

Image Reconstruction Algorithm Based on Probabilistic Graphical Model

Shanrui Zhang

csc696

1. Image Reconstruction

What is image reconstruction

Why use image reconstruction

2. What I will do

3. Compress sensing (Sparse sampling)

One image reconstruction method

4. Algorithms

6. Data

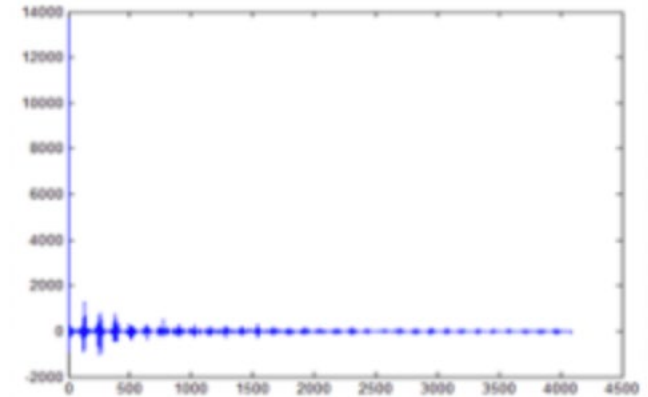


Image Reconstruction

- What is image reconstruction(in cv)

Image reconstruction is a technology that fills in the missing pixels in the image and reconstructs based on the pixel information of the background.

- Why I need image reconstruction

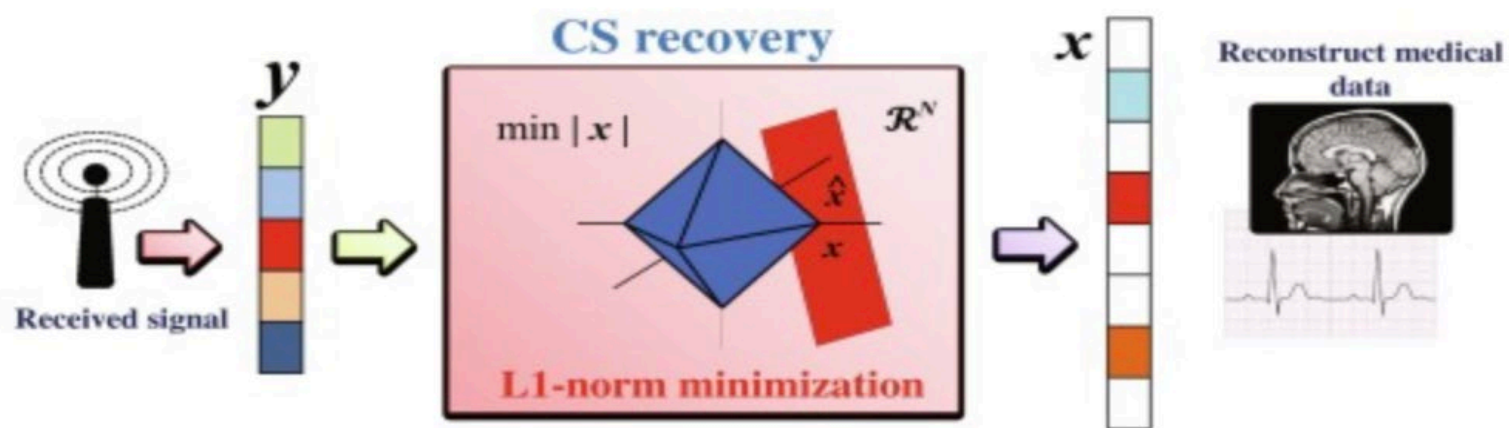
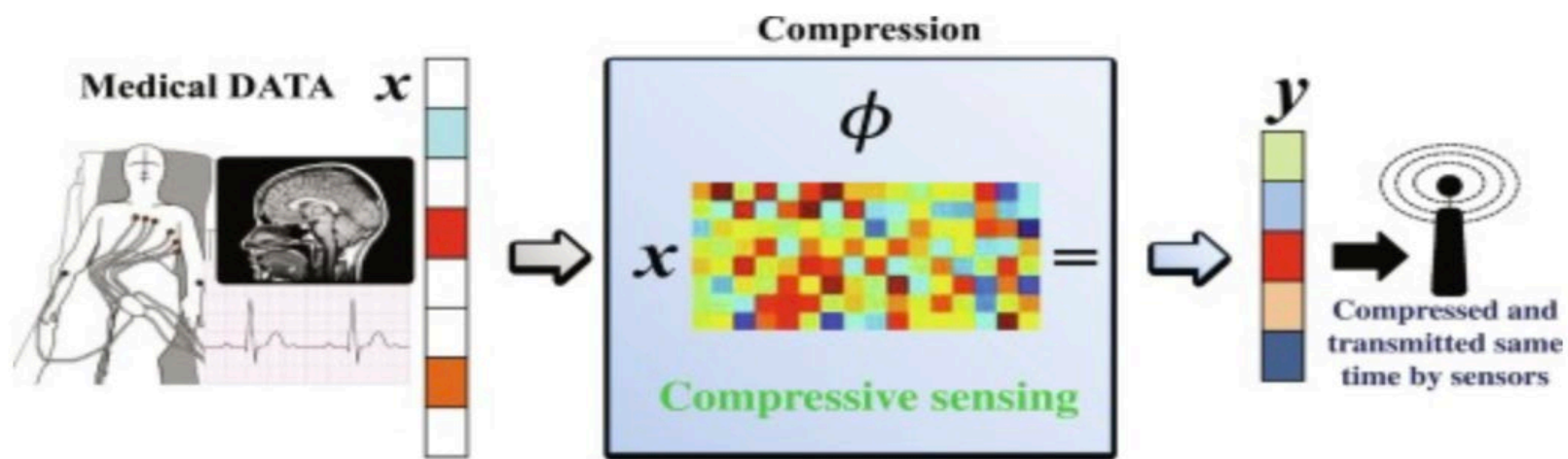
Image denoise,improve image quality. Better model, more accurate in inference.

