussed in the next section. On lines 10–12, the candidate granules are renumbered to reflect their global position. The function `renumber()` on line 11 uses  $R_s$  to find the global position of  $r$ , computing  $g$  as a single granule, or group of granules if  $R_s > 1$ . Once the  $C_{set}$  is reported, the CSO can exactly pinpoint the corrupted tuples and can thus weed out the false positives. In order to

```

// input:  $\tau_{FVF}$  is the time of first validation failure
//        $I_V$  is the validation interval
//        $k$  is used for the creation of  $C_{t,k}$ 
//        $R_s$  is the spatial detection resolution
// output:  $C_{set}$ , an array of binary numbers
function Tiled_Bitmap( $\tau_{FVF}$ ,  $I_V$ ,  $k$ ,  $R_s$ )
1:  $t \leftarrow 0$  // the target
2:  $C_{set} \leftarrow C_{temp} \leftarrow \emptyset$ 
3:  $\tau \leftarrow 1$ 
4: while  $\tau < \tau_{FVF}$  do
5:   if  $\neg \text{val\_check}(c_0(\tau))$  then
6:      $n \leftarrow \lg(I_V/R_s)$ 
7:     for  $i \leftarrow n$  to 1
8:        $t \leftarrow t + 2^{n-i} \cdot \text{val\_check}(c_i(\tau))$ 
9:        $C_{temp} \leftarrow \text{candidateSet}(t, n, k)$ 
10:      for each  $r \in C_{temp}$ 
11:         $g \leftarrow \text{renumber}(r, \tau, R_s)$ 
12:         $C_{set} \leftarrow C_{set} \cup \{g\}$ 
13:       $\tau \leftarrow \tau + I_V$ 
14: return  $C_{set}$ 

```

Fig. 2. The tiled bitmap algorithm.

achieve this, he must compare the data stored in the backup tapes with the data contained in the granules.

We now state the running time of the Tiled Bitmap Algorithm. Let  $D$  be the number of granules (hours) before the first validation failure (for the example,  $D = 64$ ). The “while” loop on line 4 takes  $\lceil D/I_V \rceil$  in the worst case. In reality, because of the “if” statement on line 5 the body of the loop gets executed only if corruption is initially detected by using  $c_0(\tau)$ . Hence, the actual number of times the loop is executed is  $\Theta(F)$  where  $F$  is the number of times the validation of a  $c_0(\tau)$  chain fails. The “for” loop on line 7 takes  $\lg(I_V/R_s)$  while the candidateSet function takes  $\Omega(\lg(I_V/R_s) + 2^{z(t)})$ , where  $z(t)$  is the number of zeros in the target binary number  $t$  (see Section 5). The loop on line 10 takes  $\Theta(2^{z(t)})$ . Hence, the runtime of this algorithm is

$$\begin{aligned}
& \Omega(F \cdot (\lg(I_V/R_s) + (\lg(I_V/R_s) + 2^{z(t)}) + 2^{z(t)})) \\
&= \Omega(F \cdot (\lg(I_V/R_s) + 2^{z(t)})) \\
&= O((D/I_V) \cdot (\lg(I_V/R_s) + 2^{z(t)})) \\
&= O((D \cdot \lg(I_V/R_s))/I_V + D),
\end{aligned}$$

given that in the worst case  $F$  takes the value  $(D/I_V)$ , which is the total number of tiles, and  $2^{z(t)}$  takes the value  $I_V$ , which the total number of granules in a tile.

There is one important aspect left unaddressed in the above algorithm: the candidateSet function. But before we can present this latter algorithm, we must formally characterize the candidate set.

## 5 CHARACTERIZING THE CANDIDATE SET

In the forensic analysis context, the parameter  $k$  passed to the algorithm represents the *actual* number of granules corrupted. In the example shown in Fig. 1,  $k = 2$ . However,

we usually have no knowledge of the value of  $k$ . What we have is only the *target* from which we have to find the possible bit patterns (each generated by the validation of the chains in the tile assuming that corruption *occurred by itself*) that when bitwise *ANDed* produce the *target*. The reason for requiring that the different bit patterns produce the *target* when *ANDed* is because this is effectively what happens when the corruptions occur *simultaneously* within a tile. This arises from the mechanics of forensic analysis. Specifically, each corruption event renders some of the chains as “failing.” A chain will succeed in the end only if it succeeds in every one of the corruption events. So, in the example, chains  $c_1$  and  $c_3$  succeed, but chains  $c_2$  and  $c_4$  fail. The set of all such bit patterns which could produce the *target* when *ANDed* is termed the *candidate set*. In order to be more rigorous in our analysis, we proceed to give a formal characterization of the candidate set.

We define the length  $l$  of a binary number  $b$ , denoted by  $|b| = l$ , as the number of its digits. From this point forward we consider  $l$  to be fixed. candidateSet essentially “sums up” the preimages of all the binary numbers of length  $l$ ,  $\mathbb{B} = \{b : |b| = l\}$ , under a family of bitwise *AND* functions whose domain is a finite Cartesian product.

$$AND_k : \mathbb{B}^k \rightarrow \mathbb{B}$$

$$AND_k((b_1, b_2, \dots, b_k)) = b_1 \wedge b_2 \wedge \dots \wedge b_k$$

Observe that the maximum number  $k$  of sets participating in the Cartesian product is  $2^l$  (i.e., every granule in the tile is corrupted), since if  $k$  is allowed to take a value beyond that, it will force a repetition of one of the binary numbers. For forensic analysis purposes this implies that the same granule has been corrupted more than once. This is not informative or useful in any way since repeated *ANDing* operations with the *same* binary number leave the result invariant (the operation is *idempotent*). This is also compatible with forensic analysis since we only care if a granule is corrupted or not—if we wanted to know more we would need to increase the resolution by choosing a smaller granule size (i.e., smaller  $R_s$ ). In other words, repetition is not allowed and hence for a given  $k$ -tuple all its components are distinct. Also note that the value of  $k$  uniquely identifies a specific  $AND_k$  function in the above family.