What will I do/have done

1. Research on how to do image reconstruction and learn different algorithms.
2. Implementation of SBL and OMP algorithm and run the algorithm on image data.(not on wireless communication data).
3. Compare and analysis MSE of different algorithms of image reconstruction on lena data set.

Compressed sensing

- Signal System: In order for the digital signal after sampling to completely retain the information in the original signal, the sampling frequency must be greater than 2 times the highest frequency in the signal.
- When we say "sampling frequency", it means that we are doing equal distant sampling. In the field of digital signals, people usually do equal distant sampling..
- In 2004, Candes, Tao Zhexuan, and Donoho proposed the theory of compressed sensing, which states that if the signal is sparse, it can be reconstructed from sampling points far below the sampling theorem
- Compressed sensing: In the process of signal sampling, a few sampling points are used to achieve the same effect as full sampling.



Compressed sensing

- Using the sparsity property of the signal, the signal can be compressed. For example, the JPEG format in the field of image compression is to transform the image into the discrete cosine domain to obtain an approximate sparse matrix, and only retain larger values to achieve compression.
- The concepts of image compression and compressed sensing are fundamentally different.
- Image compression is to perform full sampling first, and then discard small coefficients in the transform domain to complete the compression;
- Compressed sensing is different. Its idea actually borrows a lot from image compression: since the full sampling has to be discarded, why can't we just sample less points? Therefore, compressed sensing directly performs subsampling, and then uses the algorithm to eliminate the artifacts caused by subsampling. It can be said that compressed sensing completes the compression directly at the time of sampling.

Compressed sensing

- Taking pictures with a camera, full sample, the acquisition of all pixels is completed in an instant.
- MRI image usually takes tens of seconds, and the slow speed is also a major defect of MRI. After applying CS technology, the original image can be reconstructed only by collecting a fraction of the full sampling data. This can increase the imaging speed several times with little impact on image quality.

The observation matrix maps the high-dimensional signal X to the low-dimension space