We formally define the set of all binary numbers which appear as components in at least one of the preimages (i.e.,  $k$ -tuples) of a specific target binary number  $t$  the *candidate set*:

$$\begin{aligned}
C_{t,k} &= \{b \in \mathbb{B} : \exists b_1, b_2, \dots, b_{k-1} \in \mathbb{B} \text{ s.t.} \\
& \quad AND_k((b, b_1, \dots, b_{k-1})) = t\}.
\end{aligned}$$

The  $\wedge$  operation is commutative: the order of the operands does not matter, and that is why this can be defined more simply as a set of booleans rather than as a set of  $k$ -tuples of booleans. The word “candidate” was used to name this set because in forensic analysis, its elements correspond bijectively to the granules (in the example, the hours indicated in Fig. 1), which are candidates where corruption may potentially have occurred. In Dr. Dan’s case, the candidate set would be the hours 27, 28, 31, and 32 that is,  $r = 10$ ,  $r = 11$ ,  $r = 14$ , and  $r = 15$ .

Observe that, it is not the case that  $|C_{t,k}| = k$ , i.e.,  $k$  is *not* the cardinality of the candidate set. The cardinality in the example is 4: in this case, the algorithm can narrow down the possibilities only to four granules, two actual ones ( $k = 2$ ) and two false positives. The candidate set will comprise *all possible* binary numbers that could produce the target bit pattern, and not just the granules corrupted in a specific case. Hence, the candidate set will always include the actual  $k$  granules that were corrupted together with other potentially corrupted granules. This ensures correctness but allows for the existence of false positives.

For convenience, we can express these sets in decimal, though our algorithms read and write in binary. For example:  $C_{1010,1} = \{1010\} = \{10\}$ ,  $C_{1010,2} = \{1010, 1011, 1110, 1111\} = \{10, 11, 14, 15\}$ . 1001 is not in  $C_{1010,2}$  because 1001 cannot be in the preimage of 1010. Note that even though two binary target strings may have the same numerical value, if their length is different, then their candidate sets will be different. For example, the candidate set  $C_{000,2}$  is different from  $C_{0000,2}$ .

Let  $z(t)$  be the number of zeros in the binary number  $t$ , e.g.,  $z(1010) = 2$ . By definition  $1 \leq k \leq 2^l$  and  $0 \leq z \leq l$ . The behavior of  $C_{t,k}$  is interesting: as  $k$  increases the candidate set for a fixed  $t$  remains invariant and equal to the candidate set for  $k = 2$ , until some threshold value  $2^{z(t)}$  after which it becomes empty. Simply put,  $C_{t,k}$  obeys an all-or-none law.

**Lemma 1.**  $C_{t,k} = C_{t,2}$  if  $l \geq z(t) > 0$  and  $2 \leq k \leq 2^{z(t)}$ . In other words, the candidate set remains invariant given that the stated conditions are met.

**Proof.** Given in Appendix A, which can be found on the Computer Society Digital Library at <http://doi.ieeeecomputersociety.org/10.1109/TKDE.2009.121>.  $\square$

A complete characterization of the candidate sets is given below.

**Theorem 1.**

$$C_{t,k} = \begin{cases} \{t\}, & k = 1, & (1) \\ \emptyset, & z(t) = 0 \wedge k > 1, & (2) \\ C_{t,2} \neq \emptyset, & l \geq z(t) > 0 \wedge 2 \leq k \leq 2^{z(t)}, & (3) \\ \emptyset, & l \geq z(t) > 0 \wedge k > 2^{z(t)}. & (4) \end{cases}$$

**Proof.** Given in Appendix B, which can be found on the Computer Society Digital Library at <http://doi.ieeeecomputersociety.org/10.1109/TKDE.2009.121>.  $\square$

Theorem 1 is the reason for which we decided to make  $k$  user-configurable in the Tiled Bitmap pseudocode. If the CSO by some other means has any indication for the value of  $k$ , i.e., the actual number of corruptions occurred, then he can pass that information to the algorithm. If the algorithm returns an empty candidate set then the CSO can deduce that his initial knowledge/guess for the value of  $k$  was incorrect. If the CSO has no a priori knowledge about the value of  $k$ , as is usually the case, then according to Theorem 1 the CSO need only give  $k$  the default value of 2 and not worry that any other choice for  $k$  would compromise the forensic analysis results.

```

// input: t is a binary target number
//        l is the length of t
//        k is a function index for ANDk
// output: Ct,k is an array of binary numbers
//        (also created is an array of zeros, Z)
1: function candidateSet(unsigned int t, int l, int k)
2:   Ct,k ← new array()
3:   z ← 0
4:   Z ← new array()
5:   for i ← l - 1 to 0
6:     if t && (1 << i) = 0 then z ← z + 1
7:     Z[l - i - 1] ← z
8:   if k < 1 ∨ k > 2l then report NOT_DEFINED
9:   else if k = 1 then Ct,k ← {t}
10:  else if (z = 0 ∧ k > 1) ∨ (l ≥ z > 0 ∧ k > 2z)
11:    then Ct,k ← ∅
12:  else if (l ≥ z > 0) ∧ (2 ≤ k ≤ 2z) then
13:    rightmost ← createRightmost(t, l)
14:    Ct,k ← generate(t, rightmost[l], l, Ct,k)
15:    Ct,k ← funkySort(z, Ct,k)
16:  return Ct,k

```

Fig. 3. The candidateSet function.

**Corollary 1.**

$$|C_{t,k}| = \begin{cases} 1, & k = 1, \\ 0, & z(t) = 0 \wedge k > 1, \\ 2^{z(t)}, & l \geq z(t) > 0 \wedge 2 \leq k \leq 2^{z(t)}, \\ 0, & l \geq z(t) > 0 \wedge k > 2^{z(t)}. \end{cases}$$

**Proof.** This follows directly from Theorem 1.  $\square$

For example, with our target bit pattern of  $t = 1010$ , we have  $z(t) = 2$  and the candidate set is  $C_{1010,2} = \{10, 11, 14, 15\}$  with  $|C_{1010,2}| = 2^2 = 4$ .

We now turn to ways in which the candidate set may be computed. We first give an algorithm that is optimal in time, except for a very few cases. Following some further observations on candidate sets, we show how, given a candidate set, one can calculate other candidate sets with a smaller  $l$  in constant time.