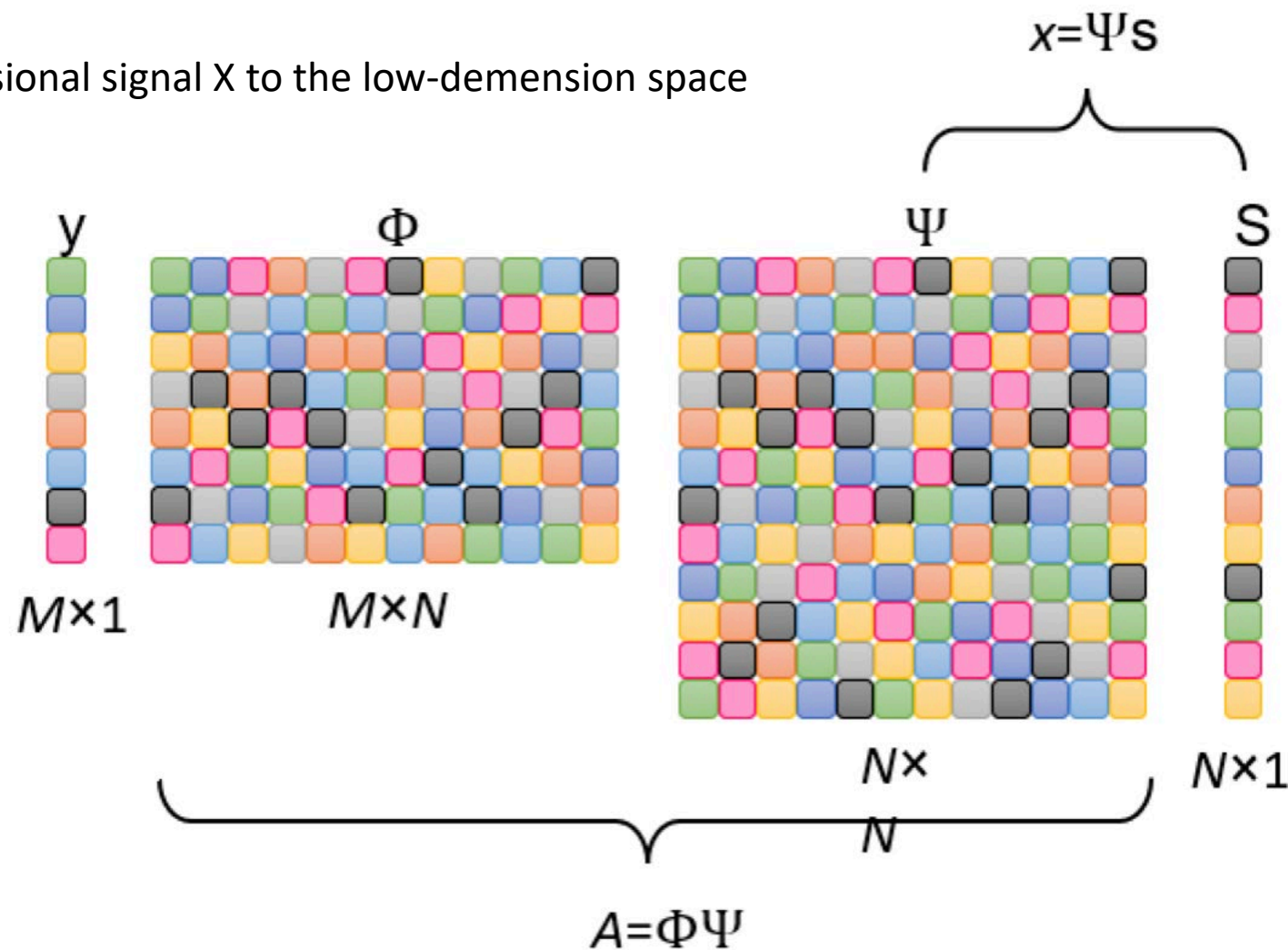
y : observation value
(known, compressed image)(dimension M)

Φ : observation matrix
(sparse sampling)

Ψ : sparse matrix
(Fourier transform)(signal \rightarrow frequency)

s : sparse index
(natural signal x not sparse)