## 6 COMPUTING THE CANDIDATE SET

Figs. 3, 4, 5, and 6 present an optimal algorithm for computing the candidate set given the target string  $t$  and  $k$ , and again assuming a fixed  $l$ . Recall that candidateSet is used in the Tiled Bitmap Algorithm. It generates the elements (bit patterns) in the candidate set from the target pattern  $t$  preserving bit positions with 1s and creating combinations of patterns of 1s and 0s using the remaining positions having 0s. Finally, it sorts the patterns in ascending order of their numeric values by using an interesting linear-time sort. (An example of the candidate set generation for our target of  $t = 1010$  can be found in Appendix G, which can be found on the Computer Society

```

// input:  $t$  is the target bit number
//        $l$  is the length of the bit representation of  $t$ 
// output: the populated rightmost array
1: function createRightmost(unsigned int  $t$ , int  $l$ )
2:   int  $i, j, flag$ 
3:    $j \leftarrow -1$ 
4:    $flag \leftarrow \text{FALSE}$ 
5:   rightmost  $\leftarrow$  new array()
6:   for  $i \leftarrow l - 1$  to  $-1$ 
7:     if  $flag$  then  $j \leftarrow l - i - 2$ 
8:     if  $t \& (1 \ll i) = 0$  then  $flag \leftarrow \text{TRUE}$ 
9:     else  $flag \leftarrow \text{FALSE}$ 
10:    rightmost[ $l - i - 1$ ]  $\leftarrow j$ 
11:  return rightmost

```

Fig. 4. The createRightmost function.

Digital Library at <http://doi.ieeecomputersociety.org/10.1109/TKDE.2009.121>.) All arrays and strings use zero-based indexing. All parameters are passed by value.

Let us now briefly examine this algorithm. We first start by looking at the candidateSet function (Fig. 3) and discuss each different function as we encounter it.

The use of the  $Z$  array on lines 4 and 7 will be explained later in the discussion following Theorem 2. Lines 8–12 follow the result of Theorem 1. Then, on line 13 the createRightmost helper function is called (Fig. 4) to preprocess the target binary number  $t$  and to fill the *rightmost* array in order to answer the “rightmost zero” query in constant time. More specifically, *rightmost*[ $p$ ] is the index (bit position) of the rightmost zero to the left of index  $p$  noninclusive. Within this function  $i$  iterates over  $t$  from left to right (high-order to low-order bits). The flag is required because we must remember what we saw in the previous iteration: if  $flag = \text{TRUE}$  we saw a 0, otherwise we saw a 1. This runs in  $\Theta(l)$ .

On line 14 (Fig. 3), the generate function (Fig. 5) is called. This is a recursive function which creates the candidate set

```

// input:  $t$  is the modified bit pattern at each stage
//       of the recursion
//        $p$  is the position of one of the zeros in  $t$ 
//        $l$  is the length of the bit representation of  $t$ 
//        $C_{t,k}$  array in which candidate granules
//       are accumulated
// output:  $C_{t,k}$  contains candidate granules (unsorted)
1: function generate(unsigned int  $t$ , int  $p$ , int  $l$ ,
                    array  $C_{t,k}$ )
2:   if  $p = -1$  then  $C_{t,k}.\text{append}(t)$ 
3:   else
4:      $C_{t,k} \leftarrow \text{generate}(t, \text{rightmost}[p], l, C_{t,k})$ 
5:      $C_{t,k} \leftarrow \text{generate}(t + (1 \ll (l - p - 1)),
                             \text{rightmost}[p], l, C_{t,k})$ 
6:   return  $C_{t,k}$ 

```

Fig. 5. The generate function.

```

// input:  $z$  is the number of zeros in  $t$ 
//        $C_{t,k}$  is the result of generate()
// output:  $C_{t,k}$  sorted in ascending order
1: function funkySort(int  $z$ , array  $C_{t,k}$ )
2:   sorted  $\leftarrow$  new array()
3:   indices  $\leftarrow$  new array()
4:   indices[0]  $\leftarrow$  0
5:   int  $i, offset, power$ 
6:    $offset \leftarrow 0$ 
7:    $power \leftarrow 1 \ll z$ 
8:   for  $i \leftarrow 1$  to  $(1 \ll z) - 1$ 
9:     if  $(i \& (i - 1)) = 0$  then
10:       $power \leftarrow power \gg 1$ 
11:       $offset \leftarrow 0$ 
12:      indices[ $i$ ]  $\leftarrow$  indices[ $offset$ ] +  $power$ 
13:       $offset \leftarrow offset + 1$ 
14:   for  $i \leftarrow 0$  to  $(1 \ll z) - 1$ 
15:     sorted[ $i$ ]  $\leftarrow C_{t,k}.\text{get}(\text{indices}[i])$ 
16:   return sorted

```

Fig. 6. The funkySort function.

elements. Given a position  $p$ , which is a specific index in the zero-based enumeration (left to right) of the binary number  $t$ , it finds the index of the rightmost zero to the left of  $p$  using the *rightmost* array. It first recurses on that index maintaining the same binary number (line 4) and then sets the digit at position  $p$  to 1 and recurses on the same index *rightmost*[ $p$ ] but with this new number (line 5). We can consider the input target string  $t$  as capturing all the  $2^{z(t)}$  numbers that must be generated during the recursion, so we can consider the input size to be  $n = 2^{z(t)}$ . Also, at each recursive call the position of the zero processed is never revisited so the input size at each call is essentially halved. Moreover, the amount of work done at each stage of the recursion is constant hence the formula that captures this recursion is  $T(n) = 2T(\frac{n}{2}) + \Theta(1)$ . The solution of this formula is  $\Theta(n)$  so the running time of the generate function is  $\Theta(2^{z(t)})$ . However, a side effect of this recursive creation of the candidate set elements is that the elements are not generated in numeric order.

On line 15 of candidateSet (Fig. 3), we call the sorting function. Even though the elements are not sorted there does exist a pattern in the order in which they are created. This funkySort function (Fig. 6) creates the sequence of indices which when used to index into the  $C_{t,k}$  array will result in the ordering of the candidate set elements. This is achieved by performing a single pass over the *indices* array and creating each new index by manipulating appropriately previous ones (lines 8–13) within the funkySort function.