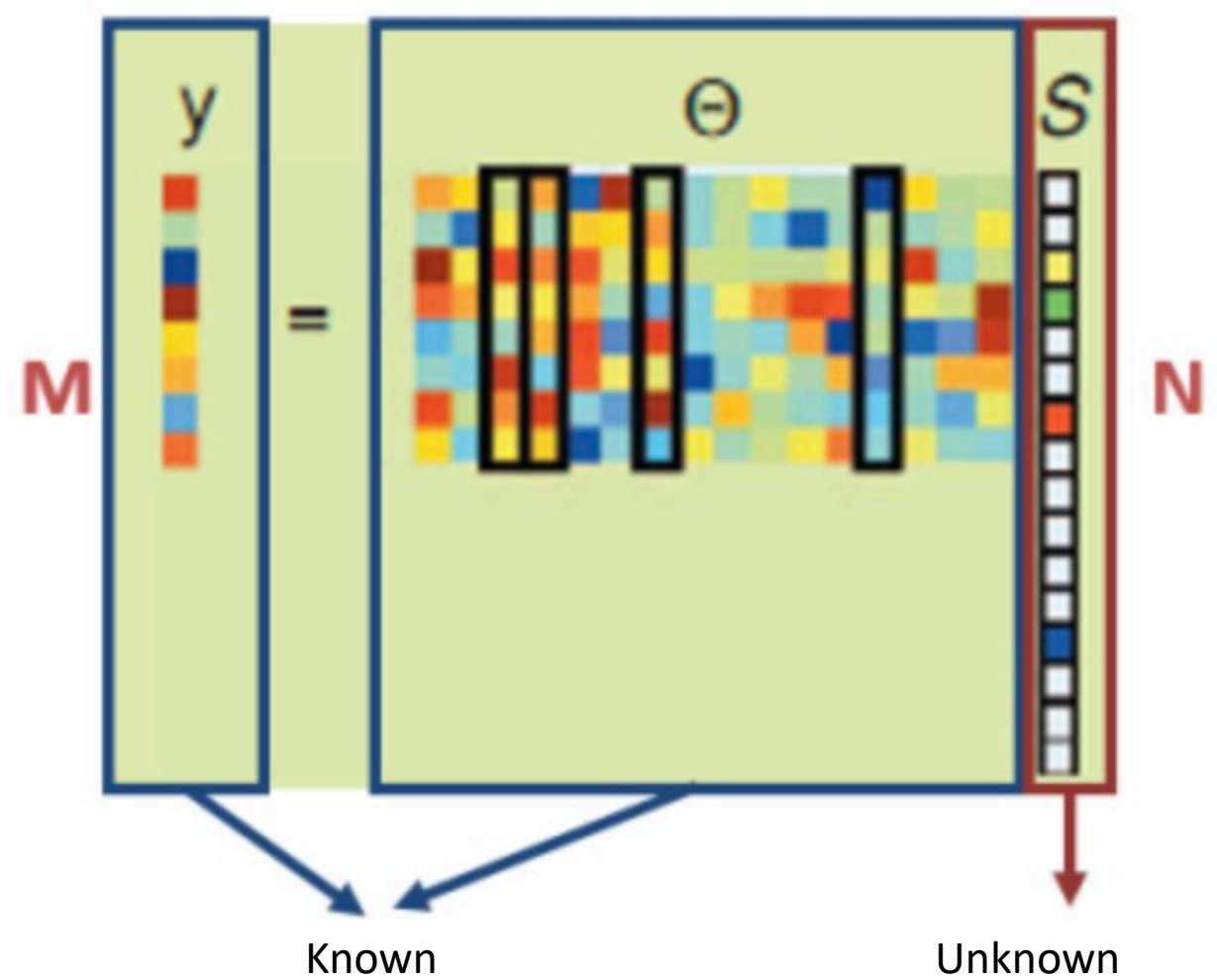
x : input signal
(unknown, we need to recover)
(dimension N)



Compressed sensing problem

- Solve the equation $y = \Phi x$, obtain original signal
- signal x not sparse, let $x = \Psi s$
- $y = \Phi \Psi s$, Knowing y , Φ , Ψ , solve for s .
- $y = \Phi \Psi s$, $\Phi \Psi$ sensing matrix. $\Theta = \Phi \Psi$, then $y = \Theta s$.
- The problem is, given y and Θ , solve for s .

$M=N, M<N$



- The number of equations $<$ the number of unknowns. However, since the signal is K -sparse, if Φ in the above formula satisfies the finite equal distant property (RIP), then the K coefficients can be accurately reconstructed from the M measured values (an optimal solution is obtained).
- The equivalent condition of RIP is that the observation matrix and the sparse representation basis are incoherent. That is, the second precondition of compressed sensing.

$$\min \left\| \Psi^T X \right\|_0$$
$$\text{s.t. } \Phi X = \Phi \Psi X = A$$

Algorithms

- Direct optimization L0 greedy algorithm

OMP

- Convex Optimization Methods
- Bayesian Optimization Algorithm

Data (lena data set)

1. Compare the reconstruction under different sampling rates

2. Compare the result of different algorithm

3. Measured by MSE



- [1] Pedersen N L, Shutin D, Manchon C N and Fleury B H .Sparse estimation using Bayesian hierarchical prior modeling for real and complex models[J].Signal Processing,2020, 115(6):94-109.
- [2]Perdersen N L,Manchon C N,Shutin D and Fleury B H. A fast iterative Bayesian inference algorithm for sparse channel estimation[C]//IEEE International Conference on Comunciations,2013.ICC'13, 2019,4591-4596.
- [3]F.V.ensen,An Introduction to Bayesian Networks. New York: Springer-Verlag, 2008.
- [4]H.-A. Loeliger, “An introduction to factor graphs,” IEEE Trans. Signal Processing, pp. 28–40, Jan. 2009.