For example, with a binary target of  $t = 10000$ , i.e., 16 in decimal, after the generate function finishes the candidate set will be  $C_{t,k} = \{16, 24, 20, 28, 18, 26, 22, 30, 17, 25, 21, 29, 19, 27, 23, 31\}$  in this order. Examining closely the set we see that in order to create the *sorted* array we must recursively visit the first element of each subsequent half

TABLE 2  
Candidate Sets for Targets  $|t| = 4$  with  $k = 2$

Binary Number $t$	$ C_{t,2} $	$C_{t,2}$
0000	16	{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
0001	8	{1, 3, 5, 7, 9, 11, 13, 15}
0010	8	{2, 3, 6, 7, 10, 11, 14, 15}
0011	4	{3, 7, 11, 15}
0100	8	{4, 5, 6, 7, 12, 13, 14, 15}
0101	4	{5, 7, 13, 15}
0110	4	{6, 7, 14, 15}
0111	2	{7, 15}
1000	8	{8, 9, 10, 11, 12, 13, 14, 15}
1001	4	{9, 11, 13, 15}
1010	4	{10, 11, 14, 15}
1011	2	{11, 15}
1100	4	{12, 13, 14, 15}
1101	2	{13, 15}
1110	2	{14, 15}
1111	0	$\emptyset$

of  $C_{t,k}$ . Line 12 creates this sequence of indices: 0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15. More specifically, by starting from 0 the 8 can be created by  $0 + 2^{z(t)-1}$  where  $z(t) = 4$ . Then, 4 and 12 can be obtained by adding  $2^2$  to each of 0 and 8. Then 2, 10, 6, and 14 are obtained by adding  $2^1$  to 0, 8, 4, and 12, respectively. Finally, the last eight numbers are obtained by adding  $2^0$  to the first eight numbers. This explains why at elements appearing at indices which are powers of 2 in the *indices* array, the *offset* is reset to zero and the *power* is halved. On lines 14-15 (Fig. 6), we use the sequence of indices we created and the actual sorting happens. This pass over the *indices* array runs in  $\Theta(2^{z(t)})$ .

The running time of candidateSet is  $\Theta(l + 2^{z(t)})$ . Thus, the algorithm is optimal most of the time, using the lower bound given in Section 5, except for the very few cases when  $l > 2^{z(t)}$ . In terms of space complexity, the algorithm given here requires  $O(l + 2^{z(t)})$  space.

## 7 AN OPTIMAL CANDIDATE SET ALGORITHM, GIVEN A SUMMARY SET

We define the *summary set* as the set of all candidate sets of all binary numbers of length  $l$ .

$$S_{l,k} = \{C_{t,k} : \forall t \in \mathbb{B} \text{ s.t. } |t| = l\}$$

For  $l = 4$  and  $k = 2$ , the last column in Table 2 provides the elements of  $S_{4,2}$ .

We now show that for fixed  $k$  and given  $S_{l,k}$  one can calculate all  $S_{l',k}$  s.t.  $l' < l$  without resorting to the algorithm given previously. This allows us to find the candidate set for a suffix of  $y$  whenever we already have the candidate set for  $y$ . The technique shown below can potentially be faster. We define the *candidate array*, denoted  $A_{t,k}$ , to be an array which contains the elements of  $C_{t,k}$  sorted in ascending numerical value. Then,  $A_{t,k}[x : y]$  selects all elements in the candidate array from index  $x$  to  $y$ . (NB:  $A_{t,k}[i] = A_{t,k}[i : i]$ )

Also, for reasons of ease and precision, we annotate  $A$  with the length of the binary number whose value was previously implicit, as a leading subscript.

Given a candidate array  ${}_l A_{y,k}$  for a specific target string  $y$ , we wish to compute the candidate array  ${}_{l-x} A_{t,k}$  where  $t$  is a suffix ( $l = |y| > l - x = |t| \geq 1$ ) of  $y$ . Each  $S_{l,k}$  captures all the candidate sets for all  $l' < l$ . This method creates each element of  $S_{l',k}$  by exploiting the fact that each of the binary numbers of length  $l'$  is a suffix of more than one corresponding binary number of length  $l$ . For example, the candidate set  $C_{1010,2}$  can be computed from the candidate sets of 01010, 001010, 101010, and so on. Let  $y = p \bullet t = \{0, 1\}^x t$  for some prefix  $p$  of length  $x$ . Let  $Suffix_i(s)$  denote the suffix of string  $s$  starting at position  $i$ .

Let us look at some examples to develop some intuition. Given  ${}_4 A_{0010,2} = [0010, 0011, 0110, 0111, 1010, 1011, 1110, 1111]$ , we wish to compute  ${}_3 A_{010,2}$ . Observe that  $t = 010$  is  $y = 0010$  with the leftmost "0" removed. Removing the leading "0" from  $y$  results in a string  $t$  which cannot encode any numbers in the range  $2^3$  to  $2^4 - 1$ . Thus the candidate array of  ${}_3 A_{010,2}$  will have the same elements as the candidate array of  ${}_4 A_{0110,2}$  except for the numbers encoded by the extra leading digit. We know that each additional "0" present in the target string doubles the cardinality of the candidate set, thus a removal of the zero will halve the number of candidate set elements. Observe also that the elements in the second half of  ${}_4 A_{0010,2}$  have essentially the same bit pattern as the elements in the first half but with a "1" at the leftmost position instead of a "0," e.g., 1110 has the same bit pattern as 0110 apart from the bit in the leftmost position.

Thus, in order to compute  ${}_3 A_{010,2}$  we can truncate the leftmost digit from all the elements in the original candidate set. By removing the leftmost digit from each of the elements in  ${}_4 A_{0010,2}$ , we get 010, 011, 110, 111, 010, 011, 110, 111. The first half of the elements will have a leading "0" removed, something which will not change their numerical value, while the second half which will have a leading "1" removed will produce identical numbers of length 3 to the truncated numbers in the first half. Since the cardinality of  ${}_3 A_{010,2}$  is half that of  ${}_4 A_{0010,2}$ , and since the two halves of  ${}_4 A_{0010,2}$  have the same elements after the truncation and by knowing that  ${}_3 A_{010,2} = [010, 011, 110, 111]$  we can verify that:

$$\begin{aligned} {}_3 A_{010,2} &= [Suffix_1({}_4 A_{0010,2}[0]), Suffix_1({}_4 A_{0010,2}[1]), \\ &\quad Suffix_1({}_4 A_{0010,2}[2]), Suffix_1({}_4 A_{0010,2}[3])] \\ &= [010, 011, 110, 111] = [2, 3, 6, 7]. \end{aligned}$$

Let us consider a different example with the original target string being  $y = 1010$  and the same suffix  $t = 010$  as before. In this case  ${}_4 A_{1010,2} = [1010, 1011, 1110, 1111]$  all elements necessarily start with a "1." Since removing the leading "1" from  $y$  to get  $t$  does not affect the number of zeros in the strings, the cardinalities of the two candidate sets is the same. Removing the leftmost "1" from all the elements of  ${}_4 A_{1010,2}$  will yield directly the desired elements of the new candidate set:

$$\begin{aligned} {}_3 A_{010,2} &= [Suffix_1({}_4 A_{1010,2}[0]), Suffix_1({}_4 A_{1010,2}[1]), \\ &\quad Suffix_1({}_4 A_{1010,2}[2]), Suffix_1({}_4 A_{1010,2}[3])] \\ &= [010, 011, 110, 111] = [2, 3, 6, 7]. \end{aligned}$$

```

// input: t is a binary target number
//        l is the length of t
//        k is a function index for ANDk
// output: Ct,k, a candidate set either computed
//         anew or derived from Cy,k
1: function candidateSetCached(unsigned int t,
                             int l, int k)
2: tstart ← -1
3: for i ← 0 to cache.length-1
4:   tstart ← findSuffix(cache[i].y, t)
5:   if tstart ≥ 0 ∧ k = cache[i].k then
6:     return candidateSetSuffix (cache[i].Cy,k,
                                tstart, k, cache[i].ly, cache[i].Z)
7: return candidateSet (t, l, k)

```

Fig. 7. The candidateSetCached function.

With these valuable observations, we can now state the theorem.

**Theorem 2.** Assume  $y = p \bullet t = \{0, 1\}^x t$ ,  $0 < x < l$ ,  $0 \leq z(t) \leq l - x$  and  $q = 2^{z(t)}$ . Then:

$$l-x A_{t,k} = \begin{cases} \text{N/A,} & k > 2^{l-x}, & (1) \\ [t], & k = 1, & (2) \\ \emptyset, & z(t) = 0 \wedge 1 < k \leq 2^{l-x}, & (3) \\ \bigcup_{0 \leq i < q} [\text{Suffix}_x(t, A_{y,2^i}[i])], & l-x \geq z(t) > 0 \wedge 2 \leq k \leq q, & (4) \\ \emptyset, & l-x \geq z(t) > 0 \wedge k > q. & (5) \end{cases}$$

**Proof.** Given in Appendix D, which can be found on the Computer Society Digital Library at <http://doi.ieeeecomputersociety.org/10.1109/TKDE.2009.121>.  $\square$

The strategy for computing the candidate sets using this new method is given below. First, we change line 9 of the original Tiled\_Bitmap function in Fig. 2

9:  $C_{temp} \leftarrow \text{candidateSet}(target, n, k)$

to

9:  $C_{temp} \leftarrow \text{candidateSetCached}(target, n, k)$

We introduce a list,  $cache[ ]$  of records  $(y, l_y, k, C_{y,k}, Z)$  which is updated with every call to the function candidateSet. Each such record stores the candidate set  $C_{y,k}$  computed by the function, the target number  $y$ , the length  $l_y$  of  $y$ , the parameter  $k$ , and the corresponding  $Z$  array for  $y$ . In order to achieve this, we change the candidateSet function in Fig. 3 to store the candidate set in the cache, before returning it.

```

15:   Ct,k ← funkySort(z, Ct,k)
16:   append(cache, (t, lt, k, Ct,k, Z))
17:   return Ct,k

```

A new function candidateSetCached (Fig. 7) checks to see if a precomputed candidate set which can be used by this new algorithm, already exists.

Note that  $t_{start}$  is the index in the original string  $y$  where the suffix  $t$  starts. The running time of the candidateSetCached function is  $O(l \cdot \text{cache.length})$ , which

```

// input: Cy,k is the original candidate set
//        tstart is the bit position at which suffix t
//        starts in y
//        ly is the length of original target string y
//        Z array for y
// output: the candidate set Ct,k
1: function candidateSetSuffix (array Cy,k,
                              int tstart, int k,
                              int ly, array Z)
2:   Ct,k ← new array()
3:   lt ← ly - tstart
4:   zt ← zy - Z[tstart - 1]
5:   mask ← (1 << lt) - 1
6:   y ← Cy,k[0]
7:   t ← y & mask
8:   if k < 1 ∨ k > 2lt then report Not_Defined
9:   else if k = 1 then Ct,k ← {t}
10:  else if (zt = 0 ∧ 1 < k ≤ 2lt) ∨
        (lt ≥ zt > 0 ∧ k > 2zt) then Ct,k ← ∅
11:  else if (lt ≥ zt > 0) ∧ (2 ≤ k ≤ 2zt) then
12:    for i ← 0 to 2zt - 1
13:      Ct,k.append(Cy,k[i] & mask)
14:  return Ct,k

```

Fig. 8. The candidateSetSuffix function.

is the worst-case running time for executing lines 3 and 4. The candidateSetSuffix function given in Fig. 8 provides the algorithm for creating the new candidate set  $C_{t,k}$  from a cached candidate set  $C_{y,k}$ .

Since creating the candidate set for  $y$  involves scanning all of  $y$  to find the zeros, we can at the same time maintain an array which accumulates the number of zeros encountered so far during the scan. This array is the  $Z$  array which was created in the function candidateSet (lines 4 and 7). We can index into this array using the position which suffix  $t$  starts in  $y$  and thus get the number of zeros in constant time. For example, for  $y = 01101010$  and  $t = 1010$  given in terms of  $t_{start}$  which is the start position of  $t$  in  $y$ , we can scan  $y$  from left to right and create the array  $Z = [1, 1, 1, 2, 2, 3, 3, 4]$ . This array gives the number zeros in every suffix of  $y$ . Thus,  $z(t) = z(y) - Z[t_{start} - 1]$ . In this case  $t_{start} = 4$ , and so  $z(1010) = z(01101010) - Z[4 - 1] = 4 - 2 = 2$ .

In addition, the  $mask$  (Fig. 8, lines 7 and 13) is used as a means of setting the first  $x$  bits of each original candidate set element to zero, which is the equivalent in a sense of taking the suffix of the corresponding binary string. For example, if the candidate set element is 18, with binary representation 10010, and we want to take the suffix starting at index 2, then the  $mask = 7$  (00111 in binary). Thus, by bitwise ANDing the  $mask$  and the element, we get 010 = 2. Note that the masking does not simply set the higher order bits to zero but it truncates the number, i.e., the length actually decreases. This is important because we seek to derive from the candidate set of 10010 the candidate set of 010 and not the set for 00010. The latter is impossible to derive in the way described in this section since  $C_{00010,2}$  is a *superset* of  $C_{10010,2}$ .



The “for” loop on line 12 dominates the running time of the above algorithm. Hence, the algorithm, in the worst case, runs in  $\Theta(2^{z(t)})$ , which is optimal.

However, we can do better by using a different representation for the candidate set of the suffix  $t$ . Since the elements of  $C_{t,k}$  are contiguous elements of  $C_{y,k}$  starting at position 0, then the candidate set of  $t$  can be given as a range of values. This is achieved just by maintaining a pointer to the position  $q - 1$  in the candidate array of  $y$  marking the last element of  $C_{t,k}$ . Thus, only two numbers  $mask$ , and  $q = 2^{z(t)}$ , both of which can be computed in constant time, are needed to capture the candidate set of any suffix of target  $y$ . To create the  $mask$ , we use  $l_t$  (as seen on line 5) which was computed from the input integer  $t_{start}$  on line 3. Obtaining  $q$  is easy since we have already computed  $z(t)$  on line 4. Thus, the first and last elements of the candidate set for the suffix can be given as  $C_{y,k}[0] \& mask$  and  $C_{y,k}[q - 1] \& mask$ , respectively. This approach avoids the expensive “for” loop on line 12 and makes the algorithm run in  $\Theta(1)$ .

It is preferable to use the candidateSetSuffix Algorithm in one particular situation: to find the candidate set for the suffix of  $y$  whenever we already have the candidate set for  $y$ . Consider the following examples: For  $l = 4$ , we want to calculate  $C_{1010,7}$  and  $C_{010,3}$ .  $C_{1010,7} = \emptyset$  since  $|C_{1010,7}| = 2^2 = 4 < k = 7$ . In the case of  $C_{010,3}$ , we have  $3 \geq z(t) = 2 > 0$  and  $2^{z(t)} = 4 > k = 3$  so

$$\begin{aligned} {}_3A_{010,3} &= \bigcup_{1 \leq i \leq 4} [Suffix_1({}_4A_{1010,2}[i])] \\ &= \bigcup_{1 \leq i \leq 4} [Suffix_1[1010, 1011, 1110, 1111]] \\ &= [010, 011, 110, 111], \end{aligned}$$

and thus  $C_{010,3} = \{2, 3, 6, 7\}$ . If we decide to use the faster constant running time approach, the result will be given as  $mask = 0111$  and  $q = 2^2 = 4$  and hence the first element in  $C_{010,3}$  is  ${}_4A_{1010,2}[0] \& 0111 = 1010 \& 0111 = 010 = 2$  while the last element is  ${}_4A_{1010,2}[4 - 1] \& 0111 = 1111 \& 0111 = 111 = 7$ .

Assume that we are auditing a variety of databases, each with a particular  $l$  value (for the example in this paper,  $l = 4$ ). Within the forensic analyzer, we could precompute a summary set for  $l_{max}$ , which is the maximum of the  $l$  values that were specified for the databases that were being audited. During forensic analysis of a specific database corruption, given the resulting target string and the  $l$  value for this particular database (with  $l \leq l_{max}$ ), this algorithm could calculate in constant time the candidate set, which consists of all the possible corrupted granules that could have yielded that target number for that value of  $l$ .

## 8 IMPLEMENTATION AND EVALUATION

Elsewhere, we have introduced the Monochromatic, the RGB, and Polychromatic Algorithms [7]. All algorithms employ the same approach of tamper detection and forensic analysis by hashing transaction data and periodically validating the resulting hash chains. The main differences between the algorithms lie in the number of hash chains used and their structure. The simplest is the Monochromatic Algorithm, which sequentially hashes all data to create a

hash chain that incrementally grows over the data of the entire database. The *cumulative nature* of this chain has two consequences. First, it limits the detection of corruption to a single event since periodic validations will yield a sequence of “success” results followed by a sequence of “failure” results. The interface in the transition between these two sequences marks the site of the first (oldest) corruption. Second, the cumulative nature of the chain enables a binary search on the sequence of “successes” and “failures” to locate the transition very quickly.

The RGB Algorithm augments the Monochromatic Algorithm by periodically superimposing (noncumulative) partial hash chains over the entire database. The name of the algorithm is derived from the color coding of the different partial hash chains. In this case, the cumulative chain can be used to perform binary search to quickly locate the oldest corruption and then switch to using the “colored” partial chains to explore the rest (more recent part) of the database. This algorithm can detect up to two corruption events.

The Polychromatic Algorithm retains the main Red, Green, and Blue partial chains of RGB and introduces more Red and Blue chains to create groups of chains similar to a tile. This has the advantage that it can arbitrarily shrink the spatial detection resolution by introducing a *logarithmic* number of hash chains as opposed to a linear number needed in RGB. The Polychromatic Algorithm, as with the RGB Algorithm, can only detect only up to two corruption events but could potentially be modified to handle multiple corruptions.

The Tiled Bitmap Algorithm introduced here can be thought of as a refinement/replacement of the Polychromatic Algorithm. The new algorithm can use the cumulative chain of the Monochromatic Algorithm (not elaborated on here). It extends the idea of the RGB Algorithm of using partial chains, and it refines the groups of hash chains of the Polychromatic Algorithm.

The advantage of the Tiled Bitmap Algorithm is that it lays down a *regular pattern* (a “tile”) of such chains over contiguous segments of the database. What is more, it inherits all the advantages of the Polychromatic Algorithm: the chains in the tile form a bitmap which can be used for easy identification of the corruption region, and a logarithmic number of chains can be used to reduce  $R_s$ .

The other advantage of the Tiled Bitmap Algorithm is that it can detect multiple corruption events (up to  $D$  of them, i.e., all granules were corrupted) something that the Monochromatic, RGB, and Polychromatic Algorithms cannot. On the other hand it suffers from false positives while the previous three algorithms do not. (More information on the rate of false positives of the Tiled Bitmap Algorithm can be found elsewhere [8].) Table 3 shows the running time for three of the forensic analysis algorithms (the Polychromatic Algorithm is omitted because it is replaced by the Tiled Bitmap Algorithm). We assume that the spatial detection resolution  $R_s$  is equal to 1 for simplicity. Observe that the algorithms become progressively slower because of the increased number of chains maintained and used during forensic analysis. The Monochromatic Algorithm, while being the fastest algorithm, suffers from the fact that only the first corruption event can be detected. As noted, the Tiled Bitmap Algorithm can be slightly optimized by

TABLE 3  
Running Time Complexity of Forensic Analysis Algorithms

Algorithm	Running Time ( $R_s = 1$ )
Monochromatic	$O(\lg(D/I_V))$
RGB	$O(D/I_V)$
Tiled Bitmap	$O((D \cdot \lg I_V)/I_V + D)$

TABLE 4  
Worst-Case Cost/Space Complexity of  
Forensic Analysis Algorithms

Algorithm	Cost ( $R_s = 1$ )
Monochromatic	$O(D)$
RGB	$O(D)$
Tiled Bitmap	$O(D \cdot (1 + \lg I_V)/I_V)$

retaining the cumulative chain of the Monochromatic in order to locate the first corrupted tile by performing binary search, although this refinement does not affect its asymptotic running time.

Recall that all algorithms rely on an external notarization service in order to validate the audit log. However, each such contact costs real money. Hence, we quantify the cost of the algorithms *as the number of contacts* with the notarization service during a specific duration of the normal operation of the system, i.e., whenever a part of the database is notarized or validated. The units of the cost are, therefore, number of notarizations plus number of validations. We chose to deal with only notarizations and validations occurring before corruption or forensic analysis, because otherwise the cost would be dependent on the number of corruptions. This would render the comparison unfair since the Monochromatic and RGB Algorithms can only detect a limited number of corruptions. More information on the mathematical formulation of the cost can be found elsewhere [8]. It is desirable to minimize this cost for each algorithm while trying to extract as much information as possible. Table 4 shows the cost for each of the forensic algorithms assuming a spatial detection resolution of one hour ( $R_s = 1$ ) and a single corruption event. In this case, we observe the opposite trend compared to the one observed for the running times of the algorithms. For a sufficiently large validation interval  $I_V$ , the Tiled Bitmap Algorithm has the smallest cost. This is because the ratio  $(1 + \lg I_V)/I_V$  becomes less than one.

This quantification of cost also reflects the space complexity of the forensic algorithms since each of the contacts with the external notarization service corresponds to a hash value (of chains) which must be initially computed (and recomputed for comparison during validation) and maintained within the system. None of algorithms in Table 4 require extra space over the collection of hash values themselves.

A 1250-line C code implementation is available at <http://www.cs.arizona.edu/projects/tau/tbdb/>. The code implements several forensic analysis algorithms, including the candidateSet and candidateSetSuffix construction algorithms, the Tiled Bitmap Algorithm, and the Monochromatic

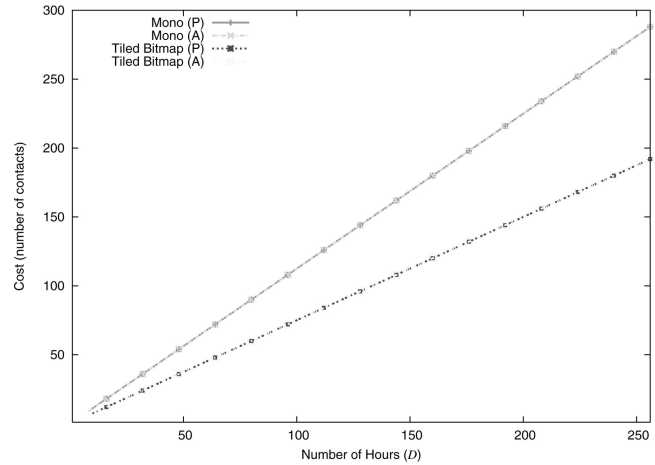


Fig. 9. The cost of the Monochromatic and Tiled Bitmap Algorithms.

Algorithm. This C code implementation uses the more efficient pass-by-reference for arrays and strings compared to the pseudocode given in Section 6. All algorithms were tested extensively and their theoretical costs were experimentally validated. Appendix F, which can be found on the Computer Society Digital Library at <http://doi.ieeecomputersociety.org/10.1109/TKDE.2009.121>, provides proofs of correctness for all functions introduced in this paper. We also have developed several graphical user interfaces which include a convenient visual representation of the spatial and temporal extent(s) of detected corruption(s).

Fig. 9 shows the results of the experimental cost validation for the Monochromatic and Tiled Bitmap Algorithms (the RGB Algorithm has not been implemented). The experiments used the following setup:  $D = 1$  to 256,  $R_s = 1$ , and  $I_V = 8$ . Rather than using the cost formulas in order notation (as given in Table 4) to create the graphs, we used more involved (and more accurate) cost functions derived for each algorithm. Note that the cost plot shows both the predicted forensic cost (denoted by “(P)” in the plot legend) and actual cost values (denoted by “(A)” in the plot legend). The actual cost values were computed by inserting appropriate counters in the C code implementation for the Monochromatic and Tiled Bitmap algorithms. The different types of symbols on the curves were added for clarity and correspond to a subset of the actual data points. As can be seen in Fig. 9, the predicted and actual cost for the two algorithms are essentially identical. A more detail explanation of the derived costs, and experimental comparisons between algorithms can be found elsewhere [8].

## 9 PREVIOUS WORK

There has been a great deal of work on records management, and indeed, an entire industry has arisen to provide solutions for these needs, motivated recently by Sarbanes-Oxley [13] and other laws requiring compliant record storage. In this context, a “record” is a version of a document. These systems utilize magnetic disks (as well as tape and optical drives) to provide WORM storage of compliant records. We wish to extend the concept of

compliant records to tuples of a table stored in a database management system.

Computer forensics is now an active field, with more than 50 books published in the last 10 years. However, these books are generally about preparing admissible evidence for a court case, through discovery, duplication, and preservation of digital evidence. There are few computer tools for these tasks, in part due to the heterogeneity of the data. One substantive example of how computer tools can be used for forensic analysis is Mena's book [6].

Goodrich et al. introduce new techniques for using main-memory indexing structures for data forensics [3]. They encode authentication information in the way a data structure is organized (not in the stored values) so that alterations can be detected. Their techniques are based on a new reduced-randomness construction for nonadaptive combinatorial group testing, using message authentication codes (MAC) built using cryptographically strong, one-way hash functions. In the database context, we have introduced in previous papers the approach of using cryptographic hash functions to detect database tampering [10] and of introducing additional hash chains to improve forensic analysis [7]. To the best of our knowledge, there are no other competing forensic analysis algorithms for high-performance databases.

Strachey has considered table lookup to increase the efficiency of bitwise operations [11]. He provides a logarithmic time/logarithmic space algorithm for reversing the bits in a word. Our second algorithm requires only constant time, but the table must be of exponential space.

Enumerating all solutions (preimages) is a key step in formal verification. Sheng and coworkers have developed efficient preimage computation algorithms [4], [9]. These algorithms are similar to the ones introduced in this paper in that they all enumerate all possible solutions. The formal verification algorithms differ in that they are computing preimages of a state transition network, rather than of bitwise *AND* functions, as in our paper.

## 10 SUMMARY

Forensic analysis commences when a crime has been detected, in this case the tampering of a database. Such analysis endeavors to ascertain when the tampering occurred, and what data were altered.

Elsewhere, we proposed several forensic analysis algorithms [7]. The present paper expands upon that work by presenting the *Tiled Bitmap Algorithm*, which is cheaper and more powerful than prior algorithms. This algorithm employs a logarithmic number of hash chains within each tile to narrow down the *when* and *what*.

Checking the hash chain values produces a binary number; it is the task of the algorithm to compute the preimage of bitwise *AND* functions of that number. This produces a *candidate set* which identifies all the potentially corrupted granules. We showed that the running time of the algorithm is linear in the length of time the database has been in existence and linear in the size of the computed candidate set. We also note that previous algorithms do not handle multiple corruption events well, whereas the Tiled Bitmap Algorithm can independently analyze corruption

events occurring both in different tiles and multiple corruption events occurring within a single tile. However, the Tiled Bitmap Algorithm suffers from false positive results while prior algorithms (Monochromatic, RGB, and Polychromatic) do not.

In the later parts of the paper, we analyzed completely the behavior of the candidate sets and developed an optimal algorithm to produce these candidate sets. We then introduced a constant-time algorithm which is preferable in the case when the target binary number is a suffix of another binary number for which a candidate set already exists. Finally, we compared prior forensic algorithms with the Tiled Bitmap Algorithm, providing a thorough space and time complexity analysis. We discussed the implementation of the algorithms and experimentally validated their cost. The Tiled Bitmap Algorithm uses additional chains (which incur a logarithmic runtime factor) to detect multiple corruption events, while requiring fewer requests of an external notarization server.

The ultimate goal is an algorithm that retains the logarithmic performance (of the additional chains) of the Tiled Bitmap Algorithm while further simplifying the analysis within a tile, furthering narrowing the bounds on when the tampering occurred, and providing additional forensic information, such as the *direction* of the tampering, i.e., whether the information was backdated or postdated.

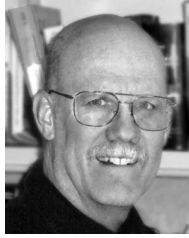
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**Richard T. Snodgrass** received the BA degree in physics from Carleton College, and the MS and PhD degrees in computer science from Carnegie Mellon University. He joined the University of Arizona in 1989, where he is a professor of computer science. He is a fellow of the ACM. He was an editor-in-chief of the *ACM Transactions on Database Systems*, was an ACM SIGMOD chair from 1997 to 2001, and has chaired the ACM Publications Board, the ACM

History Committee, and the ACM SIG Governing Board Portal Committee. He served on the editorial boards of the *International Journal on Very Large Databases* and the *IEEE Transactions on Knowledge and Data Engineering*. He chaired the Americas program committee for the 2001 International Conference on Very Large Databases and the program committee for the 1994 ACM SIGMOD Conference. He received the 2004 Outstanding Contribution to ACM Award and the 2002 ACM SIGMOD Contributions Award. He currently is a member of the ACM History Committee, the Advisory Board of ACM SIGMOD, and the Outstanding Contribution to ACM Award Committee. He chaired the TSQL2 Language Design Committee, edited the book, *The TSQL2 Temporal Query Language* (Kluwer Academic Press), and has worked with the ISO SQL3 committee to add temporal support to that language. He authored *Developing Time-Oriented Database Applications in SQL* (Morgan Kaufmann), was a coauthor of *Advanced Database Systems* (Morgan Kaufmann), and was a coeditor of *Temporal Databases: Theory, Design, and Implementation* (Benjamin/Cummings). He codirects TimeCenter, an international center for the support of temporal database applications on traditional and emerging DBMS technologies. His research interests include the science of computing, temporal databases, query language design, query optimization and evaluation, storage structures, and database design. He is a senior member of the IEEE and the IEEE Computer Society.



**Kyriacos E. Pavlou** received the BA (Hons) degree in genetics from Cambridge University in 1999, the BS degree in computer science and mathematics from the University of Arizona in 2005, and the MS degree in computer science from the University of Arizona in 2007. His research interests include temporal databases, record compliance, database forensics, computational biology, and graph theory. He is currently pursuing the PhD degree under the

guidance of Dr. Richard Snodgrass.

